XIV CONGRESS OF THE
INTERNATIONAL SOCIETY FOR PHOTOGRAMMETRY
HAMBURG 1980

Commission V
Working Group V/

Presented Paper

Dr. Iulian C. Barbalata
Civil Engineering Department
Polytechnical Institute of Iassy
Calea 23 August, 22
Iassy, 6600
Roumania

ANALYTICAL PHOTOGRAMMETRIC METHOD USED IN RESISTENCE STRUCTURE DEFORMATIONS APPLIED ON SEISMIC STRESS

Monophotogrammetric measurements to establish deformations, frequencies and amplitudes of the movements of resistence structures are applied in conjunction with a simulated earthquake on a 150 tons seismic platform.

XIV CONGRESS OF THE INTERNATIONAL SOCIETY FOR PHOTOGRAMMETRY
HAMBURG 1980

Commission V
Working Group V/
Presented Paper

Dr. Iulian C. Barbalata
Polytechnical Institute of Iassy, 6600, Roumania

ANALYTICAL PHOTOGRAMMETRIC METHOD USED IN RESISTENCE STRUCTURE DEFORMATIONS APPLIED ON SEISMIC STRESS

Abstract. Monophotogrammetric measurements to establish deformations, frequencies and amplitudes of the movements of resistence structures are applied in conjunction with a simulated earthquake on a 150 tons seismic platform.

1. INTRODUCTION

Photogrammetry provides a method for an accurate determination of small relative building movements, with the possibility of obtaining these informations at time intervals, in daylight or darkness. This capability makes photogrammetric method attractive as a means for determining building movements and deformations during a simulated earthquake.
The monitoring of such a building is accomplished by photographing at certain time intervals, object targets positioned on structures and evaluating the changes in the relative positions of the targets. The only input data required, are a few control points or line endpoints within the photograph, with known point positions or lengths. Monophotogrammetric methods are presented because the movements of the seismic platform are unidirectional. The rate of movements of the building can be investigated from the recordings of the times of the exposures.
This paper describes a test project designed to investigate the technical feasibility of the photogrammetric method for the analysis of building movements and deformations during a simulated earthquake on the seismic platform of 150 tons from the Civil Engineering Department of Polytechnical Institute of Iassy, Roumania. It includes field procedures, data reduction results, conclusions and recomandations. The results of the photogrammetric method, in the form of X,Y coordinates of object targets, make it possible to compare any one point with any or all other points in all combinations. Results of this study are expressed as coordinate differences between individual photogrammetric reductions of the 120 photographs. The consistency of the differences between individual photogrammetric reductions is an indicative of the precision of the method and therefore can be considered as a

limit for the amount of building movement and deformation can be detected by the presented method.

2. PROJECTIVE METHOD FOR RESECTION

The mathematical model employed is based on projective methods. The Projective Transformation Parameters (PTP) are computed iteratively and the X,Y coordinates corresponding to some of the measured images are computed directly.
If we assume that the film and the object are flat planes, then eight coefficients will uniquely map the object plane into the film plane. The projective equations are:

$$(2.1)\quad x_j=\frac{a_1X_j+a_2Y_j+a_3}{a_7X_j+a_8Y_j+1}\ ;\quad y_j=\frac{a_4X_j+a_5Y_j+a_6}{a_7X_j+a_8Y_j+1}\ ;\quad j=1,2,\dots,n$$

where x_j,y_j are the measured plate coordinates, properly corrected for lens distorsion;
$a_1,a_2,\dots a_8$ are the PTP coefficients;
X_j,Y_j are the Cartesian object coordinates of the j-th point.
According to (2.1), the condition equations arising from the j-th control point, are of the form:

$$(2.2)\quad Fx_j=x_j-\frac{a_1X_j+a_2Y_j+a_3}{a_7X_j+a_8Y_j+1}=0;\quad Fy_j=y_j-\frac{a_4X_j+a_5Y_j+a_6}{a_7X_j+a_8Y_j+1}=0$$

The PTP coefficients may be expressed in terms of the approximations $a_1^o,\ a_2^o,\dots a_8^o$:

$$(2.3)\quad a_k=a_k^o+\delta a_k\ ;\quad k=1,2,\dots,8$$

where δa_k are presently unknown corrections.
The adjusted film coordinates are:

