THE GENERAL BUNDLE ADJUSTMENT TRIANGULATION (GEBAT) SYSTEM

- Theory and Applications

by

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Abstract

The availability of highly accurate and dense control networks is a major requirement for large scale urban mapping, as well as for various engineering projects.

Although advanced photogrammetric systems have achieved a level of accuracy which makes their practical applications generally feasible, even in urban situations, additional data still can improve the results. Geodetic observations, even if they are not sufficient for a terrestrial network, provide excellent constraints to the photogrammetric adjustment and also reduce absolute control requirements. Although the idea of a simultaneous geodetic and photogrammetric adjustment is not new, the method used in this approach appears to be more rigorous as it does not rely on approximations. Here, a modern three dimensional geodetic mathematical model is combined with a photogrammetric bundle adjustment with self calibration. A number of tests based on real as well as simulated data is presented in this paper.

Zusammenfassung

Dichte Kontrollnetze von hoher Genauigkeit formen die Grundlage für grossmasstäbliche Karten, sowie auch für verschiedene Ingenieurprojekte.

Obwohl moderne photogrammetrische Systeme ein Genauigkeitsniveau erreicht haben, welches ihre Anwendung selbst für erhöhte Ansprüche zulässt, können zusätzliche Daten die Ergebnisse immer noch verbessern. Geodätische Beobachtungen, selbst wenn sie für ein terrestrisches Netz nicht ausreichen oder unvollständig sind, ergeben zusätzliche Bedingungen für die photogrammetrische Ausgleichung und reduzieren damit die Anzahl notwendiger Passpunkte.

Der Gedanke einer Simultanausgleichung geodätischer und photogrammetrischer Beobachtungen ist natürlich nicht neu. Das vorliegende Verfahren

scheint jedoch strenger zu sein, da es sich nicht auf Näherungen verlässt. Es wird nämlich ein modernes geodätisches Raummodell mit einer Bündelausgleichung mit Selbstkalibrierung kombiniert. Das Verfahren wurde mit praktischen sowie simultierten Daten getestet.

Introduction

Even though aerial triangulation formulations have been designed to directly accommodate auxiliary vertical control information, such as statoscope and others, the procedure remains basically a two step solution. First, the geodetic observations are adjusted to provide ground control in form of a set of coordinates and variance-covariance matrices, which in turn are utilized as input into the photogrammetric block adjustment.

In addition to creating a time lag between the two computations, this may make aerial triangulation over poorly accessible areas a difficult task. In such areas, the geodetic observations available may not be sufficient for adjusting a geodetic network to provide the necessary control point coordinates for aerial triangulation. Therefore, it is useful if instead of using geodetically adjusted control points, the available observations are adjusted simultaneously with the photogrammetric measurements. This also has the advantage of providing a realistic approach to the problem of error analysis through proper weighting of all measured quantities. The program SAPGO, developed at the University of Illinois [8] permits simultaneous adjustment of all conventional photogrammetric and geodetic observations. The mathematical model for the photogrammetric observations is based on the collinearity condition, while the geodetic observation equations are those of classical geodesy, which treats horizontal and vertical adjustments separately, and performs the horizontal adjustment on the surface of a reference ellipsoid as a function of latitude and longitude. To combine these models, a number of modifications were necessary, which led to a non-rigorous solution. Although this approach of simultaneous geodetic and photogrammetric adjustment appears occasionally in the literature, sometimes as bridging with independent control [5], a rigorous program has not been readily available. The program GEBAT (General Bundle Adjustment Triangulation) [2], developed at the University of New Brunswick is intended to fill this gap. It utilizes three dimensional geodetic mathematical models [4] together with a bundle adjustment with self calibration.

