DIFFERENTIAL PHOTOGRAMMETRY

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ABSTRACT

A novel photogrammetric mensuration procedure is described, which does not need the assumption of metrically correct photographs, or the assumption of additional parameters to compensate for deformations. The approach is based on measuring and processing appropriate differences of the image coordinate. The approach is superior to conventional photogrammetric methods and is essential for good results in satellite photogrammetry, and when using non-metric cameras.

Classical Photogrammetry Revisted

Let us start with the well known projective relationships relating the terrain coordinates N,E,H, of point P to image coordinates x,y of image (i).

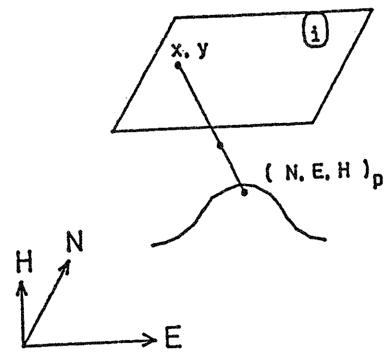

Figure 1: The Projective Relationship

$$xp^{(i)} = C* \frac{a_1*(Np-No) + a_2*(Ep-Eo) + a_3*(Hp-Ho)}{a_7*(Np-No) + a_8*(Ep-Eo) + a_9*(Hp-Ho)} = Px^{(i)}\ (Np,Ep,Hp)$$

$$yp^{(i)} = C* \frac{a_4*(Np-No) + a_5*(Ep-Eo) + a_6*(Hp-Ho)}{a_7*(Np-No) + a_8*(Ep-Eo) + a_9*(Hp-Ho)} = Py^{(i)}\ (Np,Ep,Hp)$$

Where $Px^{(i)}$, $Py^{(i)}$ denote the projective operators.

In many applications, systematic image deformations are present. These are image deformations, which are dependent on the position of the image point and repeat from image to image. The physical causes of these deformations are atmospheric refraction, optical aberration and deformation of the filmbase during processing.

The systematic deformations are usually accommodated by adding a function f(x,y) of the image coordinate to the projective relations.

$$xp^{(i)} = Px^{(i)}(Np,Ep,Hp) + Fx^{(i)}(Xp^{(i)}, Yp^{(i)})$$
$$yp^{(i)} = Py^{(i)}(Np,Eo,Hp) + Fy^{(i)}(Xp^{(i)}, Yp^{(i)}) \qquad (1)$$

For block adjustment, F is approximated by polynomials in x and y with a limited number of unknown coefficients.

This approach, although valid, poses a large number of practical problems, such as: which polynomial to choose, how to separate systematic deformations from blunders, which ground control distribution is necessary for avoiding under-determination etc. (see also Kubik, 1988).

We therefore propose here a novel and elegant approach to this problem, which also is valid for other projections such as existing in remote sensing, tomography and other fields.

The Origin of Differential Photogrammetry

We regard two image points at identical locations (x,y) in adjacent photographs, in a photogrammetric strip.

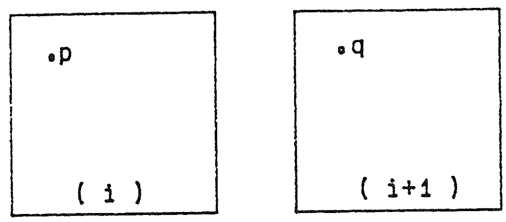

Figure 2

(see figure 2). These two points have systematic image deformations of the same magnitude. When forming the difference of their projective relations (1), the systematic deformation terms F(x,y) are eliminated:

$$\delta x^{(i)} = xq^{(i+1)} - xp^{(i)} = Px^{(i+1)}\ (Nq,Eq,Hq) - Px^{(i)}\ (Np,Ep,Hp),$$
$$\delta y^{(i)} = yq^{(i+1)} - yp^{(i)} = Py^{(i+1)}\ (Nq,Eq,Hq) - Py^{(i)}\ (Np,Ep,Hp). \quad (2)$$

We thus have a formulation of the photographic projection problem, and indeed for all projections, which is independent of the knowledge of the exact projective relationship. No assumption has to be made regarding systematic image deformation, or even on the nature of projection.

Let us consider a photogrammetic strip of 5 photographs with 60% forward overlap, and a regular grid of 15 points projected into the 9 standard positions on the photographs (see figure 3). The image coordinates were measured.