(2.4) $x_j=x_j^o+v_x$, $y_j=y_j^o+v_y$, where x_j^o, y_j^o are the measured film coordinates and vx_j, vy_j are the residuals in the condition equations.
The substitution of (2.3) and (2.4) into (2.2) and the linearization of resulting expressions by Taylor expansion, permits equations (2.2) to be replaced by equivalent pair:

$$(2.5)\quad \underset{(2.1)}{v_j}+\underset{(2.8)}{B_j}\ \underset{(8.1)}{\Delta}=\underset{(2.1)}{E_j}\qquad \text{in which:}$$

$$(2.6)\quad v_j=\begin{bmatrix}v_x & v_y\end{bmatrix}_j^T;\quad B_j=\begin{bmatrix}\dfrac{\partial Fx^o}{\partial a_k^o} & \dfrac{\partial Fy^o}{\partial a_k^o}\end{bmatrix}_j^T;\quad \Delta=\begin{bmatrix}\delta a_1 & \delta a_2 & \dots & \delta a_8\end{bmatrix}^T$$

$$Fx_j^o=x_j^o-(m/q)_j^o;\quad Fy_j^o=y_j^o-(n/q)_j^o;\quad E_j=\begin{bmatrix}-Fx^o & -Fy^o\end{bmatrix}_j^T$$

$$(2.7)\quad \begin{bmatrix}m\\ n\\ q\end{bmatrix}_j^o=\begin{bmatrix}a_1 & a_2 & a_3\\ a_4 & a_5 & a_6\\ a_7 & a_8 & 1\end{bmatrix}^o\begin{bmatrix}X\\ Y\\ 1\end{bmatrix}_j^o$$

The equations arising from all n control points are:

$$\underset{(2n.1)}{v} + \underset{(2n.8)}{B}\ \underset{(8.1)}{\Delta} = \underset{(2n.1)}{E} \qquad (2.8)$$

Four points are needed to calculate the eight resection coefficients, but line lengths are often known more accurately than the exact positions of many juxtapositioned points in a specific coordinate system.
In order to obtain the best solution, the system (2.8) was augmented with constraint equations arising from m straight-line distance measurements. Therefore they can practically be considered as conditions containing some type of observations.
The exact straight-line distance $S_i = S_{rk}$ from the end points of each line i (i=1,2,...m), can be expressed directly:

$$S_i = S_{rk} = \sqrt{(X_k - X_r)^2 + (Y_k - Y_r)^2}\,; \quad F_{Si} = S_i - \sqrt{(X_k - X_r)_i^2 + (Y_k - Y_r)_i^2} = 0 \qquad (2.9)$$

and using the same approach as before, the true distances S_{rk} can be expressed in terms of the measured quantity S_{rk}^{oo} and of the approximation S_{rk}^{o}, with the residuals $\overset{o}{v}$ and corrections δS.
From equations (2.1):

$$X_j = \frac{A_3 A_5 - A_2 A_6}{A_1 A_5 - A_2 A_4}\,; \qquad Y_j = \frac{A_3 A_4 - A_1 A_6}{A_2 A_4 - A_1 A_5}\,, \quad \text{where:} \qquad (2.10)$$

$$A_1 = a_1 - x_j a_7\,; \quad A_2 = a_2 - x_j a_8\,; \quad A_3 = x_j - a_3$$

$$A_4 = a_4 - y_j a_7\,; \quad A_5 = a_5 - y_j a_8\,; \quad A_6 = y_j - a_6$$

The substitution of equations (2.3) and $A_1 - A_6$ into (2.10) and (2.10) in (2.9), and the linearization of the resulting expressions by Taylor expansion, permit equation (2.9) to be replaced by the equivalent expression:

$$\underset{(1.1)}{\overset{o}{v}_i} + \underset{(1.8)}{\overset{o}{B}_i}\ \underset{(8.1)}{\Delta} = \underset{(1.1)}{\overset{o}{E}_i} \qquad (2.11) \quad \text{in which:}$$

$$\overset{o}{v}_i = S_i - S_i^{oo}\,; \quad \overset{o}{B}_i = \left[\partial F_{Si}^{o} / \partial a_k\right]_i^{o}\,; \quad \overset{o}{E}_i = -F_{Si}^{o}\,; \quad F_{Si}^{o} = S_i^{oo} - S_i^{o}$$

and from (2.10) $S_i^{o} = \sqrt{(X_k^{o} - X_r^{o})^2 + (Y_k^{o} - Y_r^{o})^2}$

Let there be a total of m measured distances, then the complete set of distance equations is as follows:

$$\underset{(m.1)}{\overset{o}{v}} + \underset{(m.8)}{\overset{o}{B}}\ \underset{(8.1)}{\Delta} = \underset{(m.1)}{\overset{o}{E}} \qquad (2.12)$$