Basic Formulation of GEBAT

- Photogrammetric mathematical models

The photogrammetric part consists of a bundle adjustment with additional parameters. The latter utilize an orthogonal function which is a special case of three dimensional harmonics. The photogrammetric mathematical model is thus:

$$\begin{aligned} x_A - x_0 + dV_x &= -f\,\frac{(X_p-X_c)m_{11} + (Y_p-Y_c)m_{12} + (Z_p-Z_c)m_{13}}{(X_p-X_c)m_{31} + (Y_p-Y_c)m_{32} + (Z_p-Z_c)m_{33}}\,, \\ y_Z - y_o + dV_y &= -f\,\frac{(X_p-X_c)m_{21} + (y_p-y_c)m_{22} + (Z_p-Z_c)m_{23}}{(X_p-X_c)m_{31} + (Y_p-Y_c)m_{32} + (Z_p-Z_c)m_{33}} \end{aligned} \qquad (1)$$

with

$$\begin{aligned} dV_x &= (x - x_o)\,.\,T \\ dV_y &= (y - y_o)\,.\,T \end{aligned} \qquad (2)$$

and

$$T = a_{00} + a_{11}\cos\lambda + b_{11}\sin\lambda + a_{20}r + a_{22}r.\cos 2\lambda$$
$$+ b_{22}r.\sin 2\lambda + a_{31}r^2\cos\lambda + b_{31}r^2\sin\lambda + a_{33}r^2\cos 3\lambda$$
$$+ b_{33}r^2\sin 3\lambda + \ldots\ldots \qquad (3)$$

$$r = \sqrt{(x-x_o)^2 + (y-y_o)^2}\ , \quad \text{and:} \quad \lambda = \tan^{-1}\frac{y-y_o}{x-x_o} \qquad (4)$$

Ground control points have the following observation equations:

$$X_p - X_{Gr} = 0$$
$$X_p - Y_{Gr} = 0 \qquad (5)$$
$$Z_p - Z_{Gr} = 0$$

where X_{Gr}, Y_{Gr}, Z_{Gr} are the object coordinates of the point. Depending on the available coordinates, only one or two of these may be used.

- Geodetic mathematical models

The observation equations for the geodetic observations are based on modern three dimensional geodesy ([4] and [7]) in a Cartesian (x, y, z) coordinate system.

The observations accepted in this approach are: slope distances, vertical angles, horizontal directions, astronomic azimuths, elevation differences, astronomic longitudes, and astronomic latitudes.

The equations had to be transferred into a Cartesian coordinate system to suit combination with the photogrammetric models, and are given in linearized form, for each type of observable.

a) Slope Distances S_{ij}:

$$V_{s_{ij}} = C_1(dX_j - dX_i) + C_2(dY_j - dY_i) + C_3(dZ_j - dZ_i) + (S_c - S_o) \qquad (6)$$

with

$$C_1 = (X_j - X_i)/S\ ;$$
$$C_2 = (Y_j - Y_i)/S\ ;$$
$$C_3 = (Z_j - Z_i)/S\ ;$$

where
S_c is computed from $(\Delta X^2 + \Delta Y^2 + \Delta X^2)^{1/2}$ using approximate coordinates while S_o is the observed value.

b) Vertical Angles, β_{ij}: (astronomic)

$$V_{\beta_{ij}} = b_1(dX_j - dX_i) - b_2(dY_j - dY_i) + b_3(dZ_j - dZ_i) + b_4 d\Phi_i$$
$$+ b_5 d\Lambda_i - S_{ij}/10^3.dk_i + [\beta_c - (\beta_o - S_{ij}\cos\beta_{ij}k_i)] \qquad (7)$$

where:

$$b_1 = (s_{ij}.\cos\Phi_i.\cos\Lambda_i - \Delta X\sin\beta_{ij})/S^2_{ij}.\cos\beta_{ij}$$

$$b_2 = (S_{ij}.\cos\Phi_i.\cos\Lambda_i - \Delta Y\sin\beta_{ij})/S^2_{ij}.\cos\beta_{ij}$$

$$b_3 = (S_{ij}.\sin\Phi_i - \Delta Z\ \sin\beta_{ij})/S^2_{ij}.\cos\beta_{ij}$$

$$b_4 = \cos\alpha_{ij}$$

$$b_5 = \cos\Phi_i\ .\ \sin\alpha_{ij}$$

Φ and Λ are the astronomic latitude and longitude, and α is the astronomic azimuth.

The coefficient of refraction k can be made a function of the station, or a function of the line, or even a function of the direction. The initial value of k can be given as 0.13/2R, where R is the mean radius of the earth.

