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COMPENSATION OF SYSTEMATIC ERRORS OF IMAGE AND MODEL COORDINATES (REPORT OF WORKING GROUP III/3)

Abstract

An effort has been made to evaluate different approaches in compensating systematic errors (with respect to the functional model applied) in aerotriangulation. The main object of interest has been the comparison of component calibration, test field calibration, and self calibration.

The study has been carried out empirically. Rather extensive test field data have been applied. Several institutions have participated in the measuring and computing work. The report summarizes the results obtained.

1. INTRODUCTION

The development of the methods of aerial triangulation in a more rigorous direction during the past decades has given the systematic errors of observations (with respect to the functional model applied) a decisive role in terms of the accuracy and the reliability of results. In consequence, a number of different approaches to the problem of systematic errors has been introduced in many individual studies. In order to obtain comparative knowledge of the compensation methods, a working group was established within Commission III of the ISP in accordance with a resolution made at the XIIIth Congress of the ISP in Helsinki 1976.

The WG set as its goal to supply more information on the compensation of the systematic errors of image and model coordinates in different aerotriangulation methods. An agreement was made to take into account both accuracy and, as far as possible, economic aspects. It was, also agreed that a comparative study of component, test field and self calibration methods should be carried out using real data, i.e., empirically. Further, the triangulation methods with independent models and bundles were to be the most important methods investigated.

The attainment of the goal of the WG has required a considerable effort of the participants in test field photographies, measurements and computation. The following nine organizations participated in the activities of the WG:

- South Australian Lands Department, Adelaide, Australia
- Laboratoriet for fotogrammetri og landmåling, Aalborg Universitetscenter, Denmark
- Institut für Photogrammetrie, Universität Bonn, FRG
- Institut für Photogrammetrie und Ingenieurvermessungen, Technische Universität Hannover, FRG
- Lehrstuhl für Photogrammetrie, Technische Universität München, FRG
- Institut für Photogrammetrie, Universität Stuttgart, FRG
- Institute of Photogrammetry, Helsinki University of Technology, Finland
- National Board of Survey, Helsinki, Finland
- Geodetical and Geophysical Research Institute of the Hungarian Academy of Sciences, Sopron, Hungary

For planning and coordinating its work, the WG organized the meetings in Stuttgart and Moscow in 1977 and 1978, respectively. In addition, a seminar for reviewing and evaluating the preliminary results obtained was arranged in Aalborg in 1979.

As to the execution of the study, directions for measurements and computation were prepared in order to guarantee uniform and comparable final results. However, the choice and application of triangulation and compensation methods were not touched upon; every participant was given free hands in this respect. This resulted in that all methods were not treated equally. E.g., the method of independent models had to be omitted because of an insufficient amount of results. Likewise, the treatment of component calibration remained inadequate for the same reason. Further, it was found out in the course of the study that a discussion of economical aspects would have been premature in this context, and, consequently, they have not been dealt with in this report.

In closing, it is worth mentioning that, in addition to this report, several papers which refer to WG III/3 have been submitted to this Hamburg ISP Congress. Both the theory of compensation and empirical results are discussed in them.

2. STUDY MATERIAL

2.1 Test field

Photographies over four test fields located in Australia and Finland were used in the studies of the WG (Table 1).

All test field points used as control or check points in photogrammetric adjustments have been coordinated by ground survey. The coordinates have been transformed for this study into a rectangular (cartesian) coordinate system, which makes the corrections for earth curvature and map projection superfluous.

In addition to the coordinated points, on the boarders of the Jämijärvi (large), Willunga and Kapunda test fields there are 100, 28 and 70 additional (uncoordinated) points which enlarge the test fields to cover an area of about 2,6 x 2,6 sq.km, 3 x 3 sq.km and 40 x 40 sq.km, respectively.

Both the coordinated and the additional (uncoordinated) points of the test fields were provided with targets (Figure 1). Generally, it can be stated that the targets of the Australian test fields were well visible in the photographs. As for the Jämijärvi test fields, the identification of the targets was sometimes troublesome.

2.2 Photo material

The photo material used by the WG was quite versatile; there was a fairly large range of image scales and block sizes. Further, both reseau and non-reseau photographies were available (Table 3).

With regard to the photographies, attention should be paid to the following facts:

- In each case the flight program was carried out in one day.
- Strip photographies flown over the Jämijärvi test fields are aimed only for test field calibration.
- For the Jämijärvi mission, the RMK A2 and MRB cameras were mounted in the same aircraft, which resulted in nearly simultaneous photography.
- In the Jämijärvi blocks, side overlap between some strips is only 55 % (10 %), instead of 60 % (20 %). This led to a scarcity of tie points, particularly in blocks with (nominal) 20 % side overlap.

The laboratory procedures applied were those used in standard production. The Kapunda and Willunga photo material was prepared for measurements by the South Australian Lands Department and the Jämijärvi material for measurements by the Finnish National Board of Survey and the Helsinki University of Technology.

2.3 Measurements

Altogether, six institutes participated in measuring the photographies for the WG (Tables 2 and 4). The Australian photographies have been measured for other than WG purposes already in 1975. Therefore, there are some differences in the measuring procedure as compared with that of the Jämijärvi photographies. In the following, a brief description of both procedures is given:

<u>the Kapunda and Willunga photographies (reseau)</u>

- Measurements were made from the original film negatives.
- One pointing was made on both targets and reseau crosses.
- 25 evenly or nearly evenly distributed reseau crosses were observed.

<u>the Jämijärvi photographies</u>

- Observations were made in two rounds.
- Fiducial marks were observed at the beginning of the first round and at the end of the second round.
- 4 and 20 fiducial marks were measured from the photos taken by the RMK A2 and MRB, respectively.
- The use of the ISP standard test was recommended in connection with the measurements, in order to evaluate the accuracy of the measuring instruments used.

3. COMPUTATION

3.1 Compensation methods

The data measured were computed in seven organizations. Each participant was free to choose the data he was interested in and the triangulation and compensation methods he wanted to use in processing them (Table 5). With regard to compensation, the computation carried out can be divided into four groups:

- reference adjustments, and adjustments involving
- component calibration,
- test field calibration, or
- self calibration.

These concepts are briefly defined in the following:

Reference adjustment

Reference adjustment is taken to mean an ordinary adjustment where no special effort is made to compensate systematic errors. It is to serve as a measure when evaluating the effectiveness of compensation. For reference adjustment, the following steps are taken:

1° Affine transformation with 6 parameters on 4 fiducial marks.
2° Correction of mean symmetric radial distortion according to calibration report.
3° Correction of refraction according to Bertram's formula.
4° Weight p = 1 is given to all observations and weight p = ∞ (infinity) to the ground control points.

Each participant has applied reference adjustment to every project parameter combination (overlaps, flight direction, control version) he has used.

Component calibration

Compensation measures have been referred to component calibration, if

1° data refinement differs from that for reference adjustment, and if
2° no other calibration method has been applied.

Test field calibration

In this context, test field calibration includes the cases where the calibration data applied to compensate the systematic errors of a block are determined from

1° a separate test field photography (e.g., strip photographies of the small and the large test field of Jämijärvi), or from
2° a sample of block photos by treating the test field points appearing on these photos as known points (e.g., closeness with respect to time and place).

The latter case tends to simulate an ideal case of test field calibration, in which the test field is situated in the project area.

Self calibration

In general terms, self calibration is a process, in which the compensation of systematic errors of the data is based on information extracted from the same data only, i.e., no external information is required. This definition includes simultaneous self calibration, in which calibration is accomplished by extending the functional model of the triangulation method to cover the effect of systematic errors, as well as a-posteriori self calibration, where compensation is based on the analysis of residuals of observations and the re-execution of adjustment by using improved observational data.

3.2 The practice of computation

Each participant was provided with the following material for computation:

1) Comparator (or model) coordinates

   The institutes which carried out the coordinate measurements were asked to clean them from gross errors by using their own methods. Any further elimination of gross errors in the institutes performing computations should have been avoided to obtain comparable results. However, some institutes did not fully follow this suggested course. It naturally caused some difficulties in the analysis of results.

