# PRECISION OF IMAGE MATCHING 

J.C. TRINDER, K. BECEK AND B.E. DONNELLY<br>School of Surveying<br>University of N.S.W<br>P.O. Box 1, Kensington, NSW 2033 Australia


#### Abstract

A simulation study of the precision of image matching based on the least squares and cross-correlation methods is being carried out on a variety of simulated images of typical objects on digitized photographs or satellite data. The tests in this paper will demonstrate the precision of image matching by both methods, and also the factors which influence the precision such as the amount of detail in the image, the size of the detail, the image quality, image geometry and quantization level. As the method is based on simulated images with known positions of the centres of both image windows, a reliable estimate of the precision of the matching is possible.


## Key Words: Image Matching, Image Processing, Algorithmic.

## 1. Introduction

Developments of software for machine vision applications of photogrammetry based on digital images have been under way in the School of Surveying for some time, (Trinder et al 1990). The technology is designed for medicine and quality control for manufacturing. The precisions of image matching achieved with tests of these developments to date have been of the order of 0.2 to 0.3 pixel. In addition, a check on the accuracies of the geometry of the object using analogue images observed in an analytical stereoplotter, indicated accuracies of the order of 1 mm to 2 mm , depending on the camera geometry, for a camera to object distance of 1.8 m . While these precisions are typical of those obtained by many researchers, it became necessary to investigate the precision of image matching that can be achieved with the method and the factors such as image content and quality which influence them, to determine if improvements in the results are achievable. It is noted that the precisions of template matching to well defined targets by Beyer (1992) are of the order of 0.02 pixel. The results of the study of the precision of image matching are described in this paper.

In an earlier study, investigations were carried out on the precision of target location on simulated digital images, based on the computation of the target centroid, as a function of image quality variables such as target size, extent of blur in the image, pixel size, quantization level, and noise (Trinder, 1989). This paper extends this study to investigations of the accuracy of image matching, based on the method of least squares (Foerstner 1982) of a range of simulated digital images, subject to the same variables as those above, together with variables such as displacements, scale changes and rotations between the two images.

## 2. Creation of Image Windows

The images generated in this study are designed to represent typical features in photographic or satellite images over which image matching may be undertaken. Therefore features that can be generated by the software involve a range of shapes such as circles, crosses, crosses with additional blobs, tjunctions, v -junctions and grids. Each feature is described by a grid of sample points, typically 64 by 64 points or 128 by 128. The features are then subjected to image blur by convolution by FFT of the feature with a Gaussian spread function with $2 \sigma$ widths ranging from 10 to $50 \mu \mathrm{~m}$, which are typical of those which occur for standard aerial photography or satellite images. A spread function of $2 \sigma$ value of $0 \mu \mathrm{~m}$ is also possible for testing images when no blur is present in the image. The next step is to digitize the feature with pixel sizes ranging from $10 \mu \mathrm{~m}$ to $50 \mu \mathrm{~m}$. This process has been described previously (Trinder 1989). The intensity of each pixel is a function of the volume of the solid contained by the intersection of the surface, representing the intensity distribution of the feature in the x and y directions, and the column representing the pixel. This volume is derived by
interpolating sufficient values within the area of each pixel, from the surface representing the intensity of the feature, to give adequate precision. During the digitisation process, 8 quantization levels can be selected, resulting in digital data of $2^{8}, 2^{7}, 2^{6}, 2^{5}, 2^{4}, 2^{3}, 2^{2}, 2^{1}$ grey values which means encoding into $8,7, \ldots 1$ bits, respectively. Noise can be introduced into the digitized data prior to quantization. The size of the feature in relation to the pixel size determines the size of the image window, which is therefore a dependent variable in this study.

The process of image matching clearly requires two separate images to be generated. The first image, coded as the left hand image, was derived according to the description above, while the dimensions and orientation, where appropriate, of the second or right image were varied from those of the left image to simulate changes in scale and rotations between the two images. This enabled tests to be carried out on the effects of geometric distortions in the features in the two windows on the efficiency of the image matching. The digitisation process of the right window is repeated a number of times to simulate the circumstance in practice where the position of the initial pixel with respect to the location of the feature may vary randomly. This is applied in the horizontal direction only (i.e. x-direction) by displacing the commencing pixel in the range of $\pm 1$ pixel from the initial position on the feature by a random generator. Obviously, since the features have been computer generated, the exact locations of the centres of the features are known.