β_c is computed from:

$$\beta_c = \sin^{-1} \frac{\Delta X\cos\Phi_i\ \cos\Lambda_i + \Delta Y\cos\Phi_i\sin\Lambda_i + \Delta Z\sin\Phi_i}{S_{ij}} \tag{8}$$

and

$$\alpha = \tan^{-1} \frac{\Delta Y\cos\Lambda_i - \Delta X\sin\Lambda_i}{\Delta Z\cos\Phi_i - \Delta X\sin\Phi_i\cos\Lambda_i - \Delta Y\sin\Phi_i\sin\Lambda_i} \tag{9}$$

The astronomic latitude and longitude, Φ and Λ may not be available. However, because they appear as coefficients above, the corresponding geodetic latitude ϕ and longitude λ, or any reasonable approximate can be substituted without significant loss of computational accuracy.

c) Horizontal Direction, r_{ij}:

$$V_{r_{ij}} = a_1(dX_j-dX_i) + a_2(dY_j-dY_i) + a_3(dZ_j-dZ_i)$$
$$+ a_4d\phi_i + a_5d\Lambda_i - dR_i + [\alpha_c - (r_{ij} + R_c)] \tag{10}$$

where:

$$a_1 = (\sin\alpha_{ij}\sin\Phi_i\cos\Lambda_i - \cos\alpha_{ij}\sin\Lambda_i)/S_{ij}\cos\beta_{ij};$$

$$a_2 = (\sin\alpha_{ij}\sin\Phi_i\cos\Lambda_i + \cos\alpha_{ij}\cos\Lambda_i)/S_{ij}\cos\beta_{ij};$$

$$a_3 = -\sin\alpha_{ij}\cos\Phi_i/S_{ij}\cos\beta_{ij};$$

$$a_4 = \sin\alpha_{ij}\ \tan\beta_{ij}\ ;$$

$$a_5 = \sin\Phi_i - \cos\alpha_{ij}\cos\Phi_i\tan\beta_{ij}$$

R is the initial azimuth, or station orientation, and is usually treated as unknown, with R_c as its approximation, β and α are computed from (8) and (9) respectively.

d) Astronomic Azimuth, α_{ij}:

$$V_{\alpha_{ij}} = a_1(dX_j - dX_i) + a_2(dY_j - dY_i) + a_3(dZ_j - dZ_i) + a_4 d\Phi_i + a_5 d\Lambda_i + [\alpha_c - \alpha_o] \tag{11}$$

where the coefficients are the same as in equation (10), while α_c is the observed azimuth of the line i to j.

e) Elevation Difference, ΔH_{ij}:

$$V_{\Delta H_{ij}} = e_1(dX_j - dX_i) + e_2(dY_j - dY_i) + e_3(dZ_j - dZ_i) + e_4 d\Phi_i + e_5 d\Lambda_i + e_6 d\Phi_j + e_7 d\Lambda_j + (\Delta h_c - \Delta H_o) \tag{12}$$

where

$$e_1 = (S_{ij}\cos\Phi_i \cos\Lambda_i - \Delta X \sin\beta_{ij})/2S_{ij}\cos\beta_{ij} + (S_{ij}\cos\Phi_j \cos\Lambda_j - \Delta X \sin\beta_{ji})/2S_{ij}\cos\beta_{ji} \quad ;$$

$$e_2 = (S_{ij}\cos\Phi_i \sin\Lambda_i - Y \sin\beta_{ij})/2S_{ij}\cos\beta_{ij} + (S_{ij}\cos\Phi_i \sin\Lambda_j - \Delta Y \sin\beta_{ji})/2S_{ij}\cos\beta_{ji} \quad ;$$

$$e_3 = (S_{ij}\sin\Phi_i - \Delta Z \sin\beta_{ij})/2S_{ij}\sin\beta_{ij} + (S_{ij}\sin\Phi_j - \Delta Z \sin\beta_{ji})/2S_{ij}\cos\beta_{ji} \quad ;$$

$$e_4 = (S_{ij}\cos\alpha_{ij})/2 \quad ;$$

$$e_5 = (S_{ij}\cos\Phi_i \sin\alpha_{ij})/2 \quad ;$$

$$e_6 = (S_{ij}\cos\alpha_{ji})/2 \quad ;$$

$$e_7 = (S_{ij}\cos\Phi_j \sin\alpha_{ji})/2 \quad ;$$

Δh_c is the assumed difference in ellipsoidal height, when:

$$\Delta h_c = 1/2\ S(\beta_1 - \beta_2) \quad ;$$

ΔH_o is the measured orthometric height.

