# ITERATIVE ALGORITHM FOR MULTIDIMENSIONAL IMAGE ANALYSIS

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The paper considers the method of segmentation and analysis of one class of multidimensional speckled images; the specific feature of these images is the presence of some statistical dependence between the value of one component and the validity of value assessment of the other. The proposed method provides a common approach to segmentation procedure irrespective of physical processes feature of image generation under stated conditions. For normal parameter assessment distribution based on a maximum a posteriori probability criterion. Each region being segmented is assigned an index. At the same time algorithm allows to carry out analysis of the reference scene, since its output is not only a segmented image but also characteristics of each of regions being extracted (location and parameter vector).

KEY WORDS: Algorithm, Image Analysis, Image Processing, Remote Sensing Application

### INTRODUCTION

In the automatic analysis of scenes, the main problem is the conversion of information, the image of a scene represented as a two-dimensional function, into some description of this image. This description can be stored in a memory section which is thousands times smaller than that required for storing the image. At the same time, the information contained in the original image and essential for the above analysis is retained and converted into a processable form (Duda, 1973).

An indispensable stage of image processing aimed at preparing its description is segmentation which consists in fragmenting the image into regions which are coherent by some attribute. Segmentation can be aimed at discerning scene objects and suppressing insignificant details (Борисенко, 1987).

We shall have a look at one of numerous image types - a class of 4-dimensional images which feature some statistical correlation between the observed value of one component and the degree of certainty of the estimated value of another component. This correlation is characteristic of multidimensional speckle images generated by remote sensing facilities, such as radars, radio-optical ranging and detection systems and laser radars. In the course of the generation of these images, each pixel (x,y) is assigned a detector-outputted signal/noise ratio and some parameter of the reflected signal is estimated. The fourth, estimated component of the image can be represented by such parameters as the velocity, range, radiation polarization factor, etc, or their combination, for which the degree of estimate certainty is probabilistically related to the signal/noise ratio.

Images obtained by means of coherent systems feature a specific effect. The

reflected signal which is received is a superposition of functions of scattering by a set of elementary reflecting surfaces, characterized by different phases and amplitudes. Interference results in that in each pixel the intensity of the reflected signal takes random, Weibull-distributed value, which generates the so-called speckle structure. The speckle structure of the image has a considerable effect on the accuracy of estimates of the component being evaluated, which, in the first approximation, is specified by the ratio (Dansac, 1985):

$$\sigma \sim \frac{1}{\sqrt{A / A_g}}$$
, (1)

where A is the intensity of the signal received, A<sub>g</sub> is the power of internal noises. Therefore values of different elements of the speckle image component being estimated can have essentially different estimate variances resulting from occasional fluctuations.

For the above class of images, (Lisitsyn, 1990; Лисицын, 1990) proposes an iterative algorithm for segmenting Doppler laser radar images. This algorithm formed a binary image whose pixels belonging to patterns which represented moving objects had a value of "1" and the remaining pixels had a value of "0". Drawbacks of the algorithm are instable operation when patterns representing different objects touched each other or scene objects partially shade each other. Moreover, that the segmented image is binary makes it difficult to resolve discerned patterns.

The algorithm which will be proposed below can be considered as a generalization of the algorithm for binary segmentation of Doppler laser radar images, to the case of nonbinary segmentation and its extension to other types of images belonging to the same class.

For definiteness sake, let a Doppler frequency generated by remote sensing a surface with a heterodyne-reception infrared coherent laser radar be the parameter to be estimated. Segmentation will be considered to be aimed at discerning patterns of moving objects of the scene.

### PROBLEM FORMALIZATION

Given: the Doppler image specified in the form of a M×N frequency matrix  $F=\{f_{xy}\}$ , a similar intensity matrix  $A=\{a_{xy}\}$  and image segment types numbered from 0 to R-1. Assume that each pixel (x,y) can take an arbitrary state  $s_{xy}$ , corresponding to one of the segment type Nos. Segmentation is aimed at generating the image Q consisting of the subset  $Q_i$ 