In summary, the following parameters can be varied in this study on the accuracies of image matching:-

* feature characteristics of shape, size, image quality and noise,
* pixel size used for digitizing, and therefore window size, and commencing position of the digitizing,
* quantization level,
* feature position, scale and orientation.


## 3. Image Matching

Image matching has been carried out by both the least squares and cross-correlation methods. The tests have involved the computation of the matching of the right image to the left image, commencing with the initial assumption that the correct matching position of the right window is indeed the centre of the window. Displacements, scale changes and rotations of the feature in the right window with respect to the feature in the left window can be selected by data input. It follows that the correct position of matching will be subject to these displacements, scale changes and rotations, and the matching procedure should therefore reveal the effects of these parameters. Tests will determine the maximum values of these parameters which can be accommodated in the matching process. Since the commencing position of the digitization
process is varied randomly, repeated digitizations of the feature on the right window with different starting pixel positions will generate a number of combinations of the left and right windows for matching. A precision of matching can therefore be derived from all combinations from the root mean square (RMS) of the differences between the correct matching position of the right window and that actually computed by the matching process. These results will be presented in the next section.

## 4 Precision of Matching

Results of tests on the precision of matching based on the least squares method are shown in Figures 1 to 4, while a comparison of precisions obtained by least squares and crosscorrelation is given in Table 1. In Figure 1 the precision is shown for a circular feature of $100 \mu \mathrm{~m}$ diameter, digitized by a pixel size of $12.5 \mu \mathrm{~m}$ and the same scale for the two images, expressed as a function of the quantization level. The spread functions used for the tests are shown on the appropriate lines. This figure indicates that the highest precision obtained for circles is from 0.03 to 0.05 pixel, for quantization levels of greater than 5 bits ( 32 grey levels) and that the precision deteriorates as the quantization decreases below 5 bits. Indeed, the matching becomes erratic and on occasions, it fails to converge. The magnitude of image blur, demonstrated by the size of the Gaussian spread function, has little effect on the precision of matching, although there is a tendency for the matching to improve slightly as the spread function increases. This may be partly due to the increase in the size of the window brought about by the larger image blur. In Figure 2 the influence of the feature size for 2 cases of image blur and quantization is demonstrated for a pixel size of $12.5 \mu \mathrm{~m}$. In this case the precision increases as the feature increases in size, reaching .02 pixel for a feature size of $200 \mu \mathrm{~m}$. These values agree with the precisions of template matching achieved by Beyer (1992). The effect of scale variations between the 2 images is demonstrated in Figure 3. A scale change of $10 \%$ between the 2 images has almost no effect, but for scale changes greater than $10 \%$, the effects are much more significant. For a scale difference of $30 \%$ the precision deteriorates to greater than 0.2 pixel, while further changes in scale would not result in successful matches.

The effect of rotations between the 2 images for crosses is shown in Figure 4, which indicates a similar deterioration in the precisions of matching as occurred for the circles.


Figure 1 Precision of least squares matching for circular feature $100 \mu \mathrm{~m}$ in diameter, in terms of quantization level for 3 spread function sizes (SF) for pixel size of 12.5 $\mu \mathrm{m}$.


Figure 2 Precision of least squares matching in terms of circular feature size for 2 spread functions (SF) and pixel size of $12.5 \mu \mathrm{~m}$.


Figure 3 Precision of least squares matching in terms of quantization level for circular feature $100 \mu \mathrm{~m}$ in size, for 3 cases of scale variations between the two images, and pixel size of $12.5 \mu \mathrm{~m}$.


Figure 4 Precision of least squares matching in terms of crosses with 3 cases of rotations between the two images, and pixel size of $12.5 \mu \mathrm{~m}$.

Table 1
COMPARISON OF PRECISIONS OF MATCHING OBTAINED BY LEAST SQUARES AND CROSS-CORRELATION METHODS

| Feature | $:$ | Cross $100 \times 10 \mu \mathrm{~m}$ |
| :--- | :--- | :--- |
| Spread function | $:$ | $25.0 \mu \mathrm{~m}$ |
| Pixel size | $:$ | $12.5 \mu \mathrm{~m}$ |
| Quantisation level | $:$ | $8 \mathrm{bits} / \mathrm{pixel}$. |

Precisions in terms of pixel size x 1000

| Method of <br> matching | Scale Variations |  |  | Rotation (degrees) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 38 | 0.9 | 0.8 | 0 | 10 | 20 | 30 |
| Cross- <br> Correlation | 52 | 75 | 120 | 52 | 83 | 170 | $*$ |

