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MODELLING OF PHOTOGRAMMETRIC MEASURING SYSTEMS

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ABSTRACT: Classification of measuring systems and their applications to photogrammetric purposes are discussed in the paper; a mathematical model of a general form is given. Mathematical formulae enabling one to reveal relationship between the result of measurements, parameters of measuring system and kinematics errors are suggested. The examples of the said technique application to mono-and stereocomparators to optimise their structural scheme are presented in the paper.

There are mathematical relations describing functional results of measurements together with constructive features of instruments, as well as with its kinematic errors. Those are necessary to invent and examine photogrammetric and other measurement systems.

The main principles of measurement systems invention are:

- principle of orthoscopic projection aimed to make the vector of object indication (which is a sighting axis) orthoscopic or approximately orthoscopic to the coordinate planes of the measured object;
- principle of more general kind, applied when the vector of object indication is not orthoscopic to the coordinate planes of the measured object at all.

Modern stereo-and monocomparators, analytic and automatic stereoinstruments are based on the first principle. Such different measurement systems as a theodolite and a stereoplanigraph are based upon the second principle.

Differing from the well-known methods of structural and kinematic analysis, where each kinematic pair is regarded as an errorless one, we elaborated another approach, given below.

To solve the simulation problems, it was suggested to replace a number of kinematic pairs necessary for translational and rotational movements of an object in a selected direction with single "enlarged" kinematic pair of a general type. The enlarged kinematic pair ought to have one angular or linear movement of a mobile link in space, while all the rest five degrees of movement can be considered as small attendant errors of the movement. It allows to determine all the necessary requirements for the designed assembly and at the same time it improves the stipulation of the equations.

Kinematic chains of the instruments were built from the enlarged kinematic pairs forming measuring assemblies. It is assumed that each measuring assembly has two constrained kinematic chains. For example the first one renders relative movement of the indicational axis and the object in one direction, and the second one allows to measure this movement. The latter has a closed link, which is a component vector of the measured value.

The introduced notion of "measuring assembly" facilitates structural analysis of measurement systems in the instruments. It is a base for a new classification of measurement systems.

Let us consider the pattern of measurement system of a general type in a basic coordinate system reffered to the instrument's bed (Fig.1).

The main components of the pattern are:

- the measurement object, which is radius-vector $K_{\eta e}$;
- the identification and sighting devices, including sighting axis-object indicational axis; it is simulated by an indicational object's vector $Q_{\xi K}$;
- the devices for movement of objects and indicational axis; it is simulated with enlarged kinematic pairs; they are described by coordinate system's guiding lines, accordingly indices am (m from 1 to l-1) are used for the objects while bn (n from 1 to k-1) - for indicational axis. Movement

of the mobile elements is simulated with radius-vectors $K_{\eta m}$ and $Q_{\xi n}$ for translational movements, as well as Eiler matrices for transformation of coordinates $\Pi_{\eta m}$ and $\Pi_{\xi n}$;

- the working values are simulated by coordinate systems with diacritical marks "c";
- the devices of data transform on object or indicational system movements include kinematic chains; they are concluded by index readings on object position in working measure values. They are simulated by U^{o} vectors. The diacritical mark o means the initial position, when indicational axis of object is directed to the beginning of object sector $K_{\eta e}$.

Having complete the transitions of object and axis of indication by means of opportune kinematic pairs, one brings the end of object's indication vector $Q_{\xi K}$ into a coicidence with the end of vector of $K_{\eta e}$ object, while ends of indicational readings vektor U would indicate the components of the measured value along coordinate axes- working measures.

Let us consider the equations of simulation of general type system. A chain, consisting of measuring links from the instrument bed to one of the objects would be named a branch of the measurement system. Each branch consist from two semi-branches:

a - it is related to the measurement object movements;

b - it is related to the movements of axis of object indication.