f) Astronomic Longitude, Λ_i:

$$V_{\Lambda_i} = d\Lambda_i + (\Lambda_c - \Lambda_o) \tag{13}$$

g) Astronimic Latitude, Φ_i:

$$V_{\Phi_i} = d\Phi_i + (\Phi_c - \Phi_o) \tag{14}$$

In equations (13) and (14), Λ_o and Φ_o are the observed quantities, while Λ_c and Φ_c are the computed values and can be taken as the geodetic longitude and latitude respectively.

The Combined Solution in General Terms

The photogrammetric model can be written as:

$$F_p(X_1, X_2, L_p) = 0$$

or in the linearized form as:

$$W_p + A_{p1}\hat{X}_1 + A_{p2}\hat{X}_2 + B_p\hat{V}_p = 0 \quad (15)$$

where:

X_1 is the vector of photo orientation elements and calibration parameters;
X_2 is the vector of object coordinates;
L_p is the vector of observed photo coordinates;
W_p is the misclosure vector, when:
$W_p = F_p(\overset{\circ}{X}_1, \overset{\circ}{X}_2, L_p)$, with
$\overset{\circ}{X}_1$ and $\overset{\circ}{X}_2$ as the initial values;
$\hat{V}_p$ is the vector of photo coordinate residuals; and
A_{p1}, A_{p2}, and B_p are the design matrices, namely:

$$A_{p1} = \left.\frac{\delta F_p}{\delta \hat{X}_1}\right|_{\overset{\circ}{X}_1, \overset{\circ}{X}_2 . L_p} ;$$

$$A_{p2} = \left.\frac{\delta F_p}{\delta \hat{X}_2}\right|_{\overset{\circ}{X}_1, \overset{\circ}{X}_2, L_p} \quad \text{and}$$

$$B_p = \left.\frac{\delta F_p}{\delta L_p}\right|_{\overset{\circ}{X}_1, \overset{\circ}{X}_2, L_p}$$

The geodetic model can be written as

$$F_g(\hat{X}_2, \hat{X}_3, L_g) = 0$$

or in the linearized form as:

$$W_g + A_{g1}\hat{X}_2 + A_{g2}\hat{X}_3 + B_g\hat{V}_g = 0 \quad (16)$$

where

X_3 is the vector of orientation, refraction unknowns, and astronomic coordinates;
L_g is the vector of geodetic observations;
W_g is the misclosure vector, when:

$$W_g = F_g(\hat{X}_2{}^\circ, \hat{X}_3{}^\circ, L_g) ;$$

$\hat{V}_g$ is the vector of geodetic observation residuals;

A_{g1}, A_{g2}, and B_g are the design matrices, namely

$$A_{g1} = \left.\frac{\delta F_g}{\delta X_2}\right|_{\hat{X}_2^o, \hat{X}_3^o, L_g} ;$$

$$A_{g2} = \left.\frac{\delta F_g}{\delta X_3}\right|_{\hat{X}_2^o, \hat{X}_3^o, L_g} \quad \text{and}$$

$$B_g = \left.\frac{\delta F_g}{\delta L_g}\right|_{\hat{X}_2^o, \hat{X}_3^o, L_g}$$

Applying the least squares principle, the variation function is:

$$\phi = \hat{V}_p^T P_p \hat{V}_p + \hat{V}_g^T P_g \hat{V}_g + \hat{X}_1^T P_{x_1} \hat{X}_1 + \hat{X}_2^T P_{x_2} \hat{X}_2 + \hat{X}_3^T P_{x_3} \hat{X}_3$$

$$+ 2\hat{k}_p^T (W_p + A_{p_1} \hat{X}_1 + A_{p_2} \hat{X}_2 + B_p \hat{V}_p)$$

$$+ 2\hat{k}_g^T (W_g + A_{g_1} \hat{X}_2 + A_{g_2} \hat{X}_3 + B_g \hat{V}_g) = \text{minimum}$$

where

P_p and P_g are weight matrices for the observations;
P_{x_1}, P_{x_2}, and P_{x_3} are weight matrices for the unknowns; and,
$\hat{k}_p$ and $\hat{k}_g$ are estimators for the vectors of Lagrange multipliers;

