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ACCURACY ASPECTS OF MULTIPLE FOCAL SETTING SELF-CALIBRATION APPLIED TO NON-METRIC CAMERAS

by

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ABSTRACT

Multiple focal setting self-calibration is applied in a photogrammetric test project, partially designed to ascertain the practicability of employing a non-metric camera (the Hasselblad 500C) for laboratory structural testing. A simultaneous recovery of "stable" camera calibration parameters relating to three focussed object distances is afforded by the inclusion of focal setting- and block-invariant additional parameter sets in the photogrammetric adjustment. The mathematical formulation of the multiple focal setting self-calibration technique is reviewed and shortcomings in the functional model which arise when using an "amateur" camera of unstable inner orientation are noted. Results obtained in the investigation are evaluated, and the accuracy and precision attained are discussed with reference to the image coordinate residuals, the derived camera calibration parameters, object point determinations, and invariant measures of precision.

INTRODUCTION

Numerous and varying applications of close-range photogrammetry in engineering and industry have been reported in the literature (see, for example, the review by ATKINSON, 1976). In applying photogrammetric techniques to the solution of three-dimensional measurement problems in a laboratory environment, occasions can arise where the use of metric cameras is neither feasible nor economically justifiable. This is in spite of the fact that there are a number of commercially available close-range photogrammetric cameras which possess the flexibility and versatility traditionally afforded by "amateur" cameras. One such occasion prompted the reported investigation.

This paper details a preliminary investigation carried out to ascertain the practicability of conducting engineering laboratory structural testing and monitoring by analytical photogrammetric means, employing an available Hasselblad 500C non-metric camera. The paper presented is a revised and shortened version of FRASER (1980b), and the following discussion stresses two principal aspects of the investigation: the application of multiple focal setting self-calibration (FRASER, 1980a) to a non-metric camera, and the accuracies obtained in such an application. Initially, features of the photogrammetric survey conducted are detailed. This is followed by a review of the mathematical formulation of the multiple focal setting self-calibration technique and an outline of the additional parameter model employed. Finally, a discussion of the results is presented and the accuracy and precision attained in both object point determination and in the recovery of "stable" camera calibration parameters is detailed.

DETAILS OF THE PHOTOGRAMMETRIC SURVEY

System Geometry and Data Acquisition

In the photogrammetric context, two features of the data acquisition phase of the test survey carried out for the investigation are noteworthy: the use of photographs taken at multiple focussed object distances, and the fact that the 500C was fitted with a Distagon 50 mm lens. With regard to the latter feature, although the metric quality of very wide angle photography has been shown to be most adversely affected by inner orientation instability and film unflatness (KENEFICK, 1971), a 50 mm lens does exhibit a large working depth of field which can be an asset in confined laboratory situations.

For the test project reported, an existing three-dimensional target field comprising 43 points was used. The target array, a portion of which is shown in Fig. 1, covered an area of 2.5 m^2 in the assigned XY plane, to a depth of 0.7 m. Object space control comprised solely scale; the origin and orientation of the reference coordinate system could be arbitrarily assigned. Because of the lack of an object space control configuration, developed methods of analytical data reduction for non-metric imagery, such as the Direct Linear Transformation (DLT) (ABDEL-AZIZ & KARARA, 1971) and the more rigorous treatment of BOPP & KRAUSS (1978), were deemed impractical.

The object target array was imaged from 10 exposure stations, arranged in a convergent configuration with all camera axes directed towards the centre point of the target field (see FRASER, 1980b). Photographs were taken at three magnifications: three from focussed object distances of 2 m and 3 m, and four from 1 m. The convergent configuration, coupled with the three dimensional object point distribution, was adopted to enhance the recovery of the Gaussian focal length, or principal distance, c_i, at each focal setting. Also, to counteract projective compensation between the interior orientation elements x_0, y_0 and the exposure station coordinates, nominally orthogonal kappa rotations were applied. The inclusion of 10 exposure stations may prove somewhat impractical and unnecessary for many routine engineering applications of close-range photogrammetry. However, in this investigation it was necessary to employ the camera configuration described in order to satisfy one of the aims of the project, the recovery of "stable" camera calibration parameters.