 $U Q_i = Q \text{ and } Q_i \cap Q_i = \emptyset \text{ at } \forall i \neq j, \quad (2)$ 

where  $Q_i = \{(x, y) : s_{xy} = L_i\}, L_i \sim (0, ..., R-1).$ 

The segmentation is implemented according to a posteriori probability maximum criterion

$$\mathbb{P}(\mathbb{Q}_{1}, \dots, \mathbb{Q}_{k}|\mathbb{F}) = \frac{\mathbb{P}(\mathbb{F}|\mathbb{Q}_{1}, \dots, \mathbb{Q}_{k})\mathbb{P}(\mathbb{Q}_{1}, \dots, \mathbb{Q}_{k})}{\mathbb{P}(\mathbb{F})} \xrightarrow{(3)}{\max},$$

where  $P(Q_1, \ldots, Q_k | F)$  - is the probability of  $Q_1, \ldots, Q_k$  regions presence in image on condition that F image is observed;  $p(F|Q_1, \ldots, Q_k)$  - is joint probability density of all pixels Doppler frequencies on condition that image is partitioned into regions  $Q_1, \ldots, Q_k$ ;  $P(Q_1, \ldots, Q_k)$  - is a probability of  $Q_1, \ldots, Q_k$  regions presence in image; p(F) - is unconditional joint probability density of all pixels Doppler frequencies.

Consider the cofactor  $P(Q_1, \ldots, Q_k)$  in (3). To describe laser radar images, use can be made of Markovian random-field models (Kelly,1988; Besag,1974; Derin, 1986; Hanson, 1982; Therrien, 1986) where each region is described by its own stationary random process and the transition from one image region to another is modeled by a Markovian process.

Let us use  $S_{(xy)}$  to designate a set of states of eight pixels adjacent to (x,y) and  $\Omega_{(xy)}$  to symbolize a set of states of all the pixels without (x,y). Now the Markovian model satisfies

$$\mathbb{P}(\mathbb{Q}_{j},\ldots,\mathbb{Q}_{k}) = \mathbb{P}(\mathbb{S}_{xy},\Omega_{(xy)}) = (4)$$

 $= \mathbb{P}(\mathbf{s}_{xy} | \boldsymbol{\Omega}_{(xy)}) \mathbb{P}(\boldsymbol{\Omega}_{(xy)}) = \mathbb{P}(\mathbf{s}_{xy} | \mathbf{S}_{(xy)}) \mathbb{P}(\boldsymbol{\Omega}_{(xy)})$ 

where  $P(s_{xy}|S_{(xy)})$  - is the (x,y) pixels s state probability on condition that neighbouring pixels have  $S_{(xy)}$  states, corresponding to specified image partitioning.

It is known that the Markovian model satisfies the Gibbs distribution (Derin, 1986), which can be written as

$$\mathbb{P}(\mathbb{Q}_{1},\ldots,\mathbb{Q}_{k}) = \frac{1}{\mathbb{B}_{0}} \exp \left\{ \frac{1}{\mathbb{T}} \sum_{C \in \mathbb{C}} \mathbb{V}_{C}(\mathbb{Q}) \right\}$$
(5)

where c is the pixel set termed the clique which consists either of individual pixels or of their groups, satisfying the condition that if  $(i,j) \in c$ and  $(k,l) \in C$  for  $(i,j) \neq (k,l)$ , then (i,j)and (k,l) are adjacent pixels. C is the set of cliques belonging to different types.  $V_C(Q_1, \ldots, Q_k)$  is the function depending only on pixels of type "C" cliques, intended for the specified fragmentation of the image and termed the potential function. T is the constant.  $B_0$ is the normalization factor.

To solve the above problems, it is expedient to determine clique types in accordance with Fig.1.





Fig.1. Clique Types for 8-Connection Neighbourhood of Pixel (x,y).

A potential function for the Type  $\iota$ clique consisting of two pixels  $(x_{_{1}}y_{_{1}})$ and  $(x_{_{2}}y_{_{2}})$  can be specified in the form

$$\mathbb{V}_{l}((x_{1}y_{1}), (x_{2}y_{2})) = \begin{cases} \beta_{l}, \text{if } s_{x_{1}}y_{1} = s_{x_{2}}y_{2} \\ -\beta_{l} \text{ else} \end{cases}$$
(6)

where  $\beta_l$  is the parameter corresponding to the Type *l* clique. For individual-pixel cliques, the potential function can be defined as

$$\nabla_{l}(x,y) = \alpha_{l}, \text{ if } S_{xy} = l \tag{7}$$

where  $\alpha_l$  is the parameter associated with the Type *l* clique. Then the potential function for the type "C" clique,  $V_c(Q)$ , specified all over the image Q will be a sum of potential functions (6) or (7) for the entire image.

