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FREE NET ANALYSIS OF STORAGE TANK CALIBRATION

ABSTRACT

Storage tank calibration by stereophotogrammetry is analysed following free net adjustment principles. A free net constraints elimination method, developed by the second author, combined with scale and leveling constraints are applied to the solution of tank-wall-point coordinates.
The results are free of distortions imposed normally by "hard point" adjustment, while their covariance matrix serves to correctly estimate the tank volume accuracy achieved in the process.

1. INTRODUCTION

Close range photogrammetry is similar to classical topographic photogrammetry in many respects, in particular when the solution is carried out analytically. Aerial triangulation by mono - or stereocomparator measurements and subsequent bundle adjustment call for much the same treatment as in a large number of engineering problems treated by close range photogrammetry.

An important and fundamental difference between the two photogrammetries is the definition and realization of datum. In topographic photogrammetry the datum is realized through the superior in accuracy geodetic control points which are kept fixed or are heavily weighted in the solution. In engineering close range photogrammetry, control is provided usually by precise measurements which are of a distinctly differential nature. Distances and elevation differences are measured by precise surveying methods between points which appear on the photographs. Occasionally, differences between linear exterior orientation elements of the camera stations are measured as partial control. There are cases where some of the angular exterior orientation elements can be also determined.

If the end result of the adjustment are the coordinates of points in object space there is invariably a need in close range photogrammetry to "complete" the datum by assigning more or less arbitrarily weights to a number of points or to exterior orientation elements. If the number of those complementary datum point coordinates or orientation elements is kept to a minimum (minimum constraints) the adjustment is acceptable, and no distortions are introduced into the estimated parameters although their covariance matrix depends on the particular choice of the datum quantities. If the constraints are not minimal, the solution is overconstrained, the sum of squares of the residuals is invariably larger and there are definite distortions introduced by the adjustment process into the adjusted quantities. The measurements which are weighted by the inverse of their variances have to accomodate to the more or less arbitrarily chosen datum hard points.

Our conclusion is that as far as datum is concerned, close range photogrammetry has a built-in deficiency as compared to topographic photogrammetry. It can't rely for datum and for correction of systematic residual errors on geodetic control.

Another characteristics which has to be considered is that in close range photogrammetry we are usually interested in relative positions of points i.e. we can be satisfied with partial datum which includes scale and sometimes also the direction of the vertical. However as we may be interested in using adjustment procedures (and computer programs) which solve for point coordinates rather than for their differences there is a straightforward solution to the datum problem in close range photogrammetry: by applying free net adjustment principles together with whatever differential control measurements have been made.

As a medium for applying and demonstrating our free net approach we chose the problem of calibration of storage tanks. As reported in [6] we have been using in the past years close range stereophotogrammetry for calibration of storage tanks, although so far,datum has been introduced by fixing or heavily weighting certain points or certain exterior orientation elements.

In this paper we present the ideas, mathematics and adjustment procedures developed for the solution of tank calibration by free net adjustment. It should be clear, however, that the same approach could, and to our opinion should be applied to other close range photogrammetry problems. Thus the built-in datum problem of close range photogrammetry can be turned to its advantage by providing a superior means for filtering out measurement errors and for obtaining a more realistic covariance matrix of the estimated quantities.

2. ADJUSTMENT OF FREE NETWORKS.

The basic property of a free net adjustment is that the trace of the covariance matrix of the estimated parameters is a minimum. There are two approaches for the solution of a free net: the first is based on generalized matrix algebra as in [1],[3] and [5] while the second approach is based on classical adjustment methods as in [4] and [11]. The method presented in this section and applied by us for the solution of our close range photogrammetry problem belongs to the second group and has been published in [7],[8] as free net constraint elimination method.

2.1 The linearized observation equations for the solution of point coordinates in three dimensional (3-D) space is :

$$\underset{n\times 1}{V} = \underset{n\times m}{A} \cdot \underset{m\times 1}{X} - \underset{n\times 1}{L} \tag{2.1}$$

Denoting the rank of A by R(A) we write

$$R(A) = m - d = r$$

where d is the rank defect of A. In our case d represents the number of datum quantities needed to define the network in 3-D space.