Data Reduction

As an alternative to the DLT approach it was decided to treat the 500C as a pseudo-metric camera and apply the recently developed multiple focal setting self-calibration technique which incorporates constraint equations enforcing a linear variation in radial lens distortion with changing principal distance. This method has previously been applied to close-range metric camera calibration (FRASER, 1980a).

The Hasselblad 500C had been modified so as to provide fixed image frame reference marks. Two fiducial marks had been etched into the sides of the camera's picture frame, but these were by no means geometrically stable since the image frame formed part of the removable film magazine. Further sources of instability were the lack of a film flatness control mechanism, and uncontrolled film processing.

To achieve a rigorous functional model for the self-calibration of a non-metric camera, based on the collinearity condition, additional parameters for lens distortion, interior orientation and image deformation need to be carried for each photograph separately, as in both the DLT and the formulation detailed by FAIG (1975) which is based on the coplanarity condition. For photogrammetric surveys employing more than half a dozen or so exposure stations this requirement can be computationally cumbersome. For the reported self-calibration investigation it appeared more practical - though less mathematically rigorous - to include additional parameters relating to particular image subsets. These focal setting-invariant parameters modelled the lens distortion and perturbations to the Gaussion focal length at each of the three focussed object distances. Block-invariant additional parameters were also included in the self-calibration formulation in order to compensate to some degree for the error introduced through inner orientation instability and image deformation. By the use of a flexible systematic image coordinate error correction model it was hoped that the Hasselblad 500C would yield a similar order of accuracy to that expected from an equivalent close-range metric camera.

ADDITIONAL PARAMETERS

Selection

The normal equation matrix decomposition algorithm used to factor the indefinite "bordered" matrix of the multiple focal setting self-calibration (referred to in a following section) was found to be less computationally stable in situations where other than statistically significant additional parameters were included. For this reason, a number of standard self-calibrating bundle adjustments were initially carried out to enable the determination of the most suitable focal setting- and block-invariant additional parameter sets. Parameters which did not prove to be statistically significant were suppressed.

The individual terms comprising the vector of additional parameters $\dddot{\delta}$

can be represented by the following expressions for the image coordinate corrections Δx and Δy:

$$\Delta x = \Delta x_F + \Delta x_B$$
$$\Delta y = \Delta y_F + \Delta y_B$$

Here, Δx_F and Δy_F contain terms relating to a specific principal distance c_i, whereas the expressions for Δx_B and Δy_B comprise the selected block-invariant terms. Of the additional parameter sets chosen for the preliminary self-calibration adjustments, the model yielding the most favourable results from the point of view of statistical significance and the minimisation of the variance factor estimate $\hat{\sigma}_o^2$ was the following:

$$\Delta x_F = xr^2K_{i1} + xr^4K_{i2} + (-x/c)dc_i + (3x^2+y^2)P_{i1} + 2xyP_{i2}$$
$$\Delta y_F = yr^2K_{i1} + yr^4K_{i2} + (-y/c)dc_i + 2xyP_{i1} + (x^2+3y^2)\ P_{i2}$$

and

$$\Delta x_B = -x_o + a_1xy + a_2x^2y + a_3xy^2$$
$$\Delta y_B = -y_o + b_1\ x + b_2y + b_3x^2y + b_4xy^2$$

where dc_i represents a correction to the initial estimate c_i, K_{i1} and K_{i2} are coefficients of radial lens distortion, P_{i1} and P_{i2} are the decentering distortion coefficients, and the parameters a_i, b_i largely represent empirical correction terms.