2) Calibration report

   The aerial camera RMK AR had been calibrated at Carl Zeiss, Oberkochen, three years before the test flights, i.e., in 1972. The cameras MRB as well as RMK A2, in turn, were calibrated at Carl Zeiss, Jena, and at the Helsinki University of Technology respectively just before the flights in 1977.

   Each calibration report contains the following information:

   - coordinates of fiducial marks (or reseau crosses)
   - calibrated focal length, and
   - radial distortion on four semi-diagonals.

   In addition to these, tangential distortion and some other calibration data were determined for the aerial cameras MRB and RMK A2.

3) Object coordinates referring to a rectangular (cartesian) coordinate system.

4) Recommendations for control point patterns.

   Three different control patterns were recommended for blocks (Fig. 2a).

   For test field calibration, also three point-schemes were recommended:

   - version S1: 25 evenly distributed points,
   - version S2: 64 evenly distributed points, and
   - version S3: all available points.

To guarantee a high standard of uniformity, participants were even provided with lists of control points. It was, however, impossible to follow these lists excactly in all cases, but the same pattern form was always preserved.

In addition to the control patterns given in Fig. 2a participants were free to use patterns of their own (Fig. 2b).

As seen from Figures 2a and 2b, control pattern B1 is extremely sparse with regard to height; on the contrary, patterns B3 and B6 are very heavily controlled. Patterns B2, B4 and B5 are in good correspondence with practical applications.

5) Instructions for the presentation of results

The instructions given included formulas for computing the root mean square error (RMSE) at check points (i.e., the discrepancy between geodetic and photogrammetric determination), which serves as the primary means to measure the effectiveness of compensation.

$$\mu_x = m(\sum_{i=1}^{l} (X_p - X_g)_i / l)^{1/2} \quad (\mu_y \text{ and } \mu_z \text{ accordingly})$$

$$\mu_{xy} = ((\mu_x^2 + \mu_y^2)/2)^{1/2}$$

where
- m = nominal scale of photography (cf. Table 3)
- X_p = photogrammetrically determined ground coordinate
- X_g = geodetically determined ground coordinate
- l = number of check points in the block in question

4. RESULTS

4.1 General

In the following the results achieved by the WG will be grouped in accordance with the compensation methods used. In the discussion of the results, attention has been paid to those factors that have a-priori been considered to be important for the compensation of systematic errors or that have proved to be so. When evaluating the results to be presented in this paper, the following facts should be taken into account:

- In this context, only the most evident trends seen in the results have been reported; many interesting details included in the original results have been omitted. They can be found, to some extent, in the separate reports of the different participants.
- The reliability of the findings and trends discussed varies rather widely due to the quantitative and qualitative variation in the original results from which they have been exctracted. E.g. component calibration can not be widely discussed, because there was among the participants only little interest in that method.
- A comparison of the absolute accuracies achieved with the same data in the different institutes is somewhat difficult, because even the results of the reference adjustments differ (against expectations) from each other in many cases.

4.2 Self calibration

The largest part of the results presented by the participants of the WG have been achieved by simultaneous self calibration, i.e., by introducing the additional parameters into the adjustment to compensate for systematic errors and by estimating their values simultaneously with other unknown parameters. Altogether, 14 different parameter sets were used in this study. The formulas of these parameter sets are presented in Appendix A. In the following, some comments (based on the references) are made on their typical features:

- The additional parameters of sets b and h are mutually orthogonal and also nearly orthogonal to the elements of exterior orientation. The mutual orthogonality holds exactly for an even image point distribution of 3 x 3 and 5 x 5 points, respectively.
- Parameters $a_1 \ldots a_{12}$ of set a are nearly orthogonal and also nearly orthogonal to $a_{13} \ldots a_{18}$. Among parameters $a_{13} \ldots a_{18}$ there are a few moderately high correlations.
- Also parameter set j is formulated so that the correlations between the parameters are slight.
- Parameter set n, is the purest example of an error model in which the effect of different error sources are "separately" modelled: in it there are parameters for affinity and non-orthogonality (of the image coordinate system), it includes Brown-Conrady model for decentering distortion and a polynomial of the seventh degree for radial distortion. Accordingly, this model can be referred to as a "physical model".
- The parameters in set c are the coefficients of the function of spherical harmonics of the third degree (with slight modifications: the constant term is excluded and the parameters for affinity and non-orthogonality are included). Trigonometric functions appear also in sets i, j, and k (mixed models).

On the basis of the results achieved by the WG, the following general statements can be made on self calibration:

- The accuracy improvement is on an average 20 %- 30 % as compared with the reference (conventional) adjustment. However, the variation in results is suprisingly wide. At best, the results improved

by 60 % - 70 %, but, on the other hand, in many cases they even deteriorated considerably. Table 7, which has been constructed on the basis of all the results of the WG, gives some idea of the accuracy, obtainable by self calibration.
- Against the expectations the improvements were only slightly better in height than in planimetry.
- No single parameter set has consistently produced the best results. Some examples of the performance of the different parameter sets can be found in Tables 6, 8, 9, 10 and 11.

The following discussion gives closer information on how accuracy improvement depends upon

- the way of treatment of additional parameters,
- the project parameters of the block concerned, and
- a-priori data refinement.

Blockwise versus stripwise parameters

A series of bundle adjustments with the orthogonal parameter set b, when introducing additional parameters common to

(a) all strips,
(b) every other strip (subblock with 20 % side overlap), and
(c) the whole block,

gave the following results:

- If significance testing was not applied the order of superiority was (c), (b), (a) in all cases. Particularly, stripwise introduction of additional parameters (case (a)) gave poor results: the deteriorating effect was often larger than 50 %.
- The application of significance testing clearly brought the results closer to each other, but the order remained the same in most cases.
- The results were in good agreement with expectations: The less controlled the block (control points & overlaps), the poorer the results achieved by stripwise introduction of additional parameters.

Significance testing of additional parameters

Significance tests of additional parameters have been applied in Hannover (parameter sets i and j), Stuttgart (parameter set b) and Munich (parameter sets b and h). The pure effect of testing can be assessed from the results of the first two institutes, as the results achieved without testing have also been reported.

In Stuttgart the following procedure was applied:

1° Significant parameters were determined, by applying the one-dimensional t-test, from a block adjustment, where all ground points were treated as known points.
2° Parameters found significant on the level 99,9 % were then introduced into the block adjustments of the data set concerned.

Due to the orthogonality of the parameters (set b), prerequisites for the application of the one-dimensional t-test obviously exist. However, because of the treatment of all object points as known, the procedure as such is not operational. The results achieved by this procedure turned out to be similar in all blocks adjusted: testing improved accuracy only when additional parameters were introduced stripwisely (Table 12).

In Hannover, significance testing was performed in an ordinary way by applying the one-dimensional t-test and the significance levels 67 % and 95 %. The results were not very consistent (Table 13), which may be an evidence of the strong correlations between the parameters.

Weighting of additional parameters

Weighting of additional parameters has been applied in Aalborg and Helsinki. In both institutes the principle of weighting has been the same: the weights have been so determined that the effect of an additional parameter is of the desired magnitude in a desired position on the image.

In Helsinki the weighting tests have been carried out by using the Willunga data and parameter set a. The weights used for the additional parameters correspond to displacements 1, 5, 10, 30, and 100 µm in the image point x = y = 100 mm. Table 14 shows the results, which are very favourable, in a more detailed way:

- Especially in weak blocks, the effect of weighting is very affirmative. This results, naturally. from an improvement in the condition of normal equations.
- On the contrary, it is important to notice that the results do not deteriorate in heavily controlled blocks.
- On the basis of these tests, it seems evident that the optimum weights correspond to an image point displacement of close to 5 µm.