The vector X can be partitioned into $\underset{r\times 1}{X_1}$ and $\underset{d\times 1}{X_2}$ in a way such that X_2 is a set of parameters of size d which complements the datum definition of the net. It should be pointed out that the partitioning X_1 X_2 is not unique i.e. there are many X_2 sets in X which could fulfil the datum definition. Equation (2.1) can be written now as follows :

$$V = [A_1 \; A_2] \cdot \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} - L \qquad (2.1')$$

As is well known (see [2]) an unbiased estimate of parameters in an adjustment process is possible only if the A matrix of the observation equations is of full rank, i.e. d=0. If d≠0, an unbiased solution can be obtained only for r=m-d parameters. The above can be achieved by transforming the original observation equations through the introduction of a linear relationship between X_1 and X_2 as for example.

$$X_2 = G_1^T \cdot X_1 \qquad (2.2)$$

which is equivalent to

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} I \\ G_1^T \end{bmatrix} \cdot X_1 \qquad (2.3)$$

We substitute (2.3) into (2.1') and obtain

$$V = [A_1 \; A_2] \cdot \begin{bmatrix} I \\ G^T \end{bmatrix} \cdot X_1 - L \qquad (2.4)$$

written also as

$$V = \overset{*}{A} \cdot X_1 - L \qquad (2.5)$$

where $\overset{*}{A} = A_1 + A_2 \cdot G_1^T$

and $R(\overset{*}{A}) = r$ - a full rank.

It should be pointed out that (2.1') together with (2.2) form in effect a case of observation equations with conditions between the unknowns.

As shown in [5] the condition of minimum trace of the covariance matrix of X is equivalent to the condition of $X^T \cdot X$=min. One way of obtaining a minimum for $X^T X$ is by defining the matrix G_1^T as shown in [9]

$$G_1^T = [A_2^T \cdot A_1] \cdot [A_1^T \cdot A_1]^{-1} \qquad (2.6)$$

from which we'have also

$$A_2 = A_1 \cdot G_1 \tag{2.6'}$$

Sibstituting (2.6') into (2.5) we obtain finally

$$V = \bar{A} \cdot X_1 - L \tag{2.7}$$

where $\bar{A} = A_1 \cdot [I + G_1 \cdot G_1^T] = A_1 \cdot S$

The unbiased estimate of X_1 and its weight coefficients matrix are obtained from

$$\begin{aligned} \hat{X}_1 &= (\bar{A}^T \cdot P \cdot \bar{A})^{-1} \; (\bar{A}^T \cdot P \cdot \bar{A}) \\ Q_{11} &= (\bar{A}^T \cdot P \cdot \bar{A})^{-1} = S^{-1} \cdot (A_1^T \cdot P \cdot A_1)^{-1} \cdot S^{-1} \end{aligned} \tag{2.8}$$

According to (2.2) X_2 and its weight matrix are obtained following the solution of X_1 and Q_{11} from

$$\begin{aligned} \hat{X}_2 &= G_1^T \cdot X_1 \\ Q_{22} &= G_1^T \cdot Q_{11} \cdot G_1 \end{aligned} \tag{2.9}$$

The problem treated above is a particular case of a more general situation where we seek minimum for only a part of the $X^T \cdot X$ sum. That whould mean partitioning X_1 into X_{11} and X_{12} so that the minimum condition applies to the following sum:

$$X_{12}^T \cdot X_{12} + X_2^T \cdot X_2 = \min$$

The new form of the observation and condition equations would be (see also [9])

$$\begin{bmatrix} V \\ 0 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_2 \\ 0 & \overset{*}{G}{}_1^T & -I \end{bmatrix} \cdot \begin{bmatrix} X_{11} \\ X_{12} \\ X_2 \end{bmatrix} - \begin{bmatrix} L \\ 0 \end{bmatrix} \tag{2.10}$$

The $\overset{*}{G}{}_1^T$ matrix is treated in the following subsection. Considerations which could quide us in the partitioning of X_1 are discussed in section 3 as well as in [9] and in [12].