For the 500C, calibrated values of the principal point coordinates (x_o,y_o) are only appropriate to a single attachment of the film magazine since the camera's picture frame is removable. To a lesser degree, this is also true of the image frame orientation with respect to the camera body. Principal distance is affected in a similar manner, although the values c_i at each focal setting are perturbed to a greater extent by variations in the position of the image "plane" due to the lack of a film flattenting mechanism. The treatment of the principal distances c_i as stochastic unknowns has the effect of largely compensating for any scale error in the measurement of "calibrated" fiducial mark coordinates. These coordinate values were simply determined by comparator observations on a single photographic image, where the principal point was established as being at the intersection of the two film frame diagonals. Thus, whereas the affinity term b_2y and the non-perpendicularity of axes term b_1x usually refer to shortcomings in comparator calibration, they can be interpreted more in this case as providing a correction to the systematic error introduced by the assumption that the principal point of the 500C lies at the intersection of the two film frame diagonals at one selected exposure.

Although film deformation influences cannot be expected to remain constant for each non-metric camera image, the assumption was nevertheless made that significant components of introduced systematic error could be compensated for to a large degree by the empirical terms of Δx_B and Δy_B. The investigation results indicate that this assumption was reasonably valid.

Structure of Additional Parameter Matrix

The additional parameter coefficient matrix $\dddot{B}_j^{(i)}$ for an image point observation j on a photograph taken at focussed object distance s_i can be represented in the form of two submatrices: the coefficient matrix of the focal setting -invariant parameters, $\dddot{B}_{Fj}^{(i)}$, and that for the block-invariant parameters, $\dddot{B}_{Bj}^{(i)}$. The structure of $\dddot{B}_j$, the resulting additional parameter matrix for a single image point j, will assume the following form where point j is imaged on three photographs, each taken at a different focal setting:

$$\dddot{B}_j = \begin{bmatrix} \dddot{B}^{(1)}_{Fj} & & 0 & \vert & \dddot{B}^{(1)}_{Bj} \\ 0 & \dddot{B}^{(2)}_{Fj} & 0 & \vert & \dddot{B}^{(2)}_{Bj} \\ 0 & 0 & \dddot{B}^{(3)}_{Fj} & \vert & \dddot{B}^{(3)}_{Bj} \end{bmatrix}$$

If a total of k_F focal setting-invariant and k_B block-invariant additional parameters are used in the formation of $\dddot{B}_j$, the resulting normal equation submatrix $\dddot{B}^T W \dddot{B}_j$ (see FRASER, 1980a) will be block-diagonal with dimensions $(k_F \times k_F)$, symmetrically bordered by a border of width k_B.

MULTIPLE FOCAL SETTING SELF-CALIBRATION

This self-calibration adjustment formulation is essentially a special case of multiple close-range camera self-calibration (FRASER & VERESS, 1980). The method is viewed as being most applicable in photogrammetric surveys involving photography taken at three or more finite focal settings with a long focal length camera which displays a pronounced lens distortion profile. BROWN (1972) and ABDEL-AZIZ (1973) have derived working formulae for the description of symmetric radial lens distortion variations with changing magnification. After linearization, these formulae can be recast as linear condition equations:

$$H\dddot{\delta}_h + F = 0 \qquad (1)$$

In the present investigation, preliminary self-calibrating bundle adjustments revealed that at the three focussed object distances s_i (i.e. at principal distances c_i) only the coefficients K_{i1} and K_{i2} of the distortion function

$$\Delta r_i = K_{i1}r^3 + K_{i2}r^5 + K_{i3}r^7 + \dots$$

were statistically significant. The vector $\dddot{\delta}_h$ in Eq. 1 is then given in expanded form as

$$\dddot{\delta}^T_h = [dK_{11} \quad dK_{12} \quad dc_1 \quad dK_{21} \quad dK_{22} \quad dc_2 \quad dK_{31} \quad dK_{32} \quad dc_3]$$

where dc_i, dK_{i1} and dK_{i2} represent corrections to the initial estimates for c_i, K_{i1} and K_{i2}. The matrix H has dimensions (2x9) for this case and the individual terms of this matrix are evaluated as functions of the approximate values for the principal distances and lens distortion polynomial coefficients (see FRASER, 1980a). The vector F comprises two discrepancy terms. For practical implementation the parameters dc_i can normally be suppressed without influencing the final adjustment solution.