The normalization factor  $B_0$  shall be selected on the basis of the condition

$$\sum_{i=1}^{n} (\mathbf{s}_{11}, \dots, \mathbf{s}_{NM}) = \frac{1}{B} \sum_{o i=1}^{n} \left\{ \frac{1}{T} \sum_{o \in C} (\mathbf{s}_{11}, \dots, \mathbf{s}_{NM}) \right\} = 1$$
(8)

where n is the number of possible segmentations of the image.

Now we shall make specific the potential function type for the above clique types. In accordance with (6) and using the function  $\operatorname{sign}(x)$ , which takes the values

$$\operatorname{sign}(x) = \begin{cases} 1, \text{ if } x > 0\\ 0, \text{ if } x = 0 \end{cases}$$

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we can write

$$\begin{aligned} \nabla_{k} &= a_{k} \left[ M \cdot N - \sum_{y=1}^{M} \sum_{x=1}^{N} \operatorname{sign} \left| s_{xy} - k_{zy} \right| \right] \end{aligned} \tag{9} \\ \nabla_{k+1} &= \beta_{1} \left[ M (N-1) - 2 \sum_{y=1}^{M} \sum_{x=1}^{N-1} \left| s_{xy} - s_{x+1,y} \right| \right] \\ \nabla_{k+2} &= \beta_{2} \left[ (M-1) N - 2 \sum_{y=1}^{M-1} \sum_{x=1}^{N} \operatorname{sign} \left| s_{xy} - s_{x,y+1} \right| \right] \\ \nabla_{k+3} &= \beta_{3} \left[ (M-1) (N-1) - 2 \sum_{y=1}^{M-1} \sum_{x=2}^{N-1} \left| s_{xy} - s_{x-1,y+1} \right| \right] \\ \nabla_{k+4} &= \beta_{4} \left[ (M-1) (N-1) - 2 \sum_{y=1}^{M-1} \sum_{x=1}^{N-1} \left| s_{xy} - s_{x+1,y+1} \right| \right] \end{aligned}$$

Here k is the number of image area types,  $V_k$  are potential functions for individual -pixel cliques (type "a") and for different area types,  $V_{k+1}$  to  $V_{k+4}$  are potential functions for cliques of types "b","c", "d","e".

In acccordance with (4), the probability of the specified segmentation can be split into two parts

$$P(Q_{1},\ldots,Q_{n}) = \frac{1}{D_{0}} \exp\left\{\sum_{l=1}^{6} \frac{V_{l}(S_{(x,y)})}{T}\right\} \times \exp\left\{\sum_{j=1}^{6} \frac{V_{j}(Q \setminus S_{(x,y)})}{T}\right\}$$
(10)

Then, in view of (9), we have

$$\exp\left\{\sum_{t=1}^{6} \frac{\nabla_{t} (S_{(x,y)})}{T}\right\} = \exp\left\{\beta_{1}\left[2-\operatorname{sign}|k-s_{x+1,y}|-s_{x+1,y}|\right] + \beta_{2}\left[2-\operatorname{sign}|k-s_{x,y+1}|-s_{x+1,y-1}|\right] + \beta_{3}\left[2-\operatorname{sign}|k-s_{x-1,y+1}|-s_{x+1,y-1}|\right] + \beta_{4}\left[2-\operatorname{sign}|k-s_{x-1,y-1}|-s_{x+1,y-1}|\right] + \beta_{4}\left[2-\operatorname{sign}|k-s_{x-1,y-1}|-s_{x+1,y-1}|\right] + \beta_{4}\left[2-\operatorname{sign}|k-s_{x-1,y-1}|-s_{x+1,y+1}|\right] + \alpha_{k}\left[1-\operatorname{sign}|s_{xy}-k|\right]\right\}$$

$$(11)$$

Provided certain assumptions are made, (11) can be further simplified.

## SYNTHESIS OF SEGMENTATION ALGORITHM

The sliding-window image processing (Therrien, 1986; Pratt, 1978) is an acceptable technique which can be employed for solving the problem.

Let some initial Doppler image partitioning is given and all pixels are assigned specific state values. The initial segmentation procedure will be discussed below.