In Aalborg all self-calibrating adjustments have been performed by applying weights which correspond to a displacement of 3 µm in the image point x = y = 100 mm. From the results reported, it is not possible to separate the mere effect of weighting, but, on very good grounds, it can be suspected that weighting has played an important role, when considering the good results (especially in weakly controlled blocks) obtained.

. Consideration of correlations

In spite of the importance of correlations, frequently expressed, only one participant has treated this question. Table 15 presents the few results. These offer, nevertheless, an example of the importance of the consideration of correlations, and set forth the need for further investigations.

A-priori data refinement

In order to study the effect of a-priori data refinement on the compensation of systematic errors by self calibration, a series of adjustments was carried out in Helsinki. Two data refinement levels were used in the adjustments:

1) as in reference adjustment, and
2) 4-parametric transformation, no a-priori corrections.

Two comments can be made on the results:

- On the whole, there is no consistent difference between the refinement levels; however, remarkable differences can appear in individual adjustments (Table 16).
- Parameter set n ("physical set") produced the best results with non-refined data.

Side overlap

Side overlap proved to be the most effective factor affecting the efficiency of self calibration. With 60 % side overlap the compensation of systematic errors was about twice as efficient as with 20 % side overlap (examples can be found in Table 11). Moreover, the variations of the results were much larger with 20 % side overlap. In the Jämijärvi blocks this phenomenon was accentuated obviously by the shortage of tie points (failure in navigation), but the same phenomenon appeared also in the Willunga blocks, which are geometrically very regular (five tie points per photo in the side-overlapping zone).

Single versus double (cross-flight) block

In accordance with expectations, the accuracy improvements obtained by self calibration in double (cross-flight) blocks were clearly smaller than in single blocks (Table 17). Naturally, this follows from the fact that a great part of the systematic errors has already been compensated due to the flight arrangement (cf. reference adjustments without any additional parameters). E.g., in the case of the blocks appearing in Table 17, the accuracy is in the double block by about 50 % in planimetry and 60 % in height better than in the respective single blocks.

Control point pattern

On the basis of the results achieved by the WG, the dependence of compensation and the resulting accuracy on the control pattern can be expressed as follows:

- In blocks with 60 % side overlap, the accuracy in planimetry is only a little dependent on the control pattern, that is, the weaker the control, the larger the accuracy improvement obtained by self calibration. With 20 % side overlap, compensation is still more effective in sparse blocks, but is not capable of making the final accuracy equal to the accuracy of dense blocks.
- In height, accuracy improvements were 25 % - 50 % in sparsely or moderately controlled blocks (e.g., B1, B2, B4, and B5) and 15 % - 20 % in heavily controlled blocks (e.g., B3, B6). In other words, self calibration levelled the accuracy differences somewhat, but, contrary to planimetry, the final accuracy was still dependent on the control pattern.

4.3 Test field calibration

In all, the results of appr. 120 adjustments were available for evaluating the efficiency of test field calibration. They all originate from Bonn and Munich. Regarding calibration procedures, it is worth noticing that in both institutes the data used for test field calibration have been refined as in reference adjustment. On the contrary, the procedures differ from each other in that in Bonn significance testing was not applied, while in Munich the one-dimensional t-test (significance level 95 %) was applied in including the parameters in the extended model.

Considering all the results, the accuracy improvements were on an average 20 % - 25 % in planimetry and 15 % in height. Although, based on a small sample, the variation in different blocks seems to meet with expectations: test field calibration manages especially well in poorly or moderately controlled blocks (Table 18).

There are only slight differences in the performances of the different parameter sets used in this study (Table 19).

Somewhat surprisingly, the application of test field photography at scale 1:8 000 seemed to produce slightly better results (especially in height) than the application of test field photography at scale 1:4 000, which is also the scale of block photography. If this behaviour can be interpreted as an evidence of a minor effect of the scale (within a reasonable range) on the successful performance of test field calibration, it implies greater freedom in the use of test field calibration.

A comparison of the results of the tests achieved, by varying the number of the test field points used for calibration, indicates that there are only insignificant differences between the three cases applied (Tables 9, 20, and 21). This is a very favourable finding, considering the economical aspect of calibration.

Table 22 gives some evidence of a possible loss of information that may occur in connection with test field calibration, due to the fact that the circumstances of test field photography and of block photography are not identical.

4.4 Component calibration

Considering the number of results, component calibration was the method, which was of least interest to the participants. Some results can, however, be given concerning

- the correction of film distortion by means of a reseau,
- image coordinate transformations, and
- the correction of radial distortion sectorwise.

Only the first subject has been properly treated, as for the others, the results presented serve only as an example.

Correction of film distortion by means of a reseau

In Hannover, all Willunga blocks were computed after refining the data by spline interpolation. Corrections for photo coordinates were derived from the residual errors after affine (6 parameters) transformation on 25 reseau crosses.

The results reported show that there are only insignificant differences (some per cents) in the accuracies achieved with and without the reseau corrections (see also / 6 /).

Compensation of film distortion by image coordinate transformation

Tables 23 and 24 present the results of a comparison of affine (6 parameters) transformation and of a 12-parametric transformation given by the formulas (computation in Helsinki):

$$x_p = a_1 + a_2x + a_3y + a_4xy + a_5y^2 + a_6xy^2$$

$$y_p = b_1 + b_2y + b_3x + b_4xy + b_5x^2 + b_6yx^2.$$

Two observations can be made from the results:

- 12-parametric transformation produces very good results with the Willunga data, while it is fairly inefficient with the Jämijärvi data.
- The use of 25 reseau crosses evenly distributed over the image area does not give better results than the use of 16 boarder crosses.

Correction of radial distortion sectorwise

With the same data as in the previous case, a series of adjustments, where radial distortion was corrected sectorwise based on laboratory calibration data, was carried out in Helsinki. In results showed no accuracy improvement as compared with the adjustments where mean radial distortion correction was applied.

5. CONCLUSIONS

The relatively small number of results using the different methods does not justify the establishment of recommendations for computation. Some facts, nevertheless, can be rather clearly and reliably seen in the results achieved by the WG. It is believed that, e.g. the following findings are worth consideration:

- The calibration methods applied improved the accuracy of aerial triangulation on an average 20 %. The results varied, however, considerably depending on the calibration method, the calibration procedure, the project parameters, and, of course, the data concerned.
- Generally, self calibration proved to produce better results than test field calibration, but the differences were, however, practically negligible.
- No single parameter set was found to be superior to the others. This refers both to self calibration and to test field calibration.
- In order to secure reliable and accurate results, proper attention, should be paid to the accomplishment of self calibration in poorly controlled blocks (especially, side overlap turned out to be a critical factor). Unfortunately, this study does not provide any relevant information regarding the application of significance testing and the treatment of correlations. On the other hand, the favourable effect of weighting was quite obvious. Further, it is also worth noticing that the introduction of stripinvariant - instead of blockinvariant - additional parameters was not successful.
- Regarding test field calibration, the most important finding was the independence of the results of the number of test field points used for calibration (the number of points varied from 25 to 180 in this study), which is attractive from the economical point of view.

Finally, it is felt that the matters, which still remained inadequately studied in this context and to which efforts should be directed, are, above all, the significance testing of additional parameters and the consideration of correlations both in self calibration and in test field calibration. Furthermore, concerning self calibration, it is suggested that further studies should be made on the dependence of compensation on side overlap and the distribution and number of tie points. In such investigations simulation techniques are obviously both applicable and flexible.

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APPENDIX A. Parameter sets in the studies of the WG III/3.