The normal equation system of the self-calibrating bundle adjustment is expressed by the standard matrix equation:

$$A^TPAX - A^TPL = 0 \qquad (2)$$

Here, A is the design matrix, P the weight matrix, L the vector of discrepancies, and X the vector of unknown parameters, given as

$$X^T = [\dot{\delta}^T \quad \ddot{\delta}^T \quad \dddot{\delta}^T]$$

where $\dot{\delta}$, $\ddot{\delta}$ and $\dddot{\delta}$ represent the vectors of corrections to the exterior orientation elements, the object space coordinates and the additional parameters. The parameters $\dddot{\delta}_h$ are contained in $\dddot{\delta}$, leading to the incorporation of the linear constraint, Eq. 1, into the normal equation system, Eq. 2, according to Helmert's well-known method:

$$\begin{bmatrix} A^TPA & H^T \\ H & 0 \end{bmatrix} \begin{bmatrix} X \\ -K_c \end{bmatrix} - \begin{bmatrix} A^TPL \\ -F \end{bmatrix} = 0 \qquad (3)$$

where K_c is a vector of Lagrangian multipliers.

The solution of the equation system, Eq. 3, for the object space coordinates $\ddot{\delta}$ and the camera's calibration parameters contained in $\dddot{\delta}$ can require special computational consideration since the "bordered" normal equation matrix becomes indefinite, rather than positive-definite, with the incorporation of the constraint equations. Thus, if a direct solution is required and symmetric storage is used, decomposition methods such as Cholesky and Gauss elimination with full or partial pivotting will fail. Details of the decomposition algorithm for symmetric indefinite matrices used in the present investigation are given in FRASER (1980a; b).

SELF-CALIBRATION RESULTS

The results obtained using the multiple focal setting self-calibration with linear constraints were essentially comparable with those produced in the standard self-calibration adjustment in terms of the attainable accuracy of object point coordinate determination. However, the former approach yielded results of more favourable quality when judged by the magnitude of the a posteriori variance factor estimate $\hat{\sigma}_0^2$ and the attainable precision of "stable" non-metric camera calibration parameters. In this discussion only selected results of the multiple focal setting self-calibration incorporating lens distortion constraint equations are detailed. Further results are given in FRASER (1980b).

Image Coordinate Residuals

As a result of the multiple focal setting self-calibration adjustment, the following root mean square (RMS) values of the image coordinate residuals v_{x_i} and v_{y_i} where obtained: $s_x = \pm\ 4.6\ \mu m$ and $s_y = \pm\ 5.4\ \mu m$. On examining the pattern of the residuals it was noticeable that the magnitude of the image coordinate corrections tended to increase with radial distance r. However, this trend has not as yet been quantified. In general, the magnitudes of v_{x_i} and v_{y_i} for points with $r < 10$ mm were in accord with the estimated standard error of the AP/C image coordinate observations, about 2.5 µm. For a number of points at the edges of the image format ($r > 25$ mm) magnitudes of four and five times this value were encountered. Even with this apparent bias in the distribution of the residuals v_{x_i} and v_{y_i}, it will be assumed for the remaining discussion, which stresses quantitative aspects of the obtained precision, that $\hat{\sigma}_0^2$ is a sufficiently representative estimate of the a priori "true" variance factor σ_0^2.

Radial Distortion and Gaussian Focal Length

Fig. 2 illustrates the plots of the radial lens distortion functions obtained for the focal settings s_1, s_2 and s_3, along with the adjusted Gaussian focal lengths c_1, c_2, and c_3, and their a posteriori standard errors. Tests of statistical significance involving null hypotheses Ho of rank (Ho) = 2 were carried out and all parameters K_{i1}, K_{i2} were found to be highly significant at the 5% level. The distortion function displaying the highest precision was that for focussed object distance s_1, where at a radius of r = 25 mm, the standard error $\sigma_{\Delta r}$ of the distortion function Δr, given by