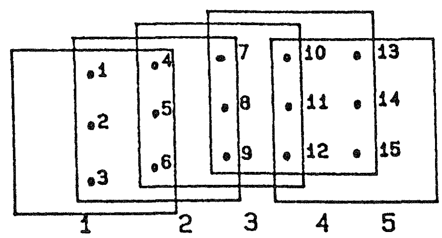

Figure 3

The strip is described in our approach by 60 difference equations of type (2), building the difference of the image coordinates:

$$
\begin{array}{llll}
x_1^{(1)}-x_4^{(2)} & x_1^{(2)}-x_4^{(3)} & x_4^{(3)}-x_7^{(4)} & x_7^{(4)}-x_{10}^{(5)} \\
x_2^{(1)}-x_5^{(2)} & x_2^{(2)}-x_5^{(3)} & x_5^{(3)}-x_8^{(4)} & x_8^{(4)}-x_{11}^{(5)} \\
x_3^{(1)}-x_6^{(2)} & x_3^{(2)}-x_6^{(3)} & x_6^{(3)}-x_9^{(4)} & x_9^{(4)}-x_{12}^{(5)} \\
x_4^{(1)}-x_7^{(2)} & x_4^{(2)}-x_7^{(3)} & x_7^{(3)}-x_{10}^{(4)} & x_{10}^{(4)}-x_{13}^{(5)} \\
x_5^{(1)}-x_8^{(2)} & x_5^{(2)}-x_8^{(3)} & x_8^{(3)}-x_{11}^{(4)} & x_{11}^{(4)}-x_{14}^{(5)} \\
x_6^{(1)}-x_9^{(2)} & x_6^{(2)}-x_9^{(3)} & x_9^{(3)}-x_{12}^{(4)} & x_{12}^{(4)}-x_{15}^{(5)} \\
 & x_7^{(2)}-x_{10}^{(3)} & x_{10}^{(3)}-x_{13}^{(4)} & \\
 & x_8^{(2)}-x_{11}^{(3)} & x_{11}^{(3)}-x_{14}^{(4)} & \\
 & x_9^{(2)}-x_{12}^{(3)} & x_{12}^{(3)}-x_{15}^{(4)} &
\end{array}
\qquad (3)
$$

Correspondingly for the y ordinates.

There are 75 parameters, which usually are unknown.

- The (N,E,H) coordinate values of 15 terrain points (45 parameters)
- The 6 orientation parameters for each of the 5 photographs (30 parameters)

In order to determine these 75 parameters from the 60 observation equations, ground control points are necessary. Assume we have given the ground control coordinates of the first 6 terrain points. This should allow us in principle to perform 2 space resections of the first and second photograph and will establish an absolute coordinate geometry in these photographs. Once the systematic image deformation and the coordinate geometry is reestablished in, say, the first photograph, the absolute image coordinates for the other photographs can be computed from the difference equations (3).

However, we cannot resect the first photographs with 6 control points only, otherwise we would get an undetermined solution. The task we are given here is the task of a full camera calibration in space. The strip computation may be regarded as a combined camera calibration/strip adjustment exercise, using a proper control distribution.

The observation equations for least squares strip adjustment are derived from (2) as

$$\delta x^{(i)}+V_{x,p}^{(i)} = P_x^{(i+1)} \quad (Nq,\ Eq,\ Hq) - P_x^{(i)} \ (N_p,\ E_p,\ H_{p,})$$

$$\delta y^{(i)}+V_{y,p}^{(i)} = P_y^{(i+1)} \quad (Nq,\ Eq,\ Hq) - P_y^{(i)} \ (N_p,\ E_p,\ H_p)$$

Block adjustment is formulated in an analogue manner. Here, difference equations should also be formulated between the last photograph of one strip and the first photograph of the next strip.

Final Remarks

The adjustment is far more robust to blunders or gross errors, as they will not be partly absorbed by the estimated coefficients for the systematic correction terms. Also, proper accuracies are obtained for the minor control points after block adjustment, unperturbed by the assumption for systematic errors, and ground control requirements can be clearly understood, avoiding potential hidden singularities as in block adjustments with additional parameters (Kubik, 1988).

Differential photogrammetry teaches us, that in the presence of systematic errors, denser ground control is required as proposed so far; full model control should be repeated at regular intervals in order to stabilise the accuracy in the block. Detailed accuracy studies for differential photogrammetry will be published by the author in this journal.

Obviously, in practical cases there is not the nice consistent 3 * 3 pattern of image points of Figure 2, but a less repeatable arrangement.

In these cases we can segment the image into a grid of rectangular meshes, and assume a constant image deformation within every mesh.

Differences are then formed between the image points in corresponding meshes. In the presence of "empty" meshes, that are meshes without measured points, we form differences to the next, nearest meash containing image points.

Reference:

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