This leads to the following system in block matrix form:

$$\begin{bmatrix} P_p & 0 & B_p^T & 0 & 0 & 0 & 0 \\ 0 & P_g & 0 & B_g^T & 0 & 0 & 0 \\ B_p & 0 & 0 & 0 & A_{p_1} & A_{p_2} & 0 \\ 0 & B_g & 0 & 0 & 0 & A_{g_1} & A_{g_2} \\ 0 & 0 & A_{p_1}^T & 0 & P_{x_1} & 0 & 0 \\ 0 & 0 & A_{p_2}^T & A_{g_1}^T & 0 & P_{x_2} & 0 \\ 0 & 0 & 0 & A_{g_2}^T & 0 & 0 & P_{x_3} \end{bmatrix} \cdot \begin{bmatrix} V_p \\ V_g \\ k_p \\ k_g \\ X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ W_p \\ W_g \\ 0 \\ 0 \\ 0 \end{bmatrix} = 0 \qquad (17)$$

After some manipulations, the solution becomes:

$$\hat{X}_2 = -[P_{x_2} + N_{p_{22}} + N_{g_{11}} - N_{g_{12}} (P_{x_3} + N_{g_{22}})^{-1} N_{g_{21}} - N_{p_{21}} (P_{x_1} + N_{p_{11}})^{-1} N_{p_{12}}]^{-1} \cdot [U_{p_2} + U_{g_1} - N_{g_{12}} (P_{x_3} + N_{g_{22}})^{-1} U_{g_2} - N_{p_{21}} (P_{x_1} + N_{p_{11}})^{-1} U_{p_1}] \qquad (18)$$

from which the variance-covarinace matrix for the adjusted coordinates is given by:

$$\hat{\Sigma}_{xx} = -\hat{\sigma}_o [P_{x_2} + N_{p_{22}} + N_{g_{11}} - N_{g_{12}} (P_{x_3} + N_{g_{22}})^{-1} N_{g_{21}} - N_{p_{21}} (P_{x_1} + N_{p_{11}})^{-1} N_{p_{11}}]^{-1} \quad (19)$$

where

$$\hat{\sigma}_o = \frac{\hat{V}^T P \hat{V} + \hat{X}_1^T P_{x_1} \hat{X} + \hat{X}_2^T P_{x_2} \hat{X}_2 + \hat{X}_3^T P_{x_3} \hat{X}_3}{df}$$

in which the degree of freedom (df) equals the number of observations since all the unknowns are weighted.

The following abbreviations are used in the above equations:

$$N_{p_{ij}} = A^T_{p_i} [B_p P_p^{-1} B^T_p]^{-1} A_{p_j} \quad (20)$$

$$N_{g_{ij}} = A^T_{g_i} [B_g P_g^{-1} B^T_g]^{-1} A_{g_j} = A^T_{g_i} P_g A_{g_j} \quad (21)$$

$$U_{p_i} = A^T_{p_i} [B_p P_p^{-1} B^T_p]^{-1} W_p \quad (22)$$

$$U_{g_i} = A^T_{g_i} P_g W_g \quad (23)$$

Tests and Practical Application of GEBAT

The use of the harmonic function was tested by adjusting six different models taken over a test area and comparing the results to adjustments with UNBASC [6], a bundle program with additional parameters in polynomial form. The resulting RMS values were consistently smaller when using the harmonic function, sometimes by a factor of two. It appears that modelling of the combined effect of all errors by an orthogonal function is more effective than modelling individual errors by empirical formulae.

A few versions of GEBAT, depending on the anticipated use, such as camera calibration, bundle adjustment with or without geodetic observation, utilization of collocation, etc. have been arranged for easier computation. GEBAT was tested with ficticious data (ISP test block [1]) as well as real data from an industrial network.