$$\sigma_{\Delta r_1}^2 = r^6\sigma_{K_{11}}^2 + r^{10}\sigma_{K_{12}}^2 + 2r^8\sigma_{K_{11}K_{12}}$$

was found to be $|\sigma_{\Delta r_1}| = 5\ \mu m$. Although not shown, the distortion curve for

s_1 obtained via the standard self-calibration technique nearly coincides with the one plotted for the 1 m object distance in Fig.2. However, this is not the case for the curves Δr_2 and Δr_3. The linear constraint equations, Eq.1, have the effect of leaving the most precise function Δr_1 unaltered, while bringing Δr_2 and Δr_3 into conformity with it, such that the functions show the required linear variation with changing magnification. At the radius r = 25 mm, values of $|\sigma_{\Delta r_2}|$ = 6 µm and $|\sigma_{\Delta r_3}|$ = 8 µm were obtained.

Precision of Object Point Determinations

A measure of the average precision of selected object point coordinates can be obtained by

$$\bar{\sigma}^2_{x_c} = \frac{1}{u_c} \operatorname{tr} \Sigma_c$$

where $\bar{\sigma}^2_{x_c}$ is an estimate of the mean variance of the chosen u_c coordinates, and Σ_c is the relevant sub-matrix of the variance-covariance matrix Σ_{xx}.

Estimates of average precision were determined for two object point coordinate samples. For the first sample, comprising 20 points in the target midfield, values of $|\bar{\sigma}_{X,Y}|$ = 140 µm (pooled estimate for planimetry) and $|\bar{\sigma}_Z|$ = 200 µm were obtained. At the target field centre the a posteriori standard error of object point coordinates was found to be $|\sigma_{X_i}| \simeq |\sigma_{Y_i}| \simeq$ 80 µm and $|\sigma_{Z_i}| \simeq$ 130 µm. For the second sample comprising the coordinates of six points lying at the vertical and lateral extremities of the target array the computed average standard errors were $|\bar{\sigma}_{X,Y}|$ = 320 µm and $|\bar{\sigma}_Z|$ = 460 µm. In the first sample, between six and ten rays intersected each of the 20 targets. However, the coordinates of points forming the latter sample were determined by only three- or four-ray intersections. This partially accounts for the lesser reliability of the adjusted coordinates of these outlying points.

Datum Invariant Measures of Precision

In close-range self-calibration projects it is useful, and in some cases computationally imperative, that all parameters - object space, exterior orientation and additional parameters - be regarded as stochastic unknowns. Initially, seven appropriate parameters need to be treated a priori as weighted observations to oversome a rank defect in the normal equation matrix A^TPA. However, the stability of the adjustment is typically considerably enhanced by applying realistic a priori variances to all parameters. With the matrix A^TPA being of full rank, the parameter estimates X are still datum biased, as are the variance estimates expressed by the variance-covariance matrix Σ_{xx} (BOSSLER, 1973).

To a large extent the computational stability of a self-calibrating bundle adjustment can rest on the adequate definition of a fixed datum. One scheme adopted to define the reference coordinate system, which is popular in geodetic adjustment applications, involves the suppression of a number of appropriate parameters, this number being equal to the rank defect of A^TPA (for a photogrammetric example, see KENEFICK, 1971). A second, perhaps more common close-range photogrammetric practice for minimum-constraint adjustment is to assign very tight a priori constraints to seven object space coordinate parameters so as to define the scale, orientation and origin of the reference coordinate system. Under the first scheme, parameter estimates and their variances are biased in what they refer to a fixed datum. A numerically similar situation arises for the second scheme.

Neglecting photogrammetric factors for the present; in one sense the notable fall off in object point coordinate precision away from the target field centre is to be expected in this case. Here, the reference coordinate system was established by fixing the XYZ coordinates of two points in the target midfield, and the Z coordinate of a third, non-collinear point. These constraints were applied by simply assigning a priori standard errors of $\pm$ 1 µm to the seven parameters. Thus, rather than being measures of "absolute" precision, the a posteriori estimates σ_{X_i}, σ_{Y_i} and σ_{Z_i} indicate the precision relative to the fixed control.

