ABSTRACT:
Digital imagery acquired from aircraft contains significant geometric errors resulting from platform instability and variation in viewing geometry. This paper proposes a correction method based on the collinearity equations and using a digital elevation model. Results are presented for a test site in Southern Spain to demonstrate the technique in comparison with traditional methods based on a polynomial fit.

KEYWORDS: Rectification, Registration, Remote Sensing.

## 1. INTRODUCTION

A Daedalus AADS 1268 Airborne Thematic Mapper (ATM) flight was carried out by the UK Natural Environment Research Council (NERC) on May 16, 1989 over the Antequera Valley, Andalusia, Spain. The survey was flown at a height of approximately 3000 m in a SE-NW direction, with a path width of approximately 4.6 km ( 716 pixels) and over a distance of 33 Km ( 5699 scan lines). It covers a flat area of intensive agriculture lying between two upland regions which form the hills around Antequera and the Sierra del Humilladero.

A portion of the image, 1000 scan lines and 716 pixels in size, was selected for a project which has the aim of investigating methods of incorporating cartographic information to improve land-cover classification. The selected area includes a hilly area to the south and a flat area to the north. The hilly area is covered by scrubland variably intermixed with bare soil and rock outcrops, pines and fields in which olives and fruits are the main crops. The flat area in the north is occupled by fields of mainly cereal crop. In between in the central area is the town of Antequera. The terrain height range over the image area is from approximately 428 m to 755 m .

A reasonably good registration of the digitally transformed map and the image is an essential prerequisite for the achievement of the aims of the project. Initially geometric correction of the image was carried out by traditional polynomial techniques. Results from this technique were not as good as expected and an alternative technique was investigated. The method suggested here is general in its application and may be applicable to other systems.

## 2. TRADITIONAL POLYNOMIAL TECHNIQUES FOR THE

 RECTIFICATION OF IMAGES
### 2.1 Introduction

The sources of geometric error in satellite imagery are mainly due to instrument error, panoramic distortion, Earth rotation and platform instability. Instrument errors include distortions in the optical system, nonlinearity of the scanning mechanism and non-uniform sampling rates. The panoramic distortion is a function of the angular field of view of the sensor. Earth rotation velocity varies with latitude and has the effect of skewing the image. The platform instability includes variation in altitude and attitude.

Mather (1987), describes two methods for geometric correction of remotely sensed images with a narrowangular field of view: the orbital geometry method and the map-based method. The first is based on the knowledge of the orbit of the satellite, the Earth's rotation and the along-scan and across-scan sampling rate. It is useful only when the desired accuracy is not high, or where sensor resolution is
low or when suitable maps of the area are not available. Otherwise, the second method, based on ground control points, is preferable.
The map-based geometric correction is accomplished by transforming the image point coordinates on to corresponding ground control point coordinates selected from a map or other source.

### 2.2 Form and Application of the Polynomlals

Let $(x, y)$ be the grid coordinates of a point on the map and ( $c, r$ ) be the row and pixel coordinates of the corresponding point in the image. For the transformation from $(x, y)$ to ( $c, r$ ) and vice versa, a polynomial relationship is established.

The form of the polynomial should describe the transformtion of coordinates including any additional systematic errors present in the resulting image position. Three forms of polynomial were considered, each representing a progressive increase in the order. The same form of polynomial was used in each direction to transform $c, r$ to $x$ and then $c, r$ to $y$, the reverse transformation of a similar form were also output, $x, y$ to $c$ and $x, y$ to $r$. The simplest form, a first order polynomial, is given by:

$$
\begin{align*}
& c=a_{0}+a_{1} x+a_{2} y  \tag{1}\\
& r=a_{0}^{\prime}+a_{1}^{\prime} x+a_{2}^{\prime} y \tag{2}
\end{align*}
$$

For higher order polynomials the following algorithm can be used:

$$
\begin{equation*}
c=\sum_{j=0}^{m} \sum_{k=0}^{m-j} a_{j k} x^{j} y^{k} \tag{3}
\end{equation*}
$$

where: $m=$ order of polynomial
a = polynomial coefficient
A similar expression for $r$ can be written
The coordinates of the control points are used to determine least squares estimates of the polynomial coefficients.