- Test using ISP - test block

For this test, 25 photographs were used, each containing 9 points. A few sets of geodetic observations of all the types together with their variances were simulated by the authors. The different distributions of the observations, as displayed in figures 1, 2, 3, were used to study the effect of introducing geodetic observations, when only a few control points are available. Table 1 shows the results of thes combined adjustments. The results show a significant improvement when introducing the geodetic observations over the case of using the few control points only. The improvement depends on the number and distribution of the geodetic observations.

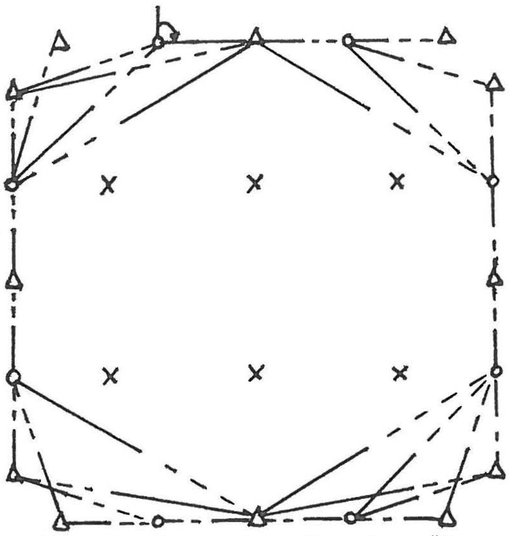

Figure 1: Distribution #1

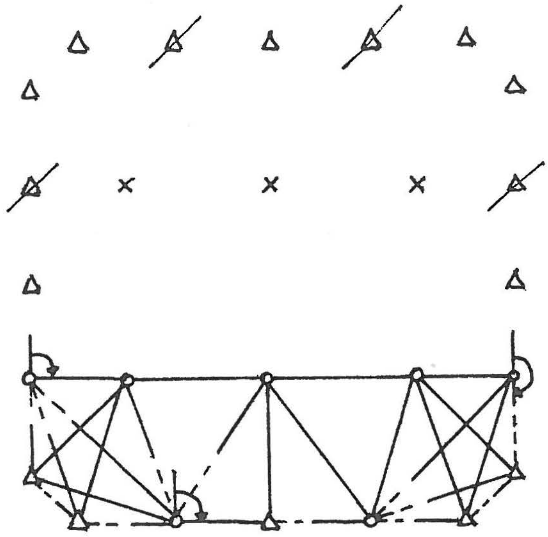

Figure 2: Distribution #2

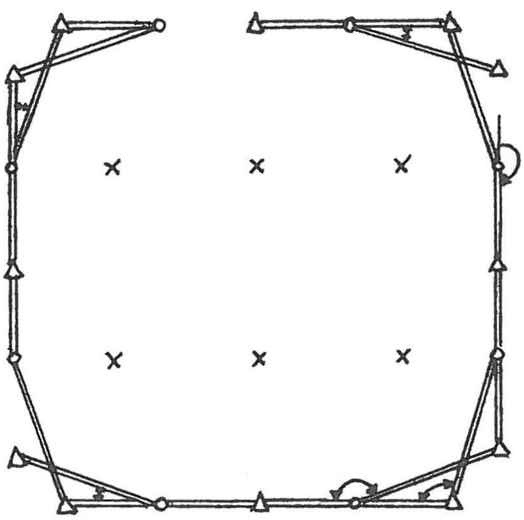

Figure 3: Distribution #3

Δ geodetic control point (coordinated)

used with zero weight

o geodetic observation point

x levelling point

NOTE: vertical angles are considered measured between geodetic observation points

Figures 1-3: Simulated Geodetic Observation for ISP-Test

Case	Control H	Control V	Iter-ations	RMS (check pts) in m X	Y	Z	R
Photogrammetry + full control	20	25	11	0.42	0.38	0.22	0.61
Photogrammetry + reduced control	12	12	20	1.20	0.62	0.45	1.43
Photogrammetry + Geodesy (distr. #1)	12	12	18	0.54	0.32	0.25	0.67
Photogrammetry + Geodesy (distr. #2)	12	12	18	0.62	0.42	0.28	0.81
Photogrammetry + Geodesy (distr. #3)	12	12	18	0.73	0.38	0.32	0.88