An unbiased estimation of the quality of the self-calibration results which relates to the object space can be gained by evaluating a reference frame invariant measure of precision. One such quantity which may prove useful for direct quality comparisons with other measuring tools is simply the a posteriori variance $\sigma^2_{d_{ij}}$ of a distance d_{ij} between i and j, given by

$$\sigma^2_{d_{ij}} = G_{ij}\ \Sigma_{ij}\ G^T_{ij}$$

where G is a row vector of the direction cosines ℓ, m and n.

$$G_{ij} = [-\ell\ \ -m\ \ -n\ \ \ell\ \ m\ \ n]_{ij}$$

and Σ_{ij} is the appropriate submatrix of Σ_{xx}. Estimates of the standard error $\sigma_{d_{ij}}$ were obtained for a number of distances within the target field. The following table lists the values $|\sigma_{d_{ij}}|$, the length d_{ij} and the ratio $|\sigma_{d_{ij}}|/d_{ij}$ for the three lines which are shown in Fig. 1.

line	$\|\sigma_{d_{ij}}\|$ (µm)	d_{ij} (m)	$\|\sigma_{d_{ij}}\|/d_{ij}$
a	186	.91	1:4900
b	107	.56	1:5200
c	210	1.28	1:6100

Throughout the midfield of the target array, a precision in distance determination of about 1 part in 5500 was achieved. However, for lines which run to the extremities of the field, the estimate falls to about 1:3800, or $|\sigma_{d_{ij}}| \simeq 260$ µm for a 1 m distance.

CONCLUSIONS

This preliminary investigation has demonstrated that it is feasible to employ non-metric cameras coupled with multiple focal setting self-calibration (both with and without linear distortion constraints) in close-range photogrammetric surveys where accuracies at the 5 µm level (RMS value of image coordinate residuals) are sought. Further, by treating the "amateur" camera as a pseudo-metric camera it is possible to successfully recover "stable" calibration parameters (lens distortion and Gaussian focal length) relating to as many focal settings as are used in the photogrammetric survey.

Notwithstanding the fact that the formulated functional model of the self-calibrating bundle adjustment adopted for this non-metric camera application lacks somewhat in mathematical rigour, the general accuracy obtained appears to be of the order expected had the DLT been applied (KARARA & ABDEL-AZIZ, 1974). The attainable precision is also only marginally lower than that anticipated for a close-range photogrammetric system employing a metric film camera (see, for example, FRASER, 1980a). In terms of relative accuracy the 500C test self-calibration adjustment yielded distance standard error estimates of about 1 part in 5500 for lines of up to 2 m in the target midfield. For object points lying in the same working area of the target array, average precisions of $|\overline{\sigma}_{X,Y}| = 140\ \mu m$ and $|\overline{\sigma}_Z| = 200\ \mu m$ were obtained.

For engineering projects involving laboratory structural testing and monitoring by means of a photogrammetric system comprising non-metric cameras, a significant advantage of applying the self-calibrating bundle adjustment is that no object space control network need be established. It is sufficient to define the scale, orientation and origin of the reference XYZ coordinate system by assigning a priori variance constraints to either the required number of selected object point coordinates or to appropriate exterior orientation parameters, thus enabling a minimum-constaint adjustment to be performed.