One branch of a measurement system is shown in Fig.1. The initial position of the system is described by a vector equation

$$Q^{o}_{\xi K} = T_e - R_K$$

The main condition of measurement is a condition when direction of indicational axis passes through the extreme point of object's vector having complete the measurement movements (Fig.2):

$$\overrightarrow{O_e A'} = R^{\Sigma}_{A'} = T^{\Sigma}_{A'}$$

This condition is a result of orthoscopic transformations. In kinematic chains of the semi-branches it can be written as follows:

$$\sum_{n=1}^{n=K} \Pi_{\xi(n-1)} \cdot (R_n - R_{(n-1)} + \Pi_{bn(a_1)} \cdot Q_{\xi n}) = R^{\Sigma}_{A'} = T^{\Sigma}_{A'} = \sum_{m=1}^{m=e} \Pi_{\eta(m-1)} \cdot (T_m - T_{(m-1)} + \Pi_{am(a_1)} \cdot K_{\eta m}). \qquad (1)$$

In equation (1) $Q_{\xi n}$ and $K_{\eta m}$ are vectors of translational movements where the main coordinate component is directed approximately in a direction, measured by the link, while the rest are small errors attendant to this movement.

$\Pi_{\xi(n-1)}$ or $\Pi_{\eta(m-1)}$ are matrices of Eiler coordinate transformation of the mobile element of kinematic pairs.

If the translational movement is the main one, then Eiler angles can be assumed to be small. For kinematic pairs with main rotational movements the matrices would include Cos and Sin of this angle, or their series, while all the components of translational movements $Q_{\xi n}$ or $K_{\eta m}$ are assumed to be small errors. Π bn(a_1) and Π am(a_1) are matrices of Eiler transformation, needed to bring the coordinate systems of motion-