The complete mathematical model of the problem is obtained by combining equations (2.8) and (2.12). Using matrix notation, the model may be written as follows:

$$\underset{[(2n+m).1]}{\bar{v}} + \underset{[(2n+m).8]}{\bar{B}}\ \underset{(8.1)}{\Delta} = \underset{[(2n+m).1]}{\bar{E}} \qquad (2.13)$$

The normal equation is:

$$\underset{(8.8)}{(\bar{B}^T \bar{W} \bar{B})}\ \underset{(8.1)}{\Delta} = \underset{(8.1)}{\bar{B}^T \bar{W} \bar{E}} \qquad (2.14)$$

where $\bar{W}$ is the weight matrix associated with the measured quantities and has the following composition:

$$\underset{(2n+m)(2n+m)}{\bar{W}} = \begin{bmatrix} W & 0 \\ 0 & \overset{o}{W} \end{bmatrix}$$

The solutions of the normal equations are thus provided by:

$$(2.15) \qquad \underset{(8.1)}{\Delta} = \underset{(8.8)}{(N + \overset{o}{N})^{-1}} \underset{(8.1)}{(C + \overset{o}{C})} \qquad \text{where:}$$

$$N = \sum_{j=1}^{n} N_j \; ; \quad N_j = B_j^T W_j B \; ; \quad C = \sum_{j=1}^{n} C_j \; ; \quad C_j = B_j^T W_j B$$

$$\overset{o}{N} = \sum_{i=1}^{m} \overset{o}{N}_i \; ; \quad \overset{o}{N}_i = \overset{o}{B}_i^T \overset{o}{W}_i \overset{o}{B} \; ; \quad \overset{o}{C} = \sum_{i=1}^{m} \overset{o}{C}_i \; ; \quad \overset{o}{C}_i = \overset{o}{B}_i^T \overset{o}{W}_i \overset{o}{B}$$

Equation (2.15) gives the vector of estimates for corrections to parameter approximations when all variables have a priori cofactor matrices. The terms N and C reflect the contribution of the conventionally known observations, and the terms $\overset{o}{N}$ and $\overset{o}{C}$ the contribution of constraints.
At the end of each iteration a criterion, established a priori is checked to see if the process is to be continued or stopped. When such criterion is satisfied and after the solution has converged, the vectors of measuring residuals may be obtained from:

$$v = E, \qquad \overset{o}{v} = \overset{o}{E}$$

in which E, $\overset{o}{E}$, denote the final discrepancy vectors of the iterative process.
The computation of the a posteriori estimate of the reference variance is similar here to the other cases. The quadratic form in the present case, however, is formed of two parts corresponding to the division of variables involved.
The estimate of the reference variance would be:

$$(2.16) \qquad \sigma_o^2 = \frac{v^T W v + \overset{o}{v}^T \overset{o}{W} \overset{o}{v}}{(2n + m) - 8}$$

Applying the propagation principle to equation (2.15), it can be readily shown that the cofactor matrix of PTP is (5):

$$(2.17) \quad Q_{\Delta\Delta} = (N+\overset{o}{N})^{-1} \quad \text{or} \quad Q_{\Delta\Delta} = N^{-1} - N^{-1}\overset{o}{B}^T(\overset{o}{W}^{-1} + \overset{o}{B}N^{-1}\overset{o}{B}^T)^{-1}\overset{o}{B}N^{-1}$$

3. DETERMINATION OF OBJECT PLANE COORDINATES

Equations (2.1) are a relation between the film coordinates measurements x_j, y_j, the unknown object plane coordinates X_j, Y_j, and the parameters a_k, determined above. They can be used for direct computation of these unknowns. Since equations (2.1) are linear functions in X_j, Y_j, we have to solve the system directly:

$$(3.2) \qquad \begin{bmatrix} a_1 - x_j a_7 & a_2 - x_j a_8 \\ a_4 - y_j a_7 & a_5 - y_j a_8 \end{bmatrix} \begin{bmatrix} X_j \\ Y_j \end{bmatrix} = \begin{bmatrix} x_j - a_3 \\ y_j - a_6 \end{bmatrix}$$