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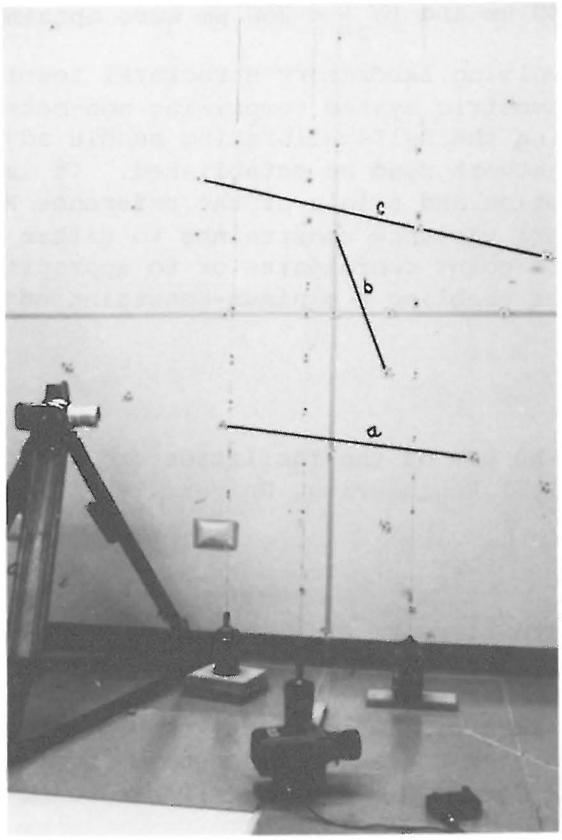

Fig. 1 : Portion of the 3-D Object Target Field.

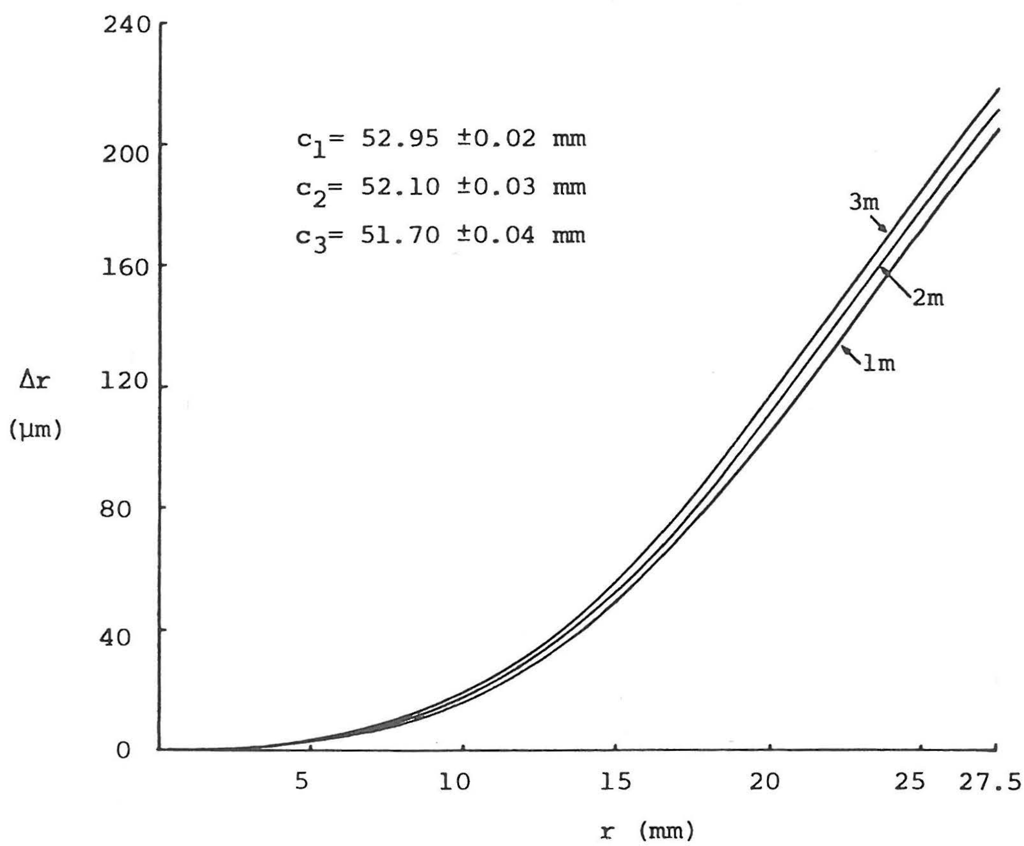

Fig. 2 : Plots of Radial Lens Distortion at Principal Distances c_1, c_2 & c_3 (i.e. at Focussed Object Distances of 1m, 2m & 3m).