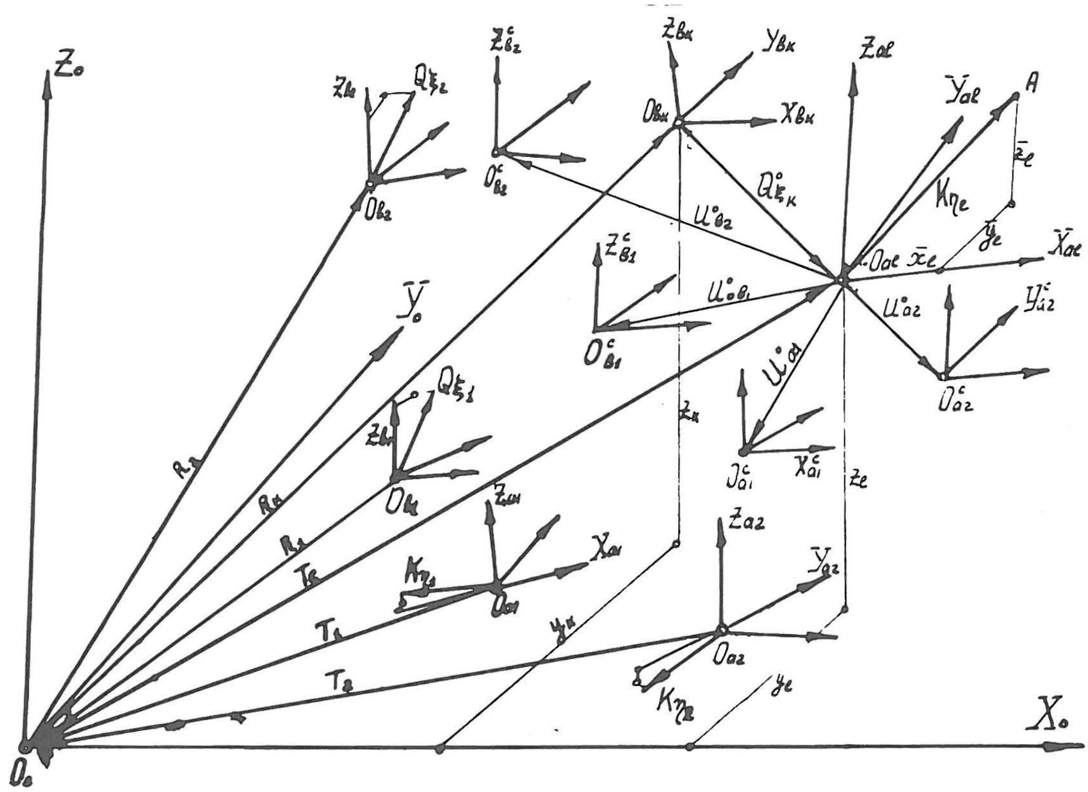

Fig.1. Simulation of general type measurement system

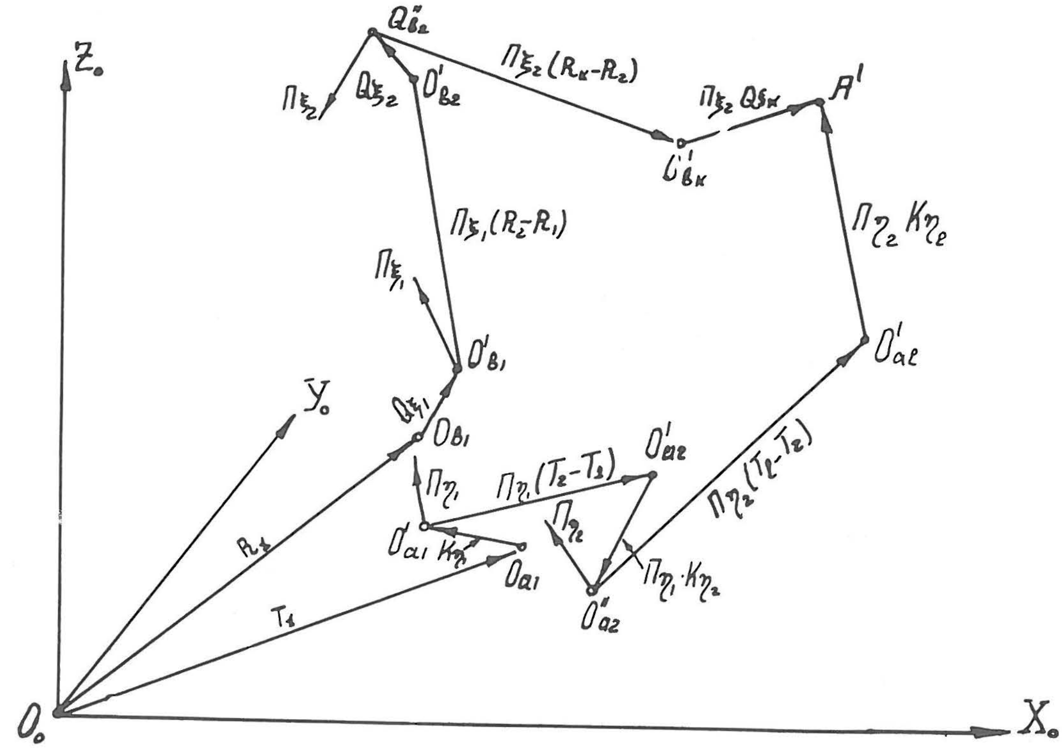

Fig.2. Vector diagram describing the main conditions of measurement

less elements to the basic system (in equation(1) a_1 system is accepted as a basic one).

Solution of equation (1) renders the values of main components of movement vectors as a function of measurement system parameters.

If n=K the indicational object vector $O^{o}_{\xi K}$ would obtain the colinear augmentation $\Delta Q_{\xi K}$. It would represent, for example, the unknown value of an unfocused optical system. If one assumes object ($K_{\eta e}$) to be the known standard, then three main components of the movements would be obtained from the solution of equation (1) when one measures single object. When two objects are measured, and if there is a common measurement link, then from solution of equation (1) there would be obtained six unknown values.

Let us consider the equations for determination of vectors of measurement correction $D_{\eta m}$ by each measuring link. For the link number m:

$$D_{\eta m} = D^{\text{ист.}}_{\eta m} - D^{\text{изм.}}_{\eta m} \qquad (2)$$

where $D^{\text{ист.}}_{\eta m}$ - is a matrice including one of coordinates of the standard object, and $D^{\text{изм.}}_{\eta m}$ is described by matrices of coordinates of the extreme points of vectors U in the beginning and the end of measurement, obtained by equation(1).