Table 1: Effect of Geodetic Observations (ISP Block)
(3-D Coordinate System)

- Adjusting a Dense Industrial Control Network

The program GEBAT has also been applied to a practical project in an industrial environment which consisted of a dense network that contained some 70 points in an area of about 15 x 15 m^2. Due to the nature of the network and the tight time schedule, that required the adjustment of the network in a relatively short period of time, it was decided to use photogrammetry rather than conventional surveying. The use of surveying would have required thousands of observations and, although the area is small, would have required a long time and thus been rather costly, especially since the required accuracy of the adjusted coordinates was 1 to 2 mm.

Convergent photographs with double coverage were taken from four elevated camera stations with a Wild photo theodolite P-31 at a photoscale ranging between 1:150 to 1:300. These were evaluated with a Zeiss PSK stereo comparator. Although there were no ground coordinates, approximately 150 slope distances had been measured within the network with an accuracy of 0.5 mm, and the elevations of all points had been obtained by precision levelling to 0.1 mm. Because of this accuracy, the elevations were kept fixed, and planimetric adjustments were performed. The following cases were evaluated:

a) Combined photogrammetric and spatial distance adjustment using all the available data.
b) Photogrammetric adjustment only, single coverage (4 photographs) and no additional parameters.
c) Same as (b) but with self calibration
d) Same as (c) but using all photographs (1 3/4 coverage, 7 photographs because one photo did not turn out and had to be discarded).

Adjustment (a) provided the required coordinates for the project, while adjustments (b), (c) and (d) were carried out for research purposes.

Since there is at least one distance observed to each point, the check accuracy is expressed by the RMS of the difference between the distances computed from the adjusted coordinates and the measured distances. The coordinates' accuracy is then equal to the distance accuracy divided by $\sqrt{2}$. The variance covariance matrix of the adjusted coordinates and the corresponding relative error ellipsoid were computed to provide the accuracy of the adjustment.

Table 2 shows the accuracy and the check accuracy for the different adjustments.

Case	RMS of Distance Discrepancies	Estimated Coordinate Absolute Error	Mean Variance of Adjusted Coordinates	Semi-major Axis of Error Ellipse (95%c.l.)
a	0.6	0.4	0.8	1.6
b	5.0	3.5	1.7	3.3
c	4.4	3.1	1.7	3.3
d	3.4	2.4	1.4	2.7

TABLE 2: Accuracy of Different Adjustments
(All dimensions are mm)

These results can be analyzed as follows:
- The combined solution (case a) provided a very high accuracy. The excellent geometry, obtained from the convergent photography, and the availability of 150 spatial distances spread within the block provided a complete control for the adjustment. It was also possible to discover some blunders in the distances after performing the adjustment once. Convergency to the solution was achieved after only three iterations, again due to the good geometry.
- The large difference in accuracy between case (a) and the other cases is understandable. This is mainly due to the large difference in control, or constraints, between them.
- The improvement by using self calibration was only by a factor 1.14, which may be due to the good quality of the camera and photography. The improvement of 0.4 mm in coordinate errors is equivalent to about 2.5 μm in photo scale (the maximum lens distortion for this camera is 4 μm).
- Multiple coverage improved the results by a factor 1.30. The expected improvement (see also [2]) for double coverage is $\sqrt{2}$ or 1.41 times, but in this project the second coverage was only 3/4 of the first, and thus the expected improvement is 1.31, which was almost achieved practically.
- The standard deviation of the adjusted coordinates givesa good indication of the accuracy. At the 95% confidence level, the semi-major axis of the error ellipse is reliable for case (c) and (d). For case (b), some systematic errors existed, and this caused the check error to be slightly outside the error ellipse.

Concluding Remarks

As shown in the test results as well as for the practical application in industry, a rigorous simultaneous bundle adjustment directly using geodetic observations provides an excellent alternative to ground surveying, even if the accuracy requirements are very high. With the inclusion of additional parameters for self calibration, the potentials of photogrammetry are fully explored. Thus we have a powerful measuring tool that can replace hundreds of angular measurements, not to mention the common advantages of photogrammetry, such as instantaneous, complete and permanent recording of a situation.

References

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