Parameter set a proposed by Brown / 1 /:

$$dx = a_1x + a_2y + a_3xy + a_4y^2 + a_5x^2y + a_6xy^2 + a_7x^2y^2 + \frac{x}{c}(a_{13}(x^2 - y^2) + a_{14}x^2y^2 + a_{15}(x^4 - y^4))$$
$$+ x(a_{16}(x^2 + y^2) + a_{17}(x^2 + y^2)^2 + a_{18}(x^2 + y^2)^3)$$

$$dy = a_8xy + a_9x^2 + a_{10}x^2y + a_{11}xy^2 + a_{12}x^2y^2 + \frac{y}{c}(a_{13}(x^2 - y^2) + a_{14}x^2y^2 + a_{15}(x^4 - y^4))$$
$$+ y(a_{16}(x^2 + y^2) + a_{17}(x^2 + y^2)^2 + a_{18}(x^2 + y^2)^3)$$

Parameter set b proposed by Ebner / 2 /:

$$dx = b_1x + b_2y - b_3(2x^2 - 4B^2/3) + b_4xy + b_5(y^2 - 2B^2/3) + b_7x(x^2 - 2B^2/3) + b_9(x^2 - 2B^2/3)y + b_{11}(x^2 - 2B^2/3)$$

$$dy = -b_1y + b_2x + b_3xy - b_4(2y^2 - 4B^2/3) + b_6(x^2 - 2B^2/3) + b_8(x^2 - 2B^2/3)y + b_{10}x(y^2 - 2B^2/3)$$
$$+ b_{12}(x^2 - 2B^2/3)(y^2 - 2B^2/3)$$

Parameter set c proposed by El-Hakim and Faig / 3 /:

$$dx = a_1x + a_2y + q\frac{x}{r}$$

$$dy = -a_1y + a_2x + q\frac{y}{r}, \text{ where}$$

$$q = a_3r\cos\lambda + a_4r\sin\lambda + a_5r^2 + a_6r^2\cos2\lambda + a_7r^2\sin2\lambda + a_8r^3\cos\lambda + a_9r^3\sin\lambda + a_{10}r^3\cos3\lambda + a_{11}r^3\sin3\lambda$$

$$r = \sqrt{x^2 + y^2} \text{ and } \lambda = \arctan\left(\frac{y}{x}\right)$$

Parameter set d proposed by Grün / 4 /:

$$dx = a_1y + a_2xy^2 + a_3x^2y$$

$$dy = b_1y + b_2xy + b_3xy^2 + b_4x^2y$$

Parameter set e proposed by Grün / 4 /:

$$dx = a_1y + a_2y^2 + a_3xy + a_4xy^2 + a_5x^2y$$

$$dy = b_1y + b_2xy + b_3xy^2 + b_4x^2 + b_5x^2y$$

Parameter set f proposed by Grün / 4 /:

$$dx = a_1y + a_2xy + a_3xy^2 + a_4x^2y + a_5y^2 + a_6x^2y^2$$

$$dy = b_1y + b_2xy + b_3xy^2 + b_4x^2y + b_5x^2 + b_6x^2y^2$$

Parameter set g proposed by Grün / 4 /:

$$dx = a_1y + a_2y^2 + a_3y^3 + a_4xy + a_5xy^2 + a_6x^2y + a_7x^3$$

$$dy = b_1y + b_2y^3 + b_3xy + b_4xy^2 + b_5x^2 + b_6x^2y + b_7x^3$$

Parameter set h proposed by Grün / 5 /:

$$dx = a_{12}x + a_{21}y + a_{22}xy + a_{31}l - b_{22}\frac{10}{7}k + a_{14}xp + a_{23}yk + a_{32}xl + a_{41}yq + a_{15}r + a_{24}xyp + a_{33}kl$$
$$+ a_{42}xyq + a_{51}s + a_{25}yr + a_{34}xlp + a_{43}ykq + a_{52}xs + a_{35}lr + a_{44}xypq + a_{53}ks + a_{45}yqr + a_{54}xps + a_{55}rs$$

$$dy = -a_{12}y + a_{21}x - a_{22}\frac{10}{7}l + b_{13}k + b_{22}xy + b_{14}xp + b_{23}yk + b_{32}xl + b_{41}yq + b_{15}r + b_{24}xyp + b_{33}kl$$
$$+ b_{42}xyq + b_{51}s + b_{25}yr + b_{34}xlp + b_{43}ykq + b_{52}xs + b_{45}yqr + b_{54}xps + b_{55}rs, \text{ where}$$

$$k = x^2 - \frac{b^2}{2};\; l = y^2 - \frac{b^2}{2};\; p = x^2 - \frac{17}{20}b^2;\; q = y^2 - \frac{17}{20}b^2;\; r = x^2(x^2 - \frac{31}{28}b^2) + \frac{9}{70}b^4;\; s = y^2(y^2 - \frac{31}{28}b^2) + \frac{9}{70}b^4$$

Parameter sets i and j (i includes the first 16 and j all 20 parameters) proposed by Jacobsen / 6 /:

$$dx = p_1x\cos(2\arctan(\tfrac{y}{x})) + p_2x\sin(2\arctan(\tfrac{y}{x})) + p_3x\cos(\arctan(\tfrac{y}{x})) + p_4x\sin(\arctan(\tfrac{y}{x})) + p_5yr\cos(\arctan(\tfrac{y}{x}))$$
$$+ p_6yr\sin(\arctan(\tfrac{y}{x})) + p_7x(r^2-A) + p_8x(r^2-Br-c) + p_9x(r^4-Dr^3-Er^2-Fr-G) + p_{10}x(r^8-Hr^7-\ldots-Nr-P)$$
$$+ p_{11}x\cos(4\arctan(\tfrac{y}{x}) + p_{12}x\sin(4\arctan\tfrac{y}{x}) + p_{13}x(r^2-12100)\cdot\cos(2\arctan\tfrac{y}{x}) + p_{14}x(r^2-12100)\cdot\sin(2\arctan\tfrac{y}{x})$$
$$+ p_{15}x(r^2-12100)\cdot\cos(4\arctan\tfrac{y}{x}) + p_{16}x(r^2-12100)\cdot\sin(4\arctan\tfrac{y}{x}) + p_{17}x + p_{18}y + p_{19}xy$$

$$dy = p_1 y\cos(2\arctan(\tfrac{y}{x})) + p_2 y\sin(2\arctan(\tfrac{y}{x})) + p_3 y\cos(\arctan(\tfrac{y}{x})) + p_4 y\sin(\arctan(\tfrac{y}{x})) + p_5 xr\cos(\arctan(\tfrac{y}{x}))$$
$$+ p_6 xr\sin(\arctan(\tfrac{y}{x})) + p_7 y(r^2-A) + p_8 y(r^2-Br-c) + p_9 y(r^4-Dr^3-Er^2-Fr-G) + p_{10} y(r^8-Hr^7-\ldots-Nr-P)$$
$$+ p_{11} y\cos(4\arctan\tfrac{y}{x}) + p_{12} y\sin(4\arctan\tfrac{y}{x}) + p_{13} y(r^2-12100)\cdot\cos(2\arctan\tfrac{y}{x}) + p_{14} y(r^2-12100)\cdot\sin(2\arctan\tfrac{y}{x})$$
$$+ p_{15} y(r^2-12100)\cdot\cos(4\arctan\tfrac{y}{x}) + p_{16} y(r^2-12100)\cdot\sin(4\arctan\tfrac{y}{x}) + p_{17} y + p_{18} x + p_{20} xy$$

Parameter set k proposed by Kölbl and modified by Juhl / 7 /:

$$dx = a_3 \frac{x}{r}(r^3 - r/r_o^2) + a_4\cdot x\cdot(\sin\frac{r\cdot\pi}{r_o})^2 + a_5\frac{x}{r}\sin(\frac{2r\pi}{r_o}) + r^{1,85}\cdot\frac{y}{r}(a_6\cos\alpha + a_7\sin\alpha + a_8\cos2\alpha + a_9\sin2\alpha)$$
$$dy = a_1 x + a_2 y + a_3\frac{y}{r}(r^3 - r/r_o^2) + a_4 y(\sin\frac{r\pi}{r_o})^2 + a_5\frac{y}{r}\sin(\frac{2r\pi}{r_o}) + r^{1,85}\cdot\frac{x}{r}(a_6\cos\alpha + a_7\sin\alpha + a_8\cos\alpha + a_9\sin2\alpha),$$

where $\alpha = \arctan(\frac{y}{x})$

r_o = a given constant (radial distance, where radial distortion is wanted to be zero for the second time)