For joint consecutive links from m=1 to m one will obtain.

$$D_{\eta m} = \begin{pmatrix} 0 \\ \bar{y}_e \\ 0 \end{pmatrix} - \Pi'_{\eta(m-1)} \left\{ \Pi_{\eta(m-1)} \Pi_{am(a1)} \cdot K_{\eta m} + \left(\Pi_{\eta m} - \Pi_{\eta(m-1)} \right) \cdot \left(T_e - T_m + \Pi_{am(a1)} U^{o}_{\eta m} \right) \right\} \qquad (3)$$

By a parallel joint of links from 1 to L , when working measures are placed at the bed of instrument, one will obtain:

$$D_{\eta e} = \begin{pmatrix} 0 \\ \bar{y}_e \\ 0 \end{pmatrix} - \left\{ \sum_{m=1}^{m=e} \Pi_{\eta(m-1)} \cdot \left(T_m - T_{(m-1)} + \Pi_{am(a1)} \cdot K_{\eta m} \right) - T_e - U^{o}_{\eta e} \right\} \qquad (4)$$

In equations (3) and (4) Π' (m-1) is a transposed matrix of m-1 link; vector $K_{\eta m}$ contains the main component, obtained from solution of equations (1).

The use of equations (1),(3) and (4) renders solution of the measurement system simulation, where measurement results are represented as components of vector corrections D_η or D_ξ, as a function of parameters of: measurement system, errors of its movements in kinematic chains, and coordinates of the measured object.

The method was tested for systems of mono-and stereocomparators.

For obviousness the results of system analyses are given up to the first order terms only (Table 1).

In optimum solution it turned out necessary to more rigidly follow Abbe's principle in three planes by modification of the design so as:

- working measures would be in horizontal plane and they continually ought to coincide with measurement lines,i.e. parameters $B_x = A_y = 0$;

Parameters and errors inherent in system and their influence on monocomparator measurement system's accuracy [2]

Table 1

Vectors of corrections	Components of vector corrections	Influencing factors: Variances of straight foreward movement of frame	Influencing factors: Turns of frames and position of reading devices	Influencing factors: Turns of frames and position of measurement lines relative the working measures
D_{x_1}	δx_{x_1}	$-\Delta L_x - \alpha_y(\Delta z_{x_1} + \Delta z_{y_2}) + \Delta B_x \cdot \varkappa_{bx}$		$+B_x \cdot \varkappa_y - \alpha_{bx}(C_x - \Delta C_x)$
	δy_{x_1}	$-\Delta B_x + \omega_y(\Delta z_{x_1} + \Delta z_{y_2}) - \Delta L_x \cdot \varkappa_{bx}$	$+(L_x + \bar{x}_{ae})\varkappa_y - \omega_{bx}(C_x - \Delta C_x)$	
	δz_{x_1}	$-\Delta C_x - \Delta L_x \cdot \alpha_{bx} + \Delta B_x \cdot \omega_{bx}$	$+(L_x + \bar{x}_{ae}) \cdot \alpha_y + B_x \cdot \omega_y$	
D_{y_2}	δx_{y_2}	$-\Delta A_y - (\Delta z_{x_1} + \Delta z_{y_2})\alpha_y + \Delta L_y \cdot \varkappa_{bx}$	$+(L_y + \bar{y}_{ae})\varkappa_y - \alpha_{bx}(C_y + \Delta C_y)$	
	δy_{y_2}	$-\Delta L_y - (\Delta z_{x_1} + \Delta z_{y_2}) \cdot \omega_y - A_y \cdot \varkappa_{bx}$		$+A_y \cdot \varkappa_y - \omega_{bx}(C_y - \Delta C_y)$
	δz_{y_2}	$+\Delta C_y + L_y \cdot \omega_{bx} - \Delta A_y \cdot \alpha_{bx}$	$-(L_y - \bar{y}_{ae}) \cdot \omega_y + A_y \cdot \alpha_y$	

- parameters C_x and C_y would be brought to minimum in the vertical planes, so as δx_{x_1} and δy_{y_2} would not to exceed $3 \div 4 \mu m$ in accordance with requirements.

Utilization of the method can be recommended when synthesis (elaboration) and analysis (testing) of photogrammetric measuring systems is carried out.

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