After that we choose an arbitrary (x,y) pixel and superpose the window center on it. Then we vary the central pixel state without other pixels state change and calculate the corresponding values of a posteriori probability (with an accuracy of up to 1/P(F)

 $\begin{array}{l} {}_{\mathbf{P}}(\mathbf{F} \mid \mathbf{Q}) \mathbf{P}(\mathbf{Q}) = {}_{\mathbf{P}}(\mathbf{F} \mid \mathbf{s}_{xy} = \mathbf{L}, \mathbf{S}_{(xy)}) \mathbf{P}(\mathbf{s}_{xy} = \mathbf{L}, \boldsymbol{\Omega}_{(xy)}) \end{array}$ Note that

$$p(F|s_{xy}=L,S_{(xy)}) = p(f_{xy}|F_{(xy)},s_{xy}=L,S_{(xy)}) = = p(F_{(xy)}|s_{xy}=L,S_{(xy)}),$$

where  $F_{(xy)}$  is the image F without the pixel (x,y). The second cofactor in the right-hand side of the relation does not depend on  $S_{xy}$ , so account can be taken only of  $P(f_{xy}|F_{(xy)},s_{xy}=L,S_{(xy)})$ .

In view of the remarks made and of (3) the search for the maximum value of a *posteriori* probability at pixel (x,y) state variation is reduced to maximizing the expression

 $\begin{array}{l} {} p \left( f \right|_{xy} | F \right|_{xy} = L, S \\ (xy) \end{array} \right) P \left( s \right|_{xy} = L | S \right) \longrightarrow \max \\ (xy) uarL \\ and the assignment of a new state to a$ (x,y) pixel. It is necessary to apply the given rule for all pixels to get a more precise image partitioning and then toiterate all the procedure. As a result we get a following rule of Doppler image segmentation

$$P_{L}(f_{xy}|F_{(xy)})P(s_{xy}^{n+1}=L|S_{(xy)}^{n})\xrightarrow{var=L} \max^{(12)}$$

where  $s_{xy}^{n+1}$  is a state of a (x,y) pixel for a current iteration step;  $S_{(xy)}^n$  are states of the window neighbouring pixels at the previous iteration step;  $p_L(f_{xy}|F_{(xy)}) - a$  conditional probability density of (x,y) pixel on condition that (x,y) pixel has  $s_{xy} = L$ , and the window neighbouring pixels which were assigned s = L at the previous step, have frequency values  $F_{(xy)}^{n}$ .

Now consider the cofactor  $P(F|Q_1, \dots, Q_k)$ of (3). Since all  $F_i$  are independent,  $p(F|Q_1, \dots, Q_k) = p(F|Q_1)p(F|Q_2) \times \dots \times p(F|Q_k)$  Doppler frequency measurement errors can be taken to be normally distributed, meaning that actual Doppler signals are described by narrow-band normal random processes (Papurt, 1981), characterized by an asymptotically normal instantaneousfrequency distribution at a high signal/noise ratio (Левин, 1974). Then, considering a set of the pixels  $F_i$ corresponding to  $Q_i$  as a vector and with an expectation  $G_i$ , the conventional joint density of the probability of Doppler frequencies of the area  $Q_i$  pixels can be written as

$$p(\mathbf{F}_{i} | \mathbf{Q}_{i}) = \frac{1}{\sqrt{(2\pi)^{n} i | \mathbf{M}_{i} |}} \exp\left\{-\frac{1}{2} (\mathbf{F}_{i} - \mathbf{G}_{i})^{\mathrm{T}} \mathbf{M}_{i}^{-1} (\mathbf{F}_{i} - \mathbf{G}_{i})\right\}$$
(13)

where  $\mathbf{M}_i$  is the correlation matrix of measurement errors,  $|\mathbf{M}_i|$  is the matrix  $\mathbf{M}_i$  determinant,  $\mathbf{N}_i$  is the vector  $\mathbf{F}_i$  number of dimensions, equal to the number of region  $\mathbf{Q}_i$  elements.

Since frequency measurement errors of different pixels do not correlate (Sullivan, 1980; Wang, 1984), the correlation matrix M is diagonal

$$\mathbf{M}_{i} = \begin{bmatrix} \sigma_{1}^{2} & & & \\ & \sigma_{2}^{2} & & \\ & & \sigma_{2}^{2} & & \\ & & & \sigma_{n_{i}}^{2} \end{bmatrix} , \qquad (14)$$

where  $\sigma_l^2$  is the Doppler frequency variance for the *l*-th pixel of the  $Q_i$  region.