Parameter set l proposed by Mauelshagen / 8 /:

$$dx = a_4 xy + a_5 y^2 + a_6 x^3 + a_7 x^2 y + a_8 xy^2 + a_9 y^3$$
$$dy = b_1 x + b_2 y + b_3 x^2 + b_4 xy + b_6 x^3 + b_7 x^2 y + b_8 xy^2 + b_9 y^3$$

Parameter set m proposed by Schut / 9 /:

$$dx = c_3 xy + c_5 y^2 + c_7 x^2 y + c_9 xy^2 + c_{11} x^2 y^2 + c_{13} x^3$$
$$dy = c_1 y + c_2 x + c_4 x^2 + c_6 xy + c_8 x^2 y + c_{10} xy^2 + c_{12} x^2 y^2 + c_{14} y^3$$

Parameter set n (cf. text):

$$dx = b_1 x + b_2 y + b_3 xr^2(1-r_o/r) + b_4 xr^4(1-r_o/r) + b_5 xr^6(1-r_o/r) + b_6\cdot 2xy + b_7(r^2 + 2x^2)$$
$$dy = -b_1 y + b_2 x + b_3 yr^2(1-r_o/r) + b_4 yr^4(1-r_o/r) + b_5 xr^6(1-r_o/r) + b_6(r^2 + 2y^2) + b_7\cdot 2xy,$$

where r_o is a given constant (first radial distance, where radial distortion is wanted to be zero).

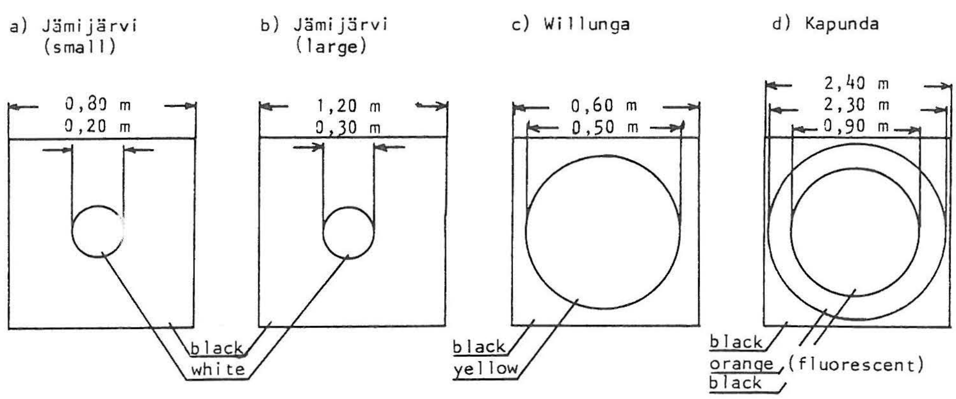

Figure 1. Target types used in the different test fields.

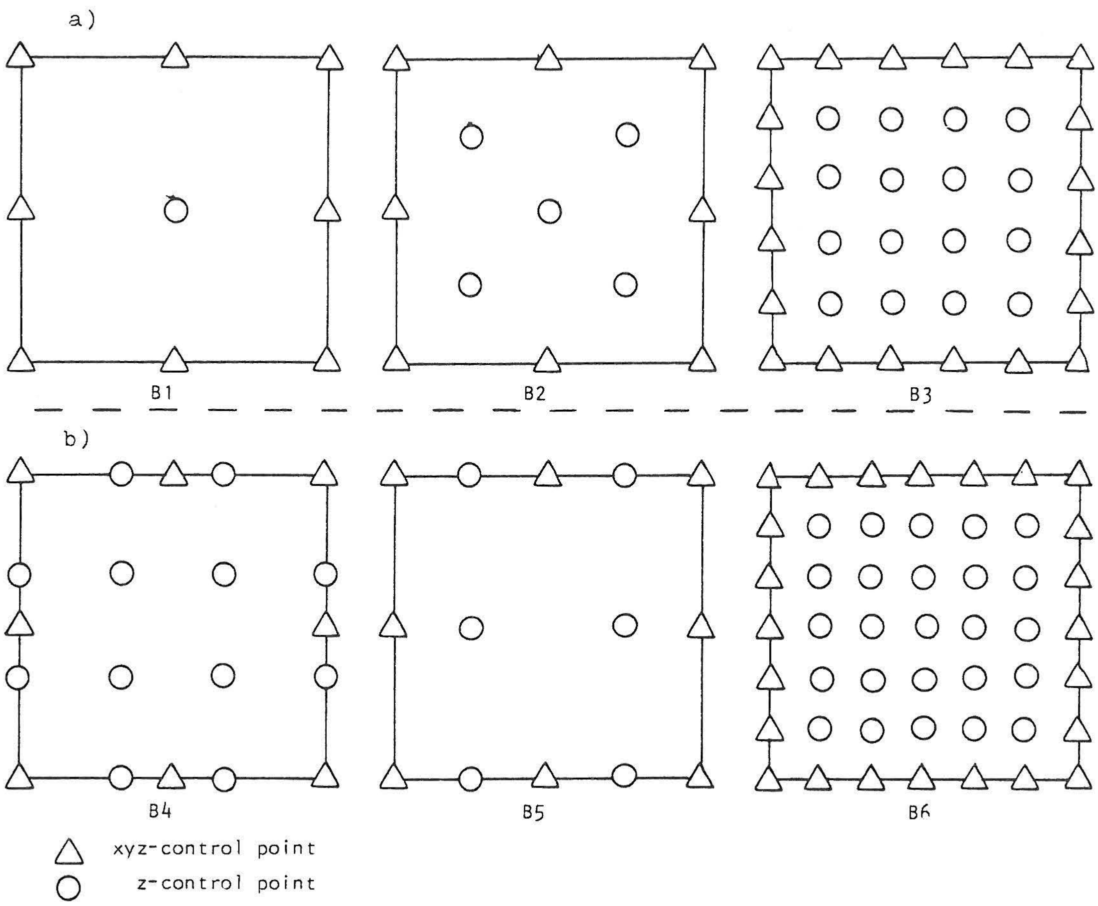

Figure 2. a) Control patterns recommended for blocks.
b) Control patterns for blocks used in addition to the recommended ones.

Test field	Country	Size[1] (sq.km)	Max. height diff. (m)	No. of points[2]	Accuracy (mm) planimetry	height
Willunga	Australia	5,8x5,8	130	96	25[3]	30[3]
Kapunda	Australia	24x24	300	43	40[3]	40[3]
Jämijärvi (small)	Finland	0,8x0,8	25	180	5[4]	0,6 mm/km
Jämijärvi (large)	Finland	2x2	60	121	3[4]	"

1) Area of the coordinated points.
2) No. of the coordinated points.
3) Average standard error of one coordinate.
4) Maximum standard error of one coordinate.

Table 1. Specifications of the test fields.