2.2 Equation (2.6) can be regarded as a general method for computing G_1^T. In cases where the parameters X are corrections to point coordinates in 3-D space there is another approach for the evaluation of G_1^T or rather $\overset{*}{G}{}_1^T$ which is geometrically meaningful (see also [4],[5] and [9]).

According to Meissl [3] a free network can be obtained from a given arbitrary network by a Helmert transformation, consisting of three translations, three rotations and a scale change which are all differentially small. The Helmert transformation matrix for a network of u points and all seven degrees of freedom is as foolows :

$$C^T = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & \cdots & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & \cdots & 1 & 0 & 0 & 1 \\ 0 & -z_1 & y_1 & 0 & -z_2 & y_2 & 0 & \cdots & y_{u-1} & 0 & -z_u & y_u \\ z_1 & 0 & -x_1 & z_2 & 0 & -x_2 & z_3 & \cdots & -x_{u-1} & z_u & 0 & -x_u \\ -y_1 & x_1 & 0 & -y_2 & x_2 & 0 & -y_3 & \cdots & 0 & -y_u & x_u & 0 \\ x_1 & y_1 & z_1 & x_2 & y_2 & z_3 & x_3 & \cdots & z_{u-1} & x_u & y_u & z_u \end{bmatrix} \quad (2.11)$$

The first three rows are associated with translations along the x,y,z axes, respectively; the next three - to rotations around the x,y,z axes, respectively and the last row - to scale change.

According to Meissl [3] a free net adjustment in all seven degrees of freedom satisfies the condition $C^T X=0$ which when added to the original observation equations (2.1) results in the following adjustment system.

$$\begin{bmatrix} V \\ 0 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_2 \\ C_{11}^T & C_{12}^T & C_2^T \end{bmatrix} \cdot \begin{bmatrix} X_{11} \\ X_{12} \\ X_2 \end{bmatrix} - \begin{bmatrix} L \\ 0 \end{bmatrix} \quad (2.12)$$

The number of rows taken from the full C^T matrix (2.11) is equal to the defect d of the equations (2.1) and their identity is determined by the particular degrees of freedom of the network. We note also that C^T is partitioned into C_{11}^T which is set to zero, C_{12}^T and C_2^T related to X_{12} and X_2 respectively. C_2^T is square and nonsingular. Its columns are shosen so that they can remove the rank defect of the original observation equations. The second row of equations in (2.12) can be multiplied from the left by the negative inverse of C_2^T the result being identical to (2.10)

where $$\overset{*}{G}{}_1^T = -(C_2^T)^{-1} \cdot C_1^T \quad (2.13)$$

As shown in section 3 we have chosen this approach for forming our $\overset{*}{G}{}_1^T$ matrix. The free net constraints elimination method can be characterized by its flexibility in allowing us the choice of leaving out part of the $X^T X$=min condition, by the ease of forming the $\overset{*}{G}{}_1^T$ matrix and finally by the reduction in the overall size of the normal matrix to be inverted : (m-d)x(m-d) instead of (m+d)x(m+d).

3. TANK CALIBRATION BY CLOSE RANGE PHOTOGRAMMERY.

We have reported in [6] on the procedure employed by us for determining the size and shape of a cylindrical storage tank. At the outset of this section and for the sake of completeness we bring a few geometrical aspects of the photography and the fifferential control measurements performed on the site.

The photographs were taken by a wide angle metric camera ZEISS-TMK ($f \approx 61mm$) from 12 stations located symmetrically with respect to the tank bottom center, where the camera principal axis was set horizontally and pointing towards the tank center. Thus the tank was photographed from the inside by a strip which closed on itself. The bottom center point appeared on all the photographs.

At a height of 1.30m. above the bottom,12 points were marked on the wall. In addition and at heights of up to 14m. above the bottom,160 more points were selected to represent the wall surface. Their coordinates were the objective of the calibration, so that horizontal cross section areas at various levels could be calculated to serve subsequently for preparing the calibration tables.