For definiteness sake, let elements of two regions,  $\mathbb{Q}_{k}$  and  $\mathbb{Q}_{m}$ , which will be designated by  $\mathbb{Q}_{k} = \mathbb{Q}_{k} \cap \mathbb{W}$  and  $\mathbb{Q}_{m}^{*} = \mathbb{Q}_{m} \cap \mathbb{W}$ , will be in the current position, in the sliding window  $\mathbb{W}$ . It is necessary to determine the state  $S_{xy}$  of the central pixel (x,y), i.e. to select one of the following hypotheses:  $\mathbb{H}_{0}^{-}(x,y) \in \mathbb{Q}_{k}$  and  $S_{xy} = \mathbb{K}$  or  $\mathbb{H}_{1}^{-}(x,y) \in \mathbb{Q}_{m}$  and  $S_{xy} = \mathbb{M}$ . Substituting (11), (13) in (12) and taking the logarithm of it, we obtain

$$\sum_{i \in Q_{k}^{*} \cup (x,y)} \left[ \ln \sigma_{i} + \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2}}{2\sigma_{i}^{2}} \right] + \sum_{i \in Q_{m}^{*}} \left[ \ln \sigma_{j} + \frac{\left(\mathbf{f}_{j} - \mathbf{g}_{j}\right)^{2}}{2\sigma_{j}^{2}} \right] - \left[ -\mathbf{E} \right]_{(x,y) \in \mathbf{Q}_{k}^{*}} \sum_{\substack{s_{xy} = \mathbf{k} \\ s_{xy} = \mathbf{k}}} \sum_{i \in \mathbf{Q}_{k}^{*}} \left[ \ln \sigma_{i} + \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2}}{2\sigma_{i}^{2}} \right] + \sum_{\substack{s_{xy} = \mathbf{k} \\ s_{xy} = \mathbf{k}}} \left[ 1 - \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2}}{2\sigma_{i}^{2}} \right] + \sum_{\substack{s_{xy} = \mathbf{k} \\ s_{xy} = \mathbf{k}}} \left[ 1 - \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2}}{2\sigma_{i}^{2}} \right] + \sum_{\substack{s_{xy} = \mathbf{k} \\ s_{xy} = \mathbf{k}}} \left[ 1 - \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2}}{2\sigma_{i}^{2}} \right] + \sum_{\substack{s_{xy} = \mathbf{k} \\ s_{xy} = \mathbf{k}}} \left[ 1 - \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2}}{2\sigma_{i}^{2}} \right] + \sum_{\substack{s_{xy} = \mathbf{k} \\ s_{xy} = \mathbf{k}}} \left[ 1 - \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2}}{2\sigma_{i}^{2}} \right] + \sum_{\substack{s_{xy} = \mathbf{k} \\ s_{xy} = \mathbf{k}}} \left[ 1 - \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2}}{2\sigma_{i}^{2}} \right] + \sum_{\substack{s_{xy} = \mathbf{k} \\ s_{xy} = \mathbf{k}}} \left[ 1 - \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2}}{2\sigma_{i}^{2}} \right] + \sum_{\substack{s_{xy} = \mathbf{k} \\ s_{xy} = \mathbf{k}}} \left[ 1 - \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2}}{2\sigma_{i}^{2}} \right] + \sum_{\substack{s_{xy} = \mathbf{k} \\ s_{xy} = \mathbf{k}}} \left[ 1 - \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2}}{2\sigma_{i}^{2}} \right] + \sum_{\substack{s_{xy} = \mathbf{k} \\ s_{xy} = \mathbf{k}}} \left[ 1 - \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2}}{2\sigma_{i}^{2}} \right] + \sum_{\substack{s_{xy} = \mathbf{k} \\ s_{xy} = \mathbf{k}}} \left[ 1 - \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2}}{2\sigma_{i}^{2}} \right] + \sum_{\substack{s_{xy} = \mathbf{k} \\ s_{xy} = \mathbf{k}}} \left[ 1 - \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2}}{2\sigma_{i}^{2}} \right] + \sum_{\substack{s_{xy} = \mathbf{k} \\ s_{xy} = \mathbf{k}}} \left[ 1 - \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2}}{2\sigma_{i}^{2}} \right] + \sum_{\substack{s_{xy} = \mathbf{k} \\ s_{xy} = \mathbf{k}}} \left[ 1 - \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2}}{2\sigma_{i}^{2}} \right] + \sum_{\substack{s_{xy} = \mathbf{k}}} \left[ 1 - \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2}}{2\sigma_{i}^{2}} \right] + \sum_{\substack{s_{xy} = \mathbf{k}}} \left[ 1 - \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2}}{2\sigma_{i}^{2}} \right] + \sum_{\substack{s_{xy} = \mathbf{k}}} \left[ 1 - \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2}}{2\sigma_{i}^{2}} \right] + \sum_{\substack{s_{xy} = \mathbf{k}}} \left[ 1 - \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2}}{2\sigma_{i}^{2}} \right] + \sum_{\substack{s_{xy} = \mathbf{k}}} \left[ 1 - \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2}}{2\sigma_{i}^{2}} \right] + \sum_{\substack{s_{xy} = \mathbf{k}}} \left[ 1 - \frac{\left(\mathbf{f}_{i} - \mathbf{g}_{i}\right)^{2$$