Inasmuch as the cofactor matrix of the adjusted PTP (a_k,k=1, 2,...8) is given by:

$$\underset{(8.8)}{Q_{\Delta\Delta}} = \left[\underset{(8.8)}{N + \overset{o}{N}}\right]^{-1} \qquad (3.3)$$

it follows that the relative covariance matrix Q_{CC} of the derived object plane coordinates X_j, Y_j, is given by:

$$\underset{(2.2)}{Q_{CC}} = \underset{(2.8)}{J_a}\ \underset{(8.8)}{Q_{\Delta\Delta}}\ \underset{(8.2)}{J_a^T} \qquad (3.4)$$

in which:

$$\underset{(2.8)}{J_a} = \frac{\partial (X,Y)_j}{\partial(a_1,a_2 \ldots a_8)} = \begin{bmatrix} \frac{\partial X_j}{\partial a_1} & \frac{\partial X_j}{a_2} & \cdots & \frac{\partial X_j}{a_8} \\ \frac{\partial Y_j}{\partial a_1} & \frac{\partial Y_j}{\partial a_2} & \cdots & \frac{\partial Y_j}{\partial a_8} \end{bmatrix} \qquad (3.5)$$

where X_j, Y_j, are the object plane coordinates expressed by (2.10) and derived from (3.2).
A computer program called SEISM was developped by the author and its formulation is based on the principle of observation equations as described in the above paragraph.