Once the transformation coefficients have been computed the corrected image can be produced. This involves the transfer of the brightness value from the original image to the corrected image. This resampling process is complicated by the fact that it is unlikely that the pixel centres of the original image will fall at the pixel centres of the correct image. A further requirement might be that the image is resampled to a different resolution from the original, then the area covered by the pixels will be different and thus the
orightness value will again be affected.
Three methods of resampling are commonly used. The first, the nearest neighbour method, takes the value of the pixel in the input image that is closest to the newly-computed centres. The second, bilinear interpolation, takes the average of the four pixels nearest to the new coordinates. The third method, cubic convolution, computes the new pixel value taking into account the 16 nearest pixels in the input image.

A more complete description of the traditional techniques for geometric rectification as well for resampling can be found in Jensen (1986) and Mather (1987).

### 2.3 Application of Traditional Techniques

A geometric rectification was carried out on the ATM image of Antequera using the least squares polynomial fit method described in section 2.2. It was based on the selection of 71 ground control points reasonably well distributed in the image. The UTM geographic coordinates of the points were taken from the 1:10 000 topographic maps and the image coordinates from a display of the image on the Nottingham Image Processing System, NIPS.

The results from the first order polynomial transformation showed some large residuals, of the order of 13 pixels, for both column and row. The Root Mean Square Error (RMSE) for columns was 5.8 and for rows was 5.7 pixels. The results from the second order polynomial showed only a small improvement with RMSE values of 5.6 and 4.5 pixels for column and row respectively. A substantial improvement was achieved with the third order polynomial with the majority of residuals being within $\pm 4$ pixels with RMSE values of 2.5 and 1.8 for column and row respectively.

A visual assessment of the effects of the transformations on the image was obtained by producing a corrected image for each order of polynomial and then overlaying the digitized map. The overlay was accomplished by clipping each corrected image by a number of rows and columns in order to get the best match with the map centre. As might be expected, the visual inspection showed increasing displacements of the corresponding features in the two images from the centre to the edges. The displacements were, as expected, bigger in the first and second order corrected images and smaller for the third order corrected image. The third order corrected image showed larger residuals than were acceptable.

### 2.4 Geometric Characteristics of the Image

The basic geometric operation of the ATM is given in section 3.2

In addition to the instrumental characteristics that affect the geometry of the image there are a number of external influences. These fall into two groups: those that are inherent in the geometry of the system and those that are due to operational difficulties and problems.

The velocity to height ratio (V/H), the scan rate of the scanning mirror and the ground resolution, as well as the swath width are defined by the flying height above the ground level. Incorrect adjustment of the scanning mirror speed will cause under or over scanning and thus influence the geometry. There will also be a change in the ground resolution element size with different scan angles (see section 3.2). A variation in the ground relief will result in image displacement, which is a similar problem to that experienced in conventional aerial photography.

The sensor platform, being aircraft-based, has
significant inherent instabilities, due to the characteristics of the aircraft motion and the atmospheric conditions in which it operates. To assist with minimising the effects of aircraft roll, up to a maximum of $\pm 15$ degrees, a gyro assembly monitors the aircraft and only allows the image to be collected within an angle of approximately 37 degrees either side of the vertical.

As mentioned above, for the polynomial technique to be fully effective it not only performs a transformation but corrects for the systematic errors present. It is extremely difficult to model the effect on the image geometry of all the above mentioned influences. Therefore, a more direct modelling technique could be employed.

## 3. TIME-DEPENDENT GEOMETRY METHOD FOR <br> RECTIFICATION OF IMAGES

### 3.1 Introduction

The basic principle of the technique discussed so far is that of modelling systematic errors in the image. The model is defined by a polynomial which should describe the systematic errors present. The polynomial should be based on an analysis of the influences affecting the geometry of the imaging system. Therefore the choice of polynomial is important to obtain the optimum results. An alternative approach to solving the problem is to attempt to model the cause rather than, as in the polynomial case, the effects on the image. The use of the collinearity equations in digital image processing is not new (Konecny, 1979), also the use of time-dependent geometry is not a new concept (El Hassin, 1981; Smith, 1989) although it is becoming more popular with developments in analytical and digital techniques. For this reason the solution presented here attempts to remain as general, for any similar sensor, as possible. The geometry of the imaging system must be analysed and modelled, which, in the case being considered here, involves the sensor geometry and the aircraft motion during the period of imaging. The principles of the sensor geometry are reasonably well documented where as the aircraft motion is not accurately known and requires certain assumptions to be made.