DATA SET SYMBOL[1]	MEASUREMENT Institute	Instr.	SPECIFICATIONS OF THE MEASURED MATERIAL Test field	Camera	Scale	Flight direct.	Side lap	No. of strips	No. of photos	Diapositive material
A1	Lands Dept.	PSK 2	Willunga	RMK AR	1:12 000	EW	20[2]	3	24	Measurements
A2	Adelaide,						20[3]	3	24	from film
A3	Australia						60	6	48	negatives
B1			Kapunda		1:50 000	EW	20[2]	4	36	
B2						WE	20[3]	5	45	
B3						EW/WE	60	9	81	
C1	TU München,	Plani-	Jämijärvi,	RMK A2	1:4 000	SN	20[2]	3	22	Film
C2	FRG	comp	large				20[3]	3	23	
C3		C-100					60	6	45	
D1		PSK 1				EW	20[2]	3	25	
D2							20[3]	3	24	
D3							60	6	49	
E1	Helsinki	PSK 1	Jämijärvi,	RMK A2	1:4 000	SN	20[2]	3	24	Film
E2	Univ. of		large				20[3]	3	24	
E3	Techn.,						60	6	48	
F1	Finland					EW	20[2]	3	24	
F2							20[3]	3	24	
F3							60	6	48	
G1	Univ.	PSK 1	Jämijärvi,	RMK A2	1:4 000	SN	20[2]	3	24	Glass
G2	Stuttgart,		large				20[3]	3	24	
G3	FRG						60	6	48	
H1						EW	20[2]	3	24	
H2							20[3]	3	24	
H3							60	6	48	
I1	Geod. and	PG 2	Jämijärvi,	MRB	1:4 000	EW	20[2]	3	24	Film
I2	Geophysical		large				20[3]	3	24	
I3	Inst.,						60	6	48	
	Sopron,									
	Hungary									
J1	Nat. Board	PSK 1	Jämijärvi,	MRB	1:4 000	EW	20[2]	3	20	Glass
J2	of Survey,		large				20[3]	3	23	
J3	Helsinki,						60	6	43	
	Finland									

1) Symbol is used to identify the data sets when presenting the results of block adjustments.
2) Odd numbered strips.
3) Even numbered strips.

Table 2. Specifications of measurements of block photographies.

Camera	Test field	Type of photogr.	Nominal scale	Flight direction	Forward/ side lap	No. of strips	No. of photos	Date of photogr.
RMK A2	Jämijärvi, large	block	1:4 000	SN & EW[1]	60/60	6 + 6	48 + 48	16.08.77
(WA)	" , large	strip	1:8 000	"	80/	1 + 1	3 + 3	"
	" , small	strip	1:4 000	"	80/	1 + 1	3 + 3	"
MRB	Jämijärvi, large	block	1:4 000	SN & EW[1]	60/60	6 + 6	48 + 48	16.08.77
(WA)	, large	strip	1:8 000	"	80/	1 + 1	3 + 3	"
	, small	strip	1:4 000	"	80/	1 + 1	3 + 3	"
RMK AR	Kapunda	block	1:50 000	EW/WE[2]	60/60	9	81	31.03.76
(WA, reseau)	Willunga	block	1:12 000	EW	60/60	6	48	15.01.75

1) Two cross-flight blocks.
2) Alternating flight directions.

Table 3. Photographies available for the WG.

DATA SET SYMBOL	MEASUREMENT		SPECIFICATIONS OF MEASURED MATERIAL					
	Institute	Instr.	Camera	Test field	Scale	Flight direction	No. of photos	Diapositive material
K1	TU Munchen	PSK 1	RMK A2	Jämijärvi, small	1:4 000	SN	3	Film
K2	FRG					EW	3	
L1				Jämijärvi, large	1:8 000	SN	3	
L2						EW	3	
M1	Helsinki	PSK 1	RMK A2	Jämijärvi, small	1:4 000	SN	3	Film
M2	Univ. of					EW	3	
N1	Techn.,			Jämijärvi, large	1:8 000	SN	3	
N2	Finland					EW	3	
P1		PK 1	MRB	Jämijärvi, small	1:4 000	SN	3	Glass
P2						EW	3	
Q1				Jämijärvi, large	1:8 000	SN	3	
Q2						EW	3	
R1	Geod. and	PG 2	MRB	Jämijärvi, small	1:4 000	SN	3	Film
R2	Geophysical					EW	3	
S1	Inst.,			Jämijärvi, large	1:8 000	SN	3	
S2	Sopron,					EW	3	
	Hungary							

Table 4. Specifications of measurements of strip photographies.

Institute	Triangulation method	Calibration method	Data used
Geod. and Geophysical Inst., Sopron, Hungary	anblock	self	E3, F3, I3
Helsinki Univ. of Technology, Finland	bundle	component, test field, self	A2, A3, J1, J3
TU München, FRG	bundle	test field, self	A3, B3, C3, D3 J3
Aalborg Univ., Denmark	bundle	self	A1-A3, E1-E3, F1-F3
Univ. Bonn, FRG	bundle	self, test field, component	A3, J3, E1-E3, F1-F3
TU Hannover, FRG	bundle	self, component	A1-A3, E1-E3, F1-F3
Univ. Stuttgart, FRG	bundle	self	A1-A3, B1-B3, G1-G3, H1-H3

Table 5. Participants in computation, and methods and data used by them.

Data set	Parameter set	Control B5 (medium)					Control B6 (dense)				
		s_o [µm]	RMSE [µm]		Impr. [%]		s_o [µm]	RMSE [µm]		Impr. [%]	
			μ_{xy}	μ_z	μ_{xy}	μ_z		μ_{xy}	μ_z	μ_{xy}	μ_z
E1	Ref.case	3,4	6,4	11,6			3,8	3,7	8,0		
	1	2,9	4,9	8,9	23	23	3,1	3,3	7,4	11	8
	a	2,8	4,4	8,3	31	28	3,0	3,2	7,2	14	10
	f	2,9	4,3	8,8	33	24	3,1	3,2	7,3	14	9
	d	2,9	4,4	8,9	31	23	3,2	3,4	7,2	8	10
E2	Ref.case	3,6	6,9	13,4			4,1	4,6	8,4		
	1	2,9	6,2	8,8	10	34	3,0	3,4	6,5	26	23
	a	2,8	4,5	8,2	35	39	2,9	3,3	6,3	28	25
	f	2,9	4,6	8,1	33	40	3,0	3,2	6,2	30	26
	d	3,0	3,9	7,6	43	43	3,1	3,1	6,2	33	26
F1	Ref.case	3,9	6,2	8,9			4,2	3,5	6,6		
	1	3,0	3,2	7,2	48	19	3,1	2,4	5,5	31	17
	a	2,9	3,1	7,8	50	12	3,0	2,4	5,1	31	23
	f	3,0	3,1	7,2	50	19	3,1	2,5	5,3	29	20
	d	3,3	3,3	7,2	47	19	3,3	2,8	5,4	20	18
F2	Ref.case	3,5	6,1	9,9			3,9	4,1	7,7		
	1	2,8	4,5	8,0	26	19	3,0	3,1	5,9	24	23
	a	2,8	4,0	8,3	34	16	2,9	3,1	5,9	24	23
	f	2,8	4,1	7,9	33	20	3,0	3,1	5,7	24	26
	d	2,9	4,2	7,8	31	21	3,1	3,1	5,6	24	27

Table 6. Comparison of the performance of the different parameter sets in blocks with 20 % side overlap and control patterns B5 or B6. Significance testing or weighting not applied. Computed in Bonn.

Side lap / Control	20 %		60 %	
	μ_{xy}	μ_z	μ_{xy}	μ_z
Sparse	4-7	10-25	2,5-3,5	6-9
Medium	4-7	7-20	2,5-3,5	5-7
Dense	3-4	5-10	2,5-3,0	4-6

Table 7. Accuracy estimates for self calibration in different types of blocks. Figures (RMSEs in μm) are based on all results obtained by the WG.

Data set	Control	Parameter set	s_o [μm]	RMSE [μm]		Impr. [%]	
				μ_{xy}	μ_z	μ_{xy}	μ_z
C3	B1	Ref.case	4,6	5,2	11,5		
	(sparse)	b	4,1	3,8	7,2	27	37
		h	4,0	3,6	6,5	31	43
	B3	Ref.case	4,8	3,9	6,7		
	(dense)	b	4,2	2,8	5,5	28	18
		h	4,1	2,6	4,5	33	33
J3	B1	Ref.case	4,1	4,3	12,0		
	(sparse)	b	4,0	2,7	10,5	37	13
		h	3,8	2,6	7,5	40	38
	B3	Ref.case	4,3	3,5	6,5		
	(dense)	b	4,1	2,6	5,7	26	12
		h	3,8	2,8	5,0	20	23

Table 8. Comparison of two orthogonal parameter sets b (12 parameters) and h (44 parameters). Parameters significant on a 95 % level have been included in the compensation model. Computed in Munich.