+ 
$$\sum_{i \in Q_m^* \cup (x,y)} \left[ \ln \sigma_j + \frac{y - \sigma_j^2}{2 \sigma_j^2} \right] - \mathbb{E} \left|_{(x,y) \in Q_m^*} \right|$$

where E is the exponential function index in (11).

Expression (15) includes the unknown parameters  $g_i$ . To use the proposed algorithm,  $g_i$  shall be substituted by the estimates  $g_i$ . In Doppler images, moving object patterns are planes. In this case  $f_{xy}$  of an object element is defined by the expression

$$\mathbf{f}_{xy} = \left[ \mathbf{g}_i + \mathbf{\varepsilon}_i x + \mathbf{\omega}_i y \right] \mathbf{q} \tag{16}$$

where  $g_i$  is a projection of an object translatory motion on to the line of sight;  $\varepsilon_i, \omega_i$  are angular rates of an object rotation relative to the orthogonal axes lying in the plane perpendicular to the line of sight; q is a proportionality factor; x, y are the pixel coordinates.

Then the vector  $\mathbf{F}_{i}$  can be represented as

$$\mathbf{F}_{i} = \mathbf{T}_{i} \mathbf{G}_{i} \mathbf{q} + \boldsymbol{\Theta}_{i} \tag{17}$$

where  $\mathbf{T}_i = (\mathbf{I}_i, \mathbf{X}, \mathbf{Y})$  is the transformation matrix of dimension  $n_i \times 3$ ;

 $\mathbf{G}_{i} = (\mathbf{g}_{i}, \mathbf{g}_{i}, \boldsymbol{\omega}_{i})^{\mathrm{T}}$  is the ith region velocity vector;  $\boldsymbol{\theta}_{i}$  is the measurement errors vector;  $\boldsymbol{n}_{i}$  is the number of pixels in ith region.

The vector  $G_i$  can be found by the least-squares technique for varying-accuracy measurements. It is easy to demonstrate that it is accomplished by solving the following linear-equation set

$$\begin{bmatrix} g_{i} \sum_{j=1}^{n_{j}} a_{j} + \varepsilon_{i} \sum_{j=1}^{n_{j}} x_{j} a_{j} + \omega_{i} \sum_{j=1}^{n_{j}} y_{j} a_{j} \end{bmatrix} q = \sum_{j=1}^{n_{j}} f_{j} a_{j}$$

$$\begin{bmatrix} g_{i} \sum_{j=1}^{n_{j}} x_{j} a_{j} + \varepsilon_{i} \sum_{j=1}^{n_{j}} x_{j}^{2} a_{j} + \omega_{i} \sum_{j=1}^{n_{j}} x_{j} y_{j} a_{j} \end{bmatrix} q = \sum_{j=1}^{n_{j}} f_{j} x_{j} a_{j}$$

$$\begin{bmatrix} g_{i} \sum_{j=1}^{n_{j}} y_{j} a_{j} + \varepsilon_{i} \sum_{j=1}^{n_{j}} x_{j} y_{j} a_{j} + \omega_{i} \sum_{j=1}^{n_{j}} y_{j}^{2} a_{j} \end{bmatrix} q = \sum_{j=1}^{n_{j}} f_{j} y_{j} a_{j}$$

Here summation is taken over all the pixels of the region  $Q_k^{\star}$  or  $Q_m^{\star}$ . Since the found value of the vector estimate  $G_i^{\star}$  has its own correlation matrix of errors, the refined variances  $\sigma_i^2$  and  $\sigma_j^2$  shall be substituted for  $\hat{\sigma}_i^2$  and  $\hat{\sigma}_j^2$  in (15). When the central pixel state varies, its value has an effect on unknown parameter estimates being computed. It results in that in (15), values of  $\sigma_i^2$ ,  $\sigma_j^2$ ,  $\hat{g}_i$ ,  $\hat{g}_j$ 