### 3.2 Analysis of the Imaging System Geometry

The sensing system consists of a rotating mirror that scans a swath of ground to either side of nadir. A scan line of data is collected as a series of 2.5 mrad instantaneous fields of view (IFOV), which therefore defines the resolution. Since the image data is captured in relation to a constant angular measure and constant pixel unit size, the ground area covered by the IFOV will vary depending on the angle from nadir, resulting in compression of the image towards the edges. This is removed from the image by the $S$-bend correction in the digitization process. So the fundamental scanning geometry being described is therefore perspective geometry if the sensor was stationary. Successive scan lines of adjoining ground swath are produced by the forward motion of the aircraft. The butting together of the scan lines is dependent largely on the velocity-to-height ratio of the aircraft, although this is controlled by the allowable scan rates and the required resolution (as mentioned above). Normally these are selected to ensure some overlap, typically $10 \%$. So the aircraft motion has an effect on the scan geometry (perspective geometry) and is used to create the series of scan lines to produce the image. It is therefore necessary to attempt to describe the aircraft motion during the image capture. considering the rate at which the image is captured, scan rates available are $12.5,25$ or 50 scans/sec, for an image of 1000 scan lines (the approximate image size being considered), the time to capture the image would be 80,40 or 20 seconds
respectively. It is, therefore, not too unrealistic to assume that the sensor platform (aircraft) is stationary for each scan line and that, for successive scan lines, the aircraft's motion has a constant rate of change.

### 3.3 Time-Dependent Geometry

If each scan line is considered as a conventional photograph with perspective geometry then the mathematical model for the image can be considered as a series of such photographs of width equal to one scan line. For any of image point coordinates the standard collinearity equations can be written:
$\frac{X_{a}}{f}=\frac{r_{11}\left(X_{A}-X_{0}\right)+r_{21}\left(Y_{A}-Y_{0}\right)+r_{31}\left(Z_{A}-Z_{0}\right)}{r_{13}\left(X_{A}-X_{0}\right)+r_{23}\left(Y_{A}-Y_{0}\right)+r_{33}\left(Z_{A}-Z_{0}\right)}$
$\frac{y_{2}}{f}=\frac{r_{12}\left(X_{A}-X_{0}\right)+r_{22}\left(Y_{A}-Y_{0}\right)+r_{32}\left(Z_{A}-Z_{0}\right)}{r_{13}\left(X_{A}-X_{0}\right)+r_{23}\left(Y_{A}-Y_{0}\right)+r_{33}\left(Z_{A}-Z_{0}\right)}$
where:
$x_{a}, y_{a} \quad=$ image coordinates of point $a$
$\mathrm{r}_{11}$ to $r_{33}$
= principal distance
$=$ elements of orthogonal rotation matrix ( $\omega, \phi, \kappa$ rotations)
$X_{0}, Y_{0}, Z_{0} \quad=\quad$ perspective centre coordinates
$X_{A}, Y_{A}, Z_{A}=$ ground coordinates of point $A$

The determination of the position and orientation of a scan line $j$, given that $x_{a}, y_{a}$, are measured and $X_{A}, Y_{A}, Z_{A}$ and $f$ are known, involves stx unknowns: $\omega_{j}, \phi_{j}, \kappa_{j}$ and $X_{0 j}, Y_{0 j}, Z_{0 j}$. The solution in this form, for all scan lines, results in an impractical situation, with far too many unknowns in the solution. A similar technique to that used in Smith (1989) is adopted, where, considering the assumption made about the aircraft motion above, a relationship exists between all like parameters:

| $\omega_{j}$ | $=\omega_{0}+x_{a} \omega_{c}$ |
| :--- | :--- |
| $\phi_{j}$ | $=\phi_{0}+x_{a} \phi_{c}$ |
| $\kappa_{j}$ | $=x_{0}+x_{a} \kappa_{c}$ |
| $X_{0 j}$ | $=X_{00}+x_{a} X_{0 c}$ |
| $Y_{0 j}$ | $=Y_{00}+x_{a} Y_{0 c}$ |
| $Z_{0 j}$ | $=Z_{00}+x_{a} Z_{0 c}$ |

where:
$x_{a}$
$=x$ coordinate of point a
$=$ scan line number
(in direction of flight)
$\omega_{0}$ to $Z_{00}=$ start values of the parameters:
$\omega_{0}=$ roll
$\phi_{0}=$ pitch
$\kappa_{0}=$ heading
$X_{00}=$ X plan location
$Y_{00}=$ Yplan location
$Z_{00}=$ flying height
$\omega_{c}$ to $Z_{0 c}=$ increment coefficients for the parameters

This reduces the number of unknowns to $\omega_{0}, \omega_{c}, \phi_{0}$, $\phi_{c}, \kappa_{0}, \kappa_{c}, X_{00}, X_{0 c}, Y_{00}, Y_{0 c}, Z_{00}, Z_{0 c}$ and a solution can now be obtained for the 12 unknowns (Smith, 1989: El Hassan, 1981; Case, 1961). Once values for $\omega_{0}$ to $Z_{0 c}$ are known, an orientation element for any scan line can be obtained, if $x$ is known, by substitution into equations (6) to (11).