Control measurements were performed in the form of distances measured between varions pairs of the 12 lower level points and also the tank bottom center. In addition, the same 12 points and the center point were precisely leveled.

As indicated in section 2 and also in [3] a network in 3-D space has 7 degrees of freedom. If the above control measurements are incorporated in the adjustment system as observations with certain weights corresponding to their variances, we can easily see that part of the quantities needed to define the datum are provided by the control measurements:

Scale is defined by the distances measured between the lower level points.
Orientation in space of the Z (vertical axis is defined by the leveling measurements of the same lower level points.

Thus the ballance of datum quantities (d) still needed to define completely the network in space is as follows:

3 quantities for defining the datum origin.
1 quantity for defining the orientation of the X-Y axes (a rotation arround Z).

According to the above we can easily select the relevant four rows from the Helmert transformation matrix C^T i.e. rows 1,2,3 and 6. We decided to apply the free net condition ($X^TX=min$) to the 13 lower level points only from practical considerations associated with computer programming.
Due to the control measurements between the 13 points, the portion of the normal matrix pertaining to their coordinates is a full matrix and so the folding-in technique employed for the rest of the (u-13) points (see section 4) can not be applied. By limiting the application of free net constraints to the same 13 points the size of the matrix to be inverted (39x39) did not increase but rather was reduced in size down to (35x35).

The observation and condition equations of our problem are written following partitioning of the unknowns as suggested in section 2 above.

$$\begin{bmatrix} V_1 \\ V_2 \\ 0 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ 0 & 0 & A_{23} & A_{24} \\ 0 & 0 & \overset{*}{G}{}_3^T & -I \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} - \begin{bmatrix} L_1 \\ L_2 \\ 0 \end{bmatrix} \qquad (3.1)$$

where

row 1 reperesents the observation equations of the comparator measurements to which a weight matrix P_1 is assigned,
row 2 represents the control measurement observation equations - with P_2 weight matrix,
row 3 represents the free net conditions as applied to X_3 and X_4 only.

X_1 - are corrections to the 6xp exterior orientation elements of the p camera stations.
X_2 - are the 3x(u-13) corrections to point coordinates.

X_3 - are the (39-4) corrections to the 13 point coordinates on which the X^TX=min condition is applied.
X_4 - are the 4 preselected datum definition quantities, namely y_{u-1}, x_u, y_u, z_u.

The first step in the solution is the transformation of the system from biased (d=4) into an unbiased (full rank) system as follows:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & \bar{A}_{13} \\ 0 & 0 & \bar{A}_{23} \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} - \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \qquad (3.2)$$

where $\bar{A}_{13} = A_{13} + A_{14} \cdot \overset{*}{G}{}_3^T$

and $\bar{A}_{23} = A_{23} + A_{24} \cdot \overset{*}{G}{}_3^T \qquad (3.2')$

The normal matrices are formed

$$\begin{bmatrix} A_{11}^T \cdot P_1 \cdot A_{11} & A_{11}^T \cdot P_1 \cdot A_{12} & A_{11}^T \cdot P_1 \cdot \bar{A}_{13} \\ A_{12}^T \cdot P_1 \cdot A_{11} & A_{12}^T \cdot P_1 \cdot A_{12} & 0 \\ \bar{A}_{13}^T \cdot P_1 \cdot A_{11} & 0 & \bar{A}_{13}^T \cdot P_1 \cdot \bar{A}_{13} + \bar{A}_{23}^T \cdot P_2 \cdot \bar{A}_{23} \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} A_{11}^T \cdot P_1 \cdot L_1 \\ A_{12}^T \cdot P_1 \cdot L_1 \\ \bar{A}_{13}^T \cdot P_1 \cdot L_1 + \bar{A}_{23}^T \cdot P_2 \cdot L_2 \end{bmatrix}$$

(3.3)

Following the solution of X_1, X_2, X_3 and the respective covariance matrices (see section 4) X_4 is evaluated from $X_4 = \overset{*}{G}{}_3^T \cdot X_3$.