in the left-hand side and right-hand side are not the same and hypothesis comparison computations are very tedious. However, if the central pixel (x,y) value is ignored in determining the above parameters, i.e. the predicted value is in fact computed, all the values of  $\sigma_i^2$ ,  $\sigma_j^2$ ,  $\hat{g}_i$ ,  $\hat{g}_j$  are the same, excluding estimates for the pixel being analysed. Then (15) will transform into

$$\left[\ln \hat{\sigma}_{xy} + \frac{(\mathbf{f}_{xy} - \hat{g}_{xy})^2}{2 \hat{\sigma}_{xy}^2} + \mathbf{E}\right] \xrightarrow[\mathbf{B}_{xy}=\mathbf{k},\mathbf{m}]{} \stackrel{(19)}{\underset{\mathbf{B}_{xy}=\mathbf{k},\mathbf{m}}{\overset{(19)}{\underset{\mathbf{B}_{xy}=\mathbf{m}}{\overset{(19)}{\underset{\mathbf{B}_{xy}=\mathbf{m}}{\overset{(19)}{\underset{\mathbf{B}_{xy}=\mathbf{m}}{\overset{(19)}{\underset{\mathbf{B}_{xy}=\mathbf{m}}{\underset{\mathbf{B}_{xy}=\mathbf{m}}{\underset{\mathbf{B}_{xy}=\mathbf{m}}{\overset{(19)}{\underset{\mathbf{$$

Now, in the case of the pixel (x,y) state variation, the estimates  $\hat{g}_{xy}$  and  $\sigma_{xy}$  are obtained on the basis of the set of the elements of the area  $Q_k^{\star}$  (at  $s_{xy} = k$ ) or of the area  $Q_m^{\star}$  (at  $s_{xy} = m$ ). As has been noted, the value of  $\hat{g}_{xy}$  can be found by solving system (18). To determine  $\hat{\sigma}_{xy}$ , a correlation matrix of vector  $G_i$  estimate errors shall be found. In accordance with (Тихонов, 1982)

$$R_{G} = (B)^{-1} = (T^{T} M^{-1} T)^{-1}$$
 (20)

where T = (I, X, Y) is the  $3 \times n$ matrix; I is the unit vector; X, Y are the vectors of the coordinates of the area  $Q^*$  elements; M is the correlation matrix of measurement errors. Substituting the above quantities in (20), we obtain

$$\mathbf{R}_{\mathrm{G}} = (\mathbf{B})^{-1} = \left\{ \begin{array}{ccc} \sum \frac{1}{\sigma_{i}^{2}} & \sum \frac{x_{i}}{\sigma_{i}^{2}} & \sum \frac{y_{i}}{\sigma_{i}^{2}} \\ \sum \frac{x_{i}}{\sigma_{i}^{2}} & \sum \frac{x_{i}}{\sigma_{i}^{2}} & \sum \frac{x_{i}y_{i}}{\sigma_{i}^{2}} \\ \sum \frac{y_{i}}{\sigma_{i}^{2}} & \sum \frac{x_{i}y_{i}}{\sigma_{i}^{2}} & \sum \frac{y_{i}^{2}}{\sigma_{i}^{2}} \end{array} \right\}^{-1}$$

and matrix  $\boldsymbol{R}_{g}$  elements are defined by the expression

$$\mathbf{r}_{ij} = \frac{(-1)^{i+j} \mathbf{m}_{ij}}{\det (\mathbf{B})}$$
(22)

where  $M_{ij}$  is the corresponding minor of the matrix **B**.

Since the pixel (x,y) coincides with the window centre and its coordinates in the sliding-window coordinate system are (0,0)

$$\hat{\sigma}_{xy}^2 = \sigma_{xy}^2 + \frac{\mathbf{m}_{11}}{\det (\mathbf{B})}$$
(23)

The result can be easily generalized to the case in which there are more than two hypotheses.

## INITIAL SEGMENTATION PROCEDURE.

Now consider the initial segmentation procedure. To implement the proposed algorithm, very stringent requirements shall be imposed on the initial segmentation since in case any segment does not get its No. during the initial segmentation, its will be impossible to further segment it, provide it with an identifying index and determine its parameters. To solve this problem, the following two-step procedure can be proposed.

The first step is the detection of planes and the second step consists in their marking. Planes shall be detected as follows.

1. The image F shall be processed with a sliding window whose size shall be deliberately smaller than the expected sizes of regions to be discerned. In each window position, regression coefficients used in plane approximation of the window by the least-squares technique for varying-accuracy measurements shall be estimated with regard for all the window pixels and proceeding from the hypothesis that all elements belong to the plane. Use shall made of expression (18).