Parameter set	Willunga A2, control B2					Jämijärvi J2, control B2				
	s_o [μm]	RMSE [μm]		Impr. [%]		s_o [μm]	RMSE [μm]		Impr. [%]	
		μ_{xy}	μ_z	μ_{xy}	μ_z		μ_{xy}	μ_z	μ_{xy}	μ_z
Ref.case	4,4	6,2	16,5			4,0	5,5	10,9		
a	3,1	6,2	17,3	0	- 5	3,6	4,0	9,1	27	17
b	3,1	6,5	9,1	- 5	45	3,9	4,0	9,8	27	10
c	3,8	5,0	12,8	19	22	3,9	3,9	9,4	29	14
h	3,7	7,7	28,0	-24	-70	3,7	4,5	8,8	18	19
m	3,5	5,6	9,2	10	44	3,9	4,0	9,9	29	9

Table 9. Comparison of the performance of the different parameter sets in two blocks with 20 % side overlap and medium control (B2). Significance testing or weighting not applied. Computed in Helsinki.

Parameter set	Willunga A3, control B2					Jämijärvi J3, control B2				
	s_o	RMSE [μm]		Impr. [%]		s_o	RMSE [μm]		Impr. [%]	
	[μm]	μ_{xy}	μ_z	μ_{xy}	μ_z	[μm]	μ_{xy}	μ_z	μ_{xy}	μ_z
Ref.case	4,8	4,5	8,5			4,4	4,3	8,7		
a	3,1	2,7	5,3	40	38	3,8	2,6	4,9	40	44
b	3,4	2,7	5,4	40	36	4,2	2,8	6,8	35	22
c	3,8	3,0	4,8	33	44	4,1	2,8	5,4	35	38
h	3,3	2,8	5,0	38	41	3,8	2,7	5,1	37	41
m	3,2	2,7	5,1	40	40	4,1	2,6	5,5	40	37

Table 10. Comparison of the performance of the different parameter sets in two blocks with 60 % side overlap and medium (B2) control. Significance testing or weighting not applied. Computed in Helsinki.

Parameter set	Side lap 20 %, Control B3		Side lap 60 % Control B1	
	μ_{xy} (Δ%)	μ_z (Δ%)	μ_{xy} (Δ%)	μ_z (Δ%)
k 1)	18	29	42	56
c 2)	12	8	35	56
b 2)	17	13	40	57
m 2)	17	18	42	57
a 2)	12	23	40	54

1) Significance testing not applied, weighting applied (cf. text).
2) Significance testing applied, weighting not applied.

Table 11. Accuracy improvements in percentages (compared with resp. reference adjustment) obtained with different parameter sets in two Willunga blocks with varying project parameters.

Control pattern	Parameters common to					
	each strip		two sub-blocks (20 %)		whole block	
	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z
B1	27	34	0	0	0	0
B2	29	20	4	2	0	0
B3	8	8	-4	0	0	0

Table 12. Accuracy improvements (%) achieved by applying significance testing on additional parameters. Testing procedure as in Stuttgart (cf. text). Data set A3.

Control pattern	Sign. level 67 %		Sign. level 95 %	
	μ_{xy}	μ_z	μ_{xy}	μ_z
B1	0	1	0	8
B2	0	0	0	7
B3	0	-18	0	-20

Table 13. Accuracy improvements (%) achieved by applying significance testing on additional parameters. Testing procedure as in Hannover (cf. text). Data set A3.

Data set	Side lap [%]	Weight 1)	Control B1				Control B2				Control B3			
			RMSE [µm]		Impr. [%]		RMSE [µm]		Impr. [%]		RMSE [µm]		Impr. [%]	
			μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z
A1	20	Ref.case	5,5	20,3			5,5	15,1			4,6	12,2		
		all free	9,4	45,6	-71	-125	10,8	9,7	-96	35	4,0	8,9	13	27
		100 µm/100 mm	9,3	35,5	-69	- 60	10,5	9,5	-91	37	4,2	8,9	9	27
		30 µm/100 mm	8,0	26,0	-45	- 28	9,9	9,4	-80	38	4,2	8,8	9	28
		10 µm/100 mm	7,2	17,7	-31	13	6,3	9,1	-15	39	4,1	8,7	11	29
		5 µm/100 mm	5,6	17,6	- 2	13	5,1	8,9	7	41	4,1	8,4	11	31
		1 µm/100 mm	4,5	17,4	18	15	4,4	10,0	20	34	4,2	8,2	9	33
A2	20	Ref.case	7,1	35,5			6,2	16,5			4,2	9,7		
		all free	6,8	46,1	4	-30	6,2	17,3	0	- 5	3,6	10,9	14	-12
		100 µm/100 mm	6,6	43,6	7	-23	5,9	16,6	5	- 1	3,4	10,6	19	- 9
		30 µm/100 mm	6,6	22,3	7	34	5,5	12,6	11	24	3,4	9,0	19	7
		10 µm/100 mm	5,8	21,4	18	40	5,5	9,1	11	45	3,4	7,2	19	26
		5 µm/100 mm	5,5	17,8	23	50	5,0	8,2	19	50	3,4	6,8	19	30
		1 µm/100 mm	5,3	19,2	25	46	4,5	11,0	27	33	3,5	6,5	17	33
A3	60	Ref.case	4,8	17,1			4,5	8,7			3,2	5,3		
		all free	2,9	7,8	40	54	2,7	5,3	40	39	2,6	5,0	19	6
		100 µm/100 mm	2,8	7,8	42	54	2,7	5,3	40	39	2,5	4,9	22	8
		30 µm/100 mm	2,8	7,7	42	55	2,7	5,3	40	39	2,5	4,9	22	8
		10 µm/100 mm	2,8	7,7	42	55	2,7	5,3	40	39	2,6	5,0	19	6
		5 µm/100 mm	2,8	7,6	42	56	2,7	5,3	40	39	2,6	5,1	19	4
		1 µm/100 mm					2,8	5,4	38	38	2,6	4,7	19	11

1) See text.

Table 14. Effect of weighting on the accuracy obtained by self calibration. Willunga data, parameter set a. Computed in Helsinki.

Data set	Control version	Improvement [%]	
		μ_{xy}	μ_z
A1	B1	2	84
	B2	2	6
	B3	4	0
A2	B1	0	-41
	B2	1	71

Table 15. Example of the improvements obtained by excluding the additional parameters correlating mutually more than 0,85. Parameter set j. Computed in Hannover.

Data set	Control version	Parameter set	Improvement[1] [%] Refinement 1[1]		Refinement 2[2]		Difference betw. 2 and 1	
			μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z
A2	B1	a	4	25	7	40	3	15
		b	1	19	-44	48	-45	29
	B2	a	0	- 5	-10	14	-10	19
		b	-5	45	-71	8	-66	-37
	B3	a	14	-12	25	-23	11	-11
		b	21	39	34	-64	13	-103
A3	B1	a	40	54	42	58	2	4
		b	42	51	40	67	- 2	16
	B2	a	40	38	40	42	0	4
		b	40	36	38	36	- 2	0
	B3	a	19	6	22	6	3	0
		b	19	25	25	6	6	-19
J2	B1	a	6	0	24	38	18	38
		b	8	-10				
	B2	a	27	17	33	29	6	12
		b	27	10	18	7	- 9	- 3
	B3	a	16	12	18	19	2	7
		b	13	1	5	-11	- 8	-12
J1	B1	a	40	36	37	34	- 3	- 2
		b	37	13	30	0	- 7	-13
	B2	a	40	44	37	38	- 3	- 6
		b	35	22	33	15	- 2	- 7
	B3	a	24	27	29	32	5	5
		b	18	9	24	23	6	14

Table 16. Examples of the effect of a-priori data refinement on accuracy. Computed in Helsinki.