4. COMPUTER PROGRAMMING.

The specific case of tank calibration discussed in this section consists of 12 photographs, a total of 166 wall and one bottom center points and 30 control measurements. The normal equations arrived at the end of section 3 (3.3) can be written as follows:

$$\begin{matrix} 72 \\ 462=3x154 \\ 35 \end{matrix} \quad \begin{bmatrix} N_{11} & N_{21}^T & N_{31}^T \\ N_{21} & N_{22} & 0 \\ N_{31} & 0 & N_{33} \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} \qquad (4.1)$$

where : $N_{11}, N_{33}, N_{21}, N_{31}$ are in general full matrices

and N_{22} is a block diagonal matrix with 3x3 blocks and quite large overal dimensions - (462x462), which is stored in the computer memory in the form of 154 (3x3) matrices.

The comparator and control measurements are read and processed sequentially and their contribution to the appropriate portions of the normal matrix and the vector of constants is summed up. Measurements which involve any of the 13 points (X_3, X_4) are treated according to (3.2') before their contribution is added in the appropriate slots.

The solution of the normal system is carried out in 3 steps:

Step 1 : folding-in of the X_2 unknowns.

$$\begin{aligned} \bar{N}_{11} &= N_{11} - N_{21}^T \cdot N_{22}^{-1} \cdot N_{21} \\ \bar{U}_1 &= U_1 - N_{21}^T \cdot N_{22}^{-1} \cdot U_2 \end{aligned} \qquad (4.2)$$

Due to the block diagonal structure of N_{22} the fold-in operation is performed on a point-by-point basis:

$$\begin{aligned} \bar{N}_{11} &= N_{11} - \sum N_{21_i}^T \cdot N_{22_i}^{-1} \cdot N_{21_i} \\ \bar{U}_1 &= U_1 - \sum N_{21_i}^T \cdot N_{22i}^{-1} \cdot U_{2i} \end{aligned} \qquad (4.2')$$

At the end of step 2 the normal system is reduced to

$$\begin{bmatrix} \bar{N}_{11} & N_{31}^T \\ N_{31} & N_{33} \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_3 \end{bmatrix} = \begin{bmatrix} \bar{U}_1 \\ U_3 \end{bmatrix} \qquad (4.3)$$

Step 2 : Folding-in of the X_3 unknowns.

$$\begin{aligned} \bar{\bar{N}}_{11} &= \bar{N}_1 - N_{31}^T \cdot N_{33}^{-1} \cdot N_{31} \\ \bar{\bar{U}}_1 &= \bar{U}_1 - N_{31} \cdot N_{33}^{-1} \cdot U_3 \end{aligned} \qquad (4.4)$$

where the result is the normal system of the exterior orientation elements

$$\bar{\bar{N}}_{11} \cdot X_1 = \bar{\bar{U}}_1 \qquad (4.5)$$

Step 3 : Solution of the unknowns in the order X_1, X_2, X_3 and X_4 including their weight coefficients matrices Q_{11}, Q_{22}, Q_{33} and Q_{44}. Because of the point-by-point fold-in operation in step 1 only the 154x(3x3) diagonal submatrices of Q_{22} are kept in the computer memory. Thus correlations between different points are not evaluated.

The adjustment procedure described above was programmed in FORTRAN on an IBM 370-168 computer. The overall size of the program for processing up to 12 photos and up to 170 points with an unlimited number of measurements was 628K bytes. An average run-time for one iteration of the solution with about 1200 comparator and 30 control measurements was 30 seconds.

5. CONCLUSIONS

As demonstrated for the case of storage tank calibration, close range photogrammetry combined with precise surveying measurements and processed by free net adjustment techniques is an extremely powerful tool which can and should be applied to a wide range of engineering problems.

The residuals of a free net adjustment can best disclose the existance of certain unmodeled systematic effects. Additional studies should be conducted on the application of free net principles for the solution of photogrammetric systems which include self-calibration parameters. It appears that high correlations between parameters or their combinations could be effectively treated by free net adjustment principles.

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