2. Squares of the distances  $d_i$  between the observed values of the window pixels F and the approximation plane shall be determined.

3. Variances of frequency estimates for the window pixels shall be determined by

$$\sigma_{xy}^{2} = \sigma_{xy}^{2} + \frac{1}{\det(B)} \left\{ m_{11} + x^{2} m_{22} + y^{2} m_{33} - 2x m_{12} + 2y m_{13} + 2xy m_{23} \right\}$$
(24)

where det(B) is the matrix B determinator,  $M_{ij}$  is the corresponding minor of the matrix B,  $\sigma_{xy}$  is deterined by (1).

4. The quantity

$$S = \sum_{i=1}^{n} d_i^2 / \hat{\sigma}_i^2$$
 (25)

(where n is the number of the window pixels) shall be determined. In case all the window pixels are in a plane, the quantity S is characterized by the distribution  $\chi^2(n)$  with n degrees of freedom.

5. The confidence interval (0,0) shall be specified in accordance with values of percentage points of the distribution  $\chi^2(n)$ . If S  $\in$  (0,0), then the window

shall be considered to be entirely in the plane. If S > C, the window shall be considered to be at a junction of different planes. In this case the central pixel shall not be marked out.

Plane marking-out shall be done parallel to plane detection and in the following order.

1. Plane approximation parameters shall be memorized as k-indexed region parameters (it is initially assumed that the number of marked-out regions is k-1). The central pixel of the segmented image shall be assigned a value of k. A correlation matrix of approximation parameter estimate  $R_G$  errors shall be memorized simultaneously. In all cases, approximation parameters shall be compared with previously recorded parameters of areas whose elements are in some window  $W_j$  and relative to the central pixel ( $W_j$  may be incoincident with W). The comparison is done with regard for  $R_G$ . If such parameters are already existing, for example, in the n-indexed region, then the central pixel is assigned a value of n.

2. If elements of  $_{1}k$  -and  $k_{2}$ -indexed regions, having the same parameters G are present in the neighbourhood, the equivalence  $k_{1}=k_{2}$  is fixed. Upon processing the entire image, these indices shall be reassigned, giving the priority to the least index.

#### ALGORITHM GENERALIZATIONS.

Upon segmenting, parameters of approximation of the discerned regions shall be refined with regard for all the pixels. Correlation matrices of errors of estimates of these parameters shall be computed simultaneously. Coordinates of each of the regions and numbers of their elements shall be determined. The algorithm outputs a segmented image in which each of the discerned regions has an identifying index, a parameters vector comprising its coordinates, regression coefficients used in the discerned-area plane approximation, a correlation matrix of estimation errors and the number of pixels in the area.

It is not difficult to apply this algorithm to other types of images of the class under consideration if the proposed models are valid for describing the images. In the case that other image models must be used, the algorithm shall be adapted. For example, in segmenting ranging images if scene objects are not describable by first-order surfaces. As a rule, such problems can be solved using a model which describes the image by firstand second- order surfaces. Then if S determined by (25) is greater than the threshold C, it is necessary to check during the initial segmentation whether it is possible to approximate sliding-window elements with a second-order surface. To this end, a sixth-order linear equation set must be solved. Then approximation parameters obtained at the initial segmentation stage shall be taken into account when determining  $\hat{g}_i$  values during iterative segmentation. Determining  $\hat{\sigma}_i$  presents some problems. However, in practice, use can be made of the lower-bound estimate of  $\sigma_i$  in accordance with (1).

#### MODELLING RESULTS.

Mathematical modelling has shown that the algorithms are workable with different types of images. The segmented image is usuallu generated in 5 to 10 algorithm iterations. Depending on the scene complexity, the labour requirements for Doppler images are 300 to 350 operations per pixel. The labour requirements for the initial segmentation procedure is about 40% of the total labour requirements for the algorithm.

Fig.2 shows an image which is a source for the Doppler image. Patterns of moving objects touch and shade each other. Fig.3 shows an image of the reflected signal intensity. Intensity generation has been based on the assumption that propagation and reflection conditions of all the pixels are the same. Fig. 4 shows the image F. All the images have been quantized into 16 levels. Fig.5 shows a segmented image, a result of the initial segmentation. Fig.6 shows a final image obtained after 8 algorithm iterations.

Good results have been also obtained in applying the algorithms to other types of images. However, when second-order surfaces are used in the model, the accuracy of estimated-component measurement shall be very high.

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Fig.6. The final segmented image.

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