1) Compared with the resp. reference adjustment.
2) Refinement 1: as in reference adjustment.
3) Refinement 2: 4-parametric transformation, no a-priori corrections.

Data set	Flight direct.	Control	Reference adj.		Self. calibration		Improvement [%]	
			μ_{xy} [µm]	μ_z [µm]	μ_{xy} [µm]	μ_z [µm]	μ_{xy}	μ_z
E1	↑	B1	8,1	15,1	4,7	9,4	42	38
		B4	7,8	14,4	4,8	10,3	38	28
		B3	3,7	8,2	3,2	7,9	14	4
F1	←	B1	5,5	14,2	3,1	10,5	44	26
		B4	5,6	10,8	3,0	8,3	46	23
		B3	3,6	7,5	2,7	5,7	25	24
E1+F1	←↑	B1	3,2	5,9	2,4	5,6	25	5
		B4	3,3	5,1	2,4	5,2	27	-2
		B3	2,5	5,1	2,0	4,0	20	28

Table 17. Comparison of two single blocks and a cross-flight block. Computed in Aalborg.

Control	Side overlap 20 %				Side overlap 60 %			
	Accuracy obtained [μm]		Improvement[1] [%]		Accuracy obtained		Improvement[1] [%]	
	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z
sparse	-	-	-	-	3-4	6-8	15-25	15-30
medium	3,5	7-8	40-45	10-20	-	-	-	-
dense	3-3,5	6-6,5	20	15	2,5-3	4,5-6	10-20	5-15

1) Compared with the resp. reference adjustment.

Table 18. The performance of test field calibration in blocks of different types estimated from the results of test field calibration performed by the participants of WG III/3.

Parameter set	No. of test field points	Scale of the test flight 1:4 000					Scale of the test flight 1:8 000				
		s_o [μm]	RMSE [μm]		Impr.[1] [%]		s_o [μm]	RMSE [μm]		Impr.[1] [%]	
			μ_{xy}	μ_z	μ_{xy}	μ_z		μ_{xy}	μ_z	μ_{xy}	μ_z
1	25	3,2	3,5	8,1	44	9	3,3	3,5	6,8	44	24
	64	"	3,4	8,1	45	9	3,3	3,6	6,9	42	22
	180/130	"	3,3	7,8	47	12	3,2	3,5	6,7	44	25
a	25	4,2	4,3	14,9	31	-67	3,2	3,6	8,0	42	10
	64	3,1	3,5	8,7	44	2	3,2	3,8	7,9	39	11
	180/130	3,1	3,3	8,4	47	6	3,1	3,8	7,8	39	12
f	25	3,1	3,4	7,7	45	13	3,3	3,5	7,0	44	21
	64	"	3,3	7,7	47	13	3,3	3,6	7,1	42	20
	180/130	"	3,3	7,8	47	12	3,2	3,4	6,8	45	24
d	25	3,5	3,6	8,0	42	10	3,3	3,5	7,1	44	20
	64	3,4	3,6	7,9	42	11	3,3	3,6	7,2	42	19
	180/130	3,4	3,5	7,7	44	13	3,3	3,5	6,9	44	22

1) Compared with the resp. reference adjustment.

Table 19. Comparison of the performance of the different parameter sets in test field calibration. Calibration parameters have been determined from cross-flight test strips K1+K2 (1:4 000) and L1+L2 (1:8 000) and utilized in the block adjustment of data set F1 with the control B5. Significance testing or weighting have not been applied. Computed in Bonn.

Data used for calibr. (scale)	No. of test field points	Control B1 (sparse)					Control B3 (dense)				
		s_o [µm]	RMSE [µm]		Impr.[1] [%]		s_o [µm]	RMSE [µm]		Impr.[1] [%]	
			μ_{xy}	μ_z	μ_{xy}	μ_z		μ_{xy}	μ_z	μ_{xy}	μ_z
L1 + L2	25	3,9	2,4	6,0	27	17	3,9	2,4	4,5	14	10
(1:8 000)	64	3,9	2,4	5,8	27	19	3,9	2,4	4,5	14	10
	130	3,8	2,4	5,8	27	19	3,9	2,3	4,5	18	10
K1 + K2	25	4,1	2,8	6,2	15	14	4,1	2,4	5,0	14	0
(1:4 000)	64	4,1	2,9	6,2	12	14	4,2	2,6	5,0	7	0
	180	4,2	2,9	6,2	12	14	4,2	2,6	5,0	7	0

1) Compared with the resp. reference adjustment.

Table 20. Dependence of the efficiency of test field calibration on the scale of test strip photography, the no. of test field points used for calibration and the control of the block. Data set D3, orthogonal parameter set b (significance level 95 % applied). Computed in Munich.

Data used for calibr. (scale)	No. of test field points	Control B1 (sparse)					Control B3 (dense)				
		s_o [µm]	RMSE [µm]		Impr.[1] [%]		s_o [µm]	RMSE [µm]		Impr.[1] [%]	
			μ_{xy}	μ_z	μ_{xy}	μ_z		μ_{xy}	μ_z	μ_{xy}	μ_z
L1 + L2	25	4,3	4,3	8,2	17	29	4,4	3,1	5,5	21	18
(1:8 000)	64	4,3	4,4	8,0	15	30	4,4	3,3	5,5	15	18
	130	4,2	4,3	8,2	17	29	4,4	3,1	5,5	21	18
K1 + K2	25	4,4	4,2	8,0	19	30	4,5	3,0	5,7	23	15
(1:4 000)	64	4,3	4,1	7,7	21	33	4,4	3,0	6,0	23	10
	180	4,3	4,1	7,2	21	37	4,4	3,0	5,7	23	15

1) Compared with the resp. reference adjustment.

Table 21. Dependence of the efficiency of test field calibration on the scale of test strip photography, the no. of test field points used for calibration and the control of the block. Data set C3, orthogonal parameter set b (significance level 95 % applied). Computed in Munich.

Parameter set	Procedure A[1]		Procedure B[2]	
	μ_{xy}	μ_z	μ_{xy}	μ_z
1	44	24	55	50
a	42	10	55	47
f	44	21	55	49
d	44	20	51	53

Table 22. Comparison of the improvements (in percentages with respect to resp. reference adjustment) obtained by two different test field calibration procedures. Data set F1, control B5. Computed in Bonn.

1) Calibration parameters determined from the separate test field flight (data set L1+L2, 25 test field points used).
2) Calibration parameters determined by using 3 photos picked from the block (= "ideal case of test field calibration").

Data set	Control	6 parameters & 25 crosses[1]		12 parameters & 16 crosses[2]		12 parameters & 25 crosses[1]		Improvement[3] [%]					
								6 & 25		12 & 16		12 & 25	
		μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z
A1	B1	5,8	32,0	5,4	14,3	5,3	12,0	19	6	25	58	26	65
	B3	3,8	8,4	3,7	6,9	3,7	6,6	10	13	12	29	12	32
A3	B1	3,7	17,7	3,4	10,4	3,4	10,2	24	10	31	47	31	48
	B3	2,9	5,1	2,9	4,4	2,8	4,2	15	7	15	20	18	24

1) Evenly distributed.
2) Locating on the image boarders.
3) Compared with the resp. ref. adjustment.

Table 23. Comparison of the results of bundle adjustments after different image coordinate transformation. Willunga reseau data. Computed in Helsinki.

Data set	Control	6 parameters & 20 fiducials		12 parameters & 20 fiducials		Improvement[1] [%]			
						6 & 20		12 & 20	
		μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z
J1	B1	4,2	11,3	3,8	15,0	0	0	10	33
	B3	3,4	9,4	3,6	8,8	11	-3	5	3
J3	B1	3,1	10,2	3,0	10,5	28	2	30	-1
	B3	2,8	6,2	2,8	6,3	18	6	18	5

1) Compared with the resp. reference adjustment.

Table 24. Comparison of the results of bundle adjustments after different image coordinate transformations. Jämijärvi/MRB data. Computed in Helsinki.