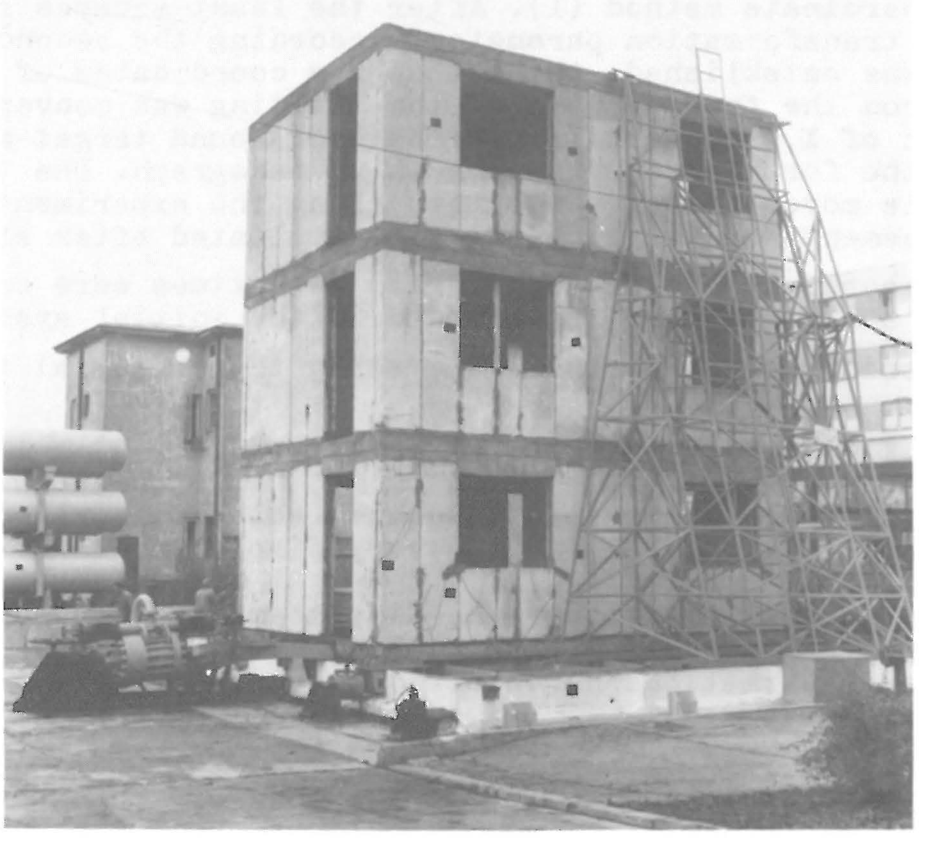
4. TESTS AND PRACTICAL EXPERIENCES

The SEISM program has been tested with fictious data for a single photograph. In essence the input to this program consists of the film measurements as reduced by the phototrilaterated coordinates (1), and corrected for symmetric radial and decentering distortions (2),(3). The input to this program consists also in the geodetically determined X,Y coordinates of a few control points and the straight-line distance from the end points of a few lines.
The primary purpose of this series of tests was to evaluate the computational accuracy of the SEISM solution. Hence, unperturbed image coordinates were used as input data in all tests and the coordinates were assigned a standard deviation of $\pm$ 3 micrometers. All of the rectangular control points were also weighed with a standard deviation of $\pm$ 0,5 mm in X,Y coordinates. Unperturbed data were also used for distance measurements, but they were assumed to have first-order accuracy and were weighed with $\pm 5\text{x}10^{-6}$ meter. Furthermore, the approximations to the PTP were made with $\pm$ 10%.
Because the exact object coordinates of all n object points evaluated in the foregoing paragraph were known, the accuracy of the SEISM solution can be evaluated directly by comparing the computed point coordinates with the corresponding known values. Various configurations of controls were used for the photography which was generated at a scale of 1:60 - 1:100. The results show that the SEISM solution has computational accuracy and sensitivity relative to the accuracy of the measured data.
The standard errors of the computed coordinates are involved between $\pm$ 0.005 mm and $\pm$ 0.03 mm. Likewise, the control points and the distances should be well distributed across

the area to be resected, to avoid excessive errors due to extrapolation outside of the periphery established by the control coordinates. A rejection technique was established and was proved quite adequate. A rejection criteria C_r is calculated, where:

$$(4.1) \quad C_r = 1,5\left(\sum_{i=1}^{m} |\Delta S_i| / m\right), \text{ in which } \quad S_i = E_i^o$$

and E_i^o was evaluated after the final corrections have been applied to the elements of PTP.
The above method was performed under operational conditions, during a simulated earthquake on the seismic platform of 150 tons(Fig. 1).



A control net was established in the background plane. This net provided the necessary means for accurate reconstruction of the bundle of projection rays without the need for information on the interior orientation of the camera (4),(6).
In order to determine the position of the points on the building, it was also necessary to attach a system of targets to the building.
The camera used in this project was a TELEMAR 130x180 mm,with a focal length of 400 mm. The camera which is normally designed for infinity setting, was refocused to operate at a distance of approximately 25 m. This was accomplished by inserting a 6,5 mm shim in front of the focal plane. This modifi-

cation made it necessary to recalibrate the camera in order to determine the corresponding distortion properties. This was accomplished by applying the analytical plumb line calibration (3),(4). The camera was modified as well in cinephototheodolite with a speed of 8 frames per second. The camera was located at 32 m from the system of reference targets and its optical axis was perpendicular to both planes, the background plane and the plane of the frontal wall of the building. In figure 2 are presented two consecutive frames taken at 0,125 second intervals, representing the movements of the building during the simulated earthquake on the platform.
The photogrammetric measurements were made on the liniar spectrophotometric IZA-2 comparator and the image coordinates x,y were obtained by the least squares using a trilaterated photocoordinate method (1). After the least squares solution of the transformation parameters according the second paragraph was established, the set of x,y coordinates of the targets from the frontal wall of the building was converted into the set of X,Y coordinates of the background target system, using the formulation from the third paragraph. Due to the possible movements of the camera along the experiment, the displacements of the building were evaluated after all the X^t, Y^t coordinates corresponding to t-th times were calculated as above and were transformed in the initial system $\overline{X}^t, \overline{Y}^t$(prior to the earthquake), using the conformal transformation:

$$(4.2) \quad \overline{X}_j^t = (a_o + a_1 X_j - b_1 Y_j)^t; \quad \overline{Y}_j^t = (b_o + b_1 X_j + a_1 Y_j)^t$$

The misclosures in the least squares solution applied on equations (4.2), represent the very displacements of the targets.
In the final computations, the displacements were scaled using the subtense bar measurements.
Detailed informations on this research project are given in (1).

5. CONCLUSIONS

The results presented in this paper indicate that precise geometrical informations can be obtained from the photographs taken along a simulated earthquake. The obvious advantage of this method is that only a few control data are used to calculate the PTP coefficients. No internal camera or external exposure station parameters are needed as input data. The use of straight-line distances as control data, enhances the value of this method.
The author express his thanks to the ICCPDC-Iassy for the possibility to try experiments upon the seismic platform, and to the Computation Center of Polytechnical Institute.