*     - no results

Table 2
COMPARISON OF MATCHING PRECISIONS OBTAINED FOR DIFFERENT SHAPED FEATURES FOR THE LEAST SQUARES METHOD

| Pixel size $:$ $12.5 \mu \mathrm{~m}$ <br> Precisions in terms of pixel size $\times 1000$   |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| TYPE <br> OF | SCALE |  | ROTATION (degrees) |  | SPREAD FUNCTION ( $\mu \mathrm{m}$ ) |  |
| FEATURE | 1.0 | 0.8 | 10 | 30 | 10 | 25 |
| CIRCLE | 46 | 69 | * | * | 44 | 38 |
| CROSS | 38 | 42 | 31 | 180 | 36 | 32 |
| ELLIPSE | 41 | 75 | 68 | 160 | 50 | 48 |

*     - no results

Table 1 reveals that the cross-correlation method, for images in which there are no scale distortions or rotations, will result in marginally worse results than the least squares method. However, as the scale distortions and rotations increase, the precisions obtained by cross-correlation rapidly deteriorate and are a factor 3 to 4 larger than those obtained by the least squares method, while matching fails for a rotation between the two images of $30^{\circ}$. This result confirms the widely held views amongst photogrammetrists that the cross-correlation method will be significantly affected by distortions between the two images, and indeed may result in completely erroneous matches.

In Table 2 are shown comparisons between 3 different features subject to scale variations and rotations, and different levels of blur. As previously demonstrated, this table confirms that better results are generally obtained for a cross than for a circle or ellipse, and also the small influence that image quality has on the precision of least squares matching.

Foerstner (1982) has given a formula for the theoretical precision of matching 2 digital images by the least squares method as follows:

$$
\sigma_{\mathrm{x}}^{2}=\frac{2}{\mathrm{~N} \cdot \mathrm{SNR}^{2}} \cdot \frac{\sigma_{\mathrm{g}}^{2}}{\sigma_{\mathrm{g}}^{2}}
$$

where $\quad \sigma_{\mathrm{x}}^{2}$ is the variance of estimating the shift
parameter in the matching,
N is the number of pixels containing the information about the feature,

SNR is the signal to noise ratio in the image,
$\sigma_{g}^{2}$ is the variance of the signal,
$\sigma_{\mathrm{g}^{\prime}}^{2}$ is the variance of the gradient of the image.
Adopting $\mathrm{N}=50$ for a circle of $100 \mu \mathrm{~m}$ with pixel size of $12.5 \mu \mathrm{~m}, \mathrm{SNR}=10, \sigma_{\mathrm{g}}^{2}=3600$, and $\sigma_{\mathrm{g}^{\prime}}^{2}=900$ for an image
quantized to 8 bits, $\sigma_{\mathrm{x}}$ is calculated to be 0.04 pixel, as indeed has been obtained for the feature in Figure 1. Similarly, for a larger circular feature of $200 \mu \mathrm{~m}$, the precision will be a factor 2 less or 0.02 pixel which also agrees with that shown in Figure 2. As stated above, a blurred image will be larger than a sharp image of the same nominal size because of the effects of blur, and therefore, the factor N of a blurred image in equation 1 will be larger than for a sharp image, while $\sigma_{g^{\prime}} \quad$ will be slightly smaller. This appears to $\mathrm{ex}_{\mathrm{k}}$ the better results obtained for blurred images than for shi $\omega$ images. These results demonstrate an agreement between the theoretical precisions and those obtained by simulation.

## 5. Conclusions

## (a) Least Squares Method

(i) Tests on the precision of image matching by the method of least squares indicate values as high as . 02 pixel can be achieved with cross and circular features, but precisions will deteriorate for quantization levels less than 5 bits.
(ii) The extent of image blur has only a marginal effect on the precision of matching, with precisions improving slightly as the blur increases up to that described by a Gaussian spread function with a $2 \sigma$ width of $50 \mu \mathrm{~m}$.
(iii) The overall differences in the quality of matching for the cross and circle features are only marginal, with generally better results being obtained for crosses than for circles.
(iv) Scale variations between the two images greater than $10 \%$ will lead to a significant decrease in the precision of image matching.
(v) There is good agreement between the previously published theoretical precisions and those obtained by simulation in this study.

## (b) Cross-Correlation

(i) The precisions of matching by cross-correlation for images in which no distortions exist are only marginally worse than for the least squares method.
(ii) The effects of distortions between the two images result in a rapid deterioration in the quality of matching by the cross-correlation method.

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