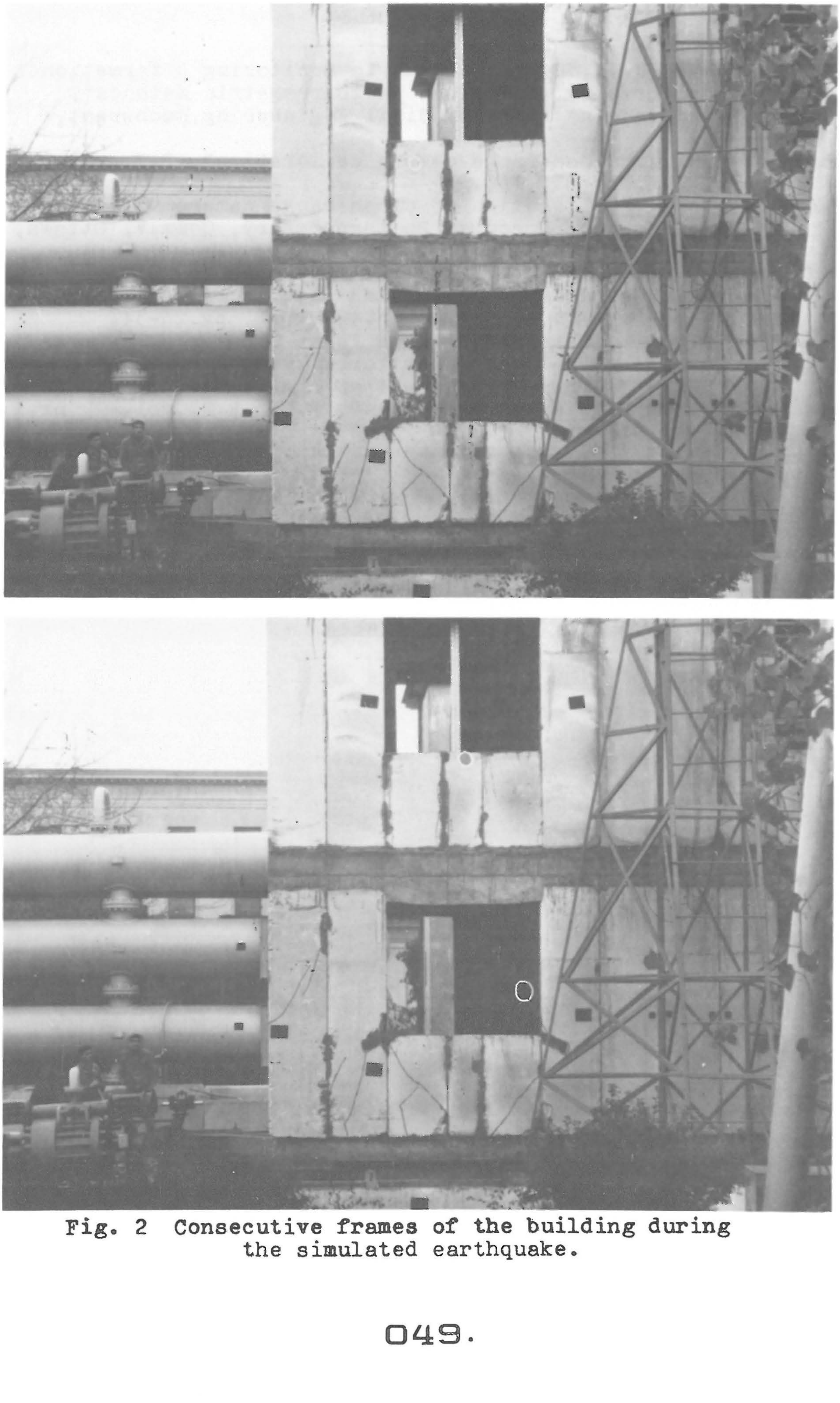


Fig. 2 Consecutive frames of the building during the simulated earthquake.

REFERENCES

1. Barbalata C.I.,"Contributions to monitoring deformations of structures by analytical photogrammetric methods", Ph.D.Thesis, Institute of Civil Engineering,Bucharest, 198 pg.
2. Brown D.C.,"Close-range camera calibration", P.E.,vol.37, nr.8, 1971, pg.855-866.
3. Brown D.C.,"Calibration of close-range cameras", XII-th International Congress of Photogrammetry, Comm.V, Ottawa, 1972, 26 pg.
4. Kratky V., Wijk M.C.,"Photogrammetry used to determine the motion of a vehicle crashing into a highway barrier", Bul.de la Soc.Fr. de Phot., Nr.42, 1971, pg.23-34.
5. Mikhail E.M.,"Observations and Least Squares", IEP - A Dun-Donnelley Publisher, New York, 1976.
6. Wijk M.C., Ziemann H.,"The use of non-metric cameras in monitoring high speed processes", P.E.& R.S.,vol.42, nr.1, 1976, pg. 91-102.