The $\phi$ element, the pitch of a scan line, has a similar effect on the image to the $Z_{0}$ term, which defines the height of the aircraft. Therefore in the solution $\phi_{0}$ and $\phi_{c}$ are held fixed.

As can be seen from the equations (4) to (11), in computing image coordinates from ground coordinates an iterative process must be adopted because the required $x$ pixel coordinate is needed to compute the orientation values. Therefore, an approximate value of the $x$ pixel is initially used, the values of $\omega_{j}$ to $Z_{0 j}$ are computed, a new value of $x$ pixel is determined and the process iterates until there is a negligible change in successive $x$ pixel values.

### 3.4 Application of the Technique

Since the collinearity equations (4) and (5) are based on conventional photograph geometry they contain the principal distance (f) of the camera. In the case of the ATM a principal distance does not physically exist. However, a theoretical principal distance was determined based on the angle subtended at the rotating mirror by one pixel, this gives a 'principal distance' of 400.0 pixel units.

The computation to determine the effects of Earth curvature and atmospheric refraction showed them to be insignificant for this application.

## 4. ANALYSIS OF THE RESULTS

It was decided to use a mathematical analysis technique rather then trying to assess the quality of superimposed images which could be very subjective. A standard technique, and the simplest, is to compute residuals at the control points. However, as the polynomial technique results show, this is not indicating the effects in the areas between the control points, hence check points are introduced. The 71 points with known image coordinates and ground coordinates were divided into two groups; control points and check points. Control point locations were chosen so as to give a reasonably even distribution over the image, although, in one area in particular, suitable points were sparse. However, the distribution was the same for both techniques. To provide a simple overall comparison of results the RMSE and the SUM of the residuals was computed. To analyse the control requirements for each technique, differing numbers of control points were used. Table 1 presents a summary of results.

The RMSE values for the time-dependent geometry and the first and second order polynomial solutions are approximately the same, mainly in the order of 4 to 6 pixels. The 3 rd order polynomial values show a small improvement on this, the RMSE being mainly in the order of 2 to 3 pixels. These results are equally applicable for the control points as well as the check points and for the different quantities of control points.

This is surprising, since a deterioration might be expected for the check point values and the smaller amounts of control, and is probably indicative of the reasonably well-distributed control, giving good values for the time-dependent geometry parameters and the polynomial coefficients.

It can be seen that the check point summation values are, in general, of similar magnitudes for each set of results and are large compared with those for the control points. The summation values for the 51 check points indicate that there is significant systematic error. This is reduced with the increase in control points as can be seen from Table 1, but there is a decrease in the number of check points.

Table 1. Summary of results


With a small amount of control, the distribution and quality of the control points can significantly effect the results. The values that appear to be most sensitive to this problem are the summation values. Therefore, it was felt that the minimum amount of control that should be considered is 20 control points, which provides a small amount of redundancy.

The time-dependent geometry approach does provide parameters which describe the sensors motion, equations (6) to (11). The parameter values obtained for the 71 control point solution are as follows:
(rotations in radians, coordinates in km)

| $\omega_{0}$ | $=$ | -0.063687 | $\omega_{0 \mathrm{c}}$ | $=$ | 0.000095240 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\phi_{0}$ | = | 0.000000 | $\phi_{0 c}$ | = | 0.000000000 |
| $\kappa_{0}$ |  | 0.819822 | $\kappa_{0 c}$ |  | 0.000035075 |
| $\mathrm{X}_{00}$ | - | 363.901 | $\mathrm{X}_{0}$ |  | 0.000143 |
| $Y_{00}$ | = | 95.925 | $Y_{0 c}$ |  | -0.001039 |
| $\mathrm{Z}_{00}$ | = | 3.018 | $Z_{0 c}$ |  | 0.000191 |

It is interesting to note that the $Z_{00}$ value compares very well with the nominal flying height of 3.000 km

## 5. CONCLUSIONS

This preliminary study has shown that the timedependent geometry technique is feasible and, in general, has produced results at least comparable with those obtained from the first and second order polynomial solutions. An improvement in the results from the time-dependent geometry might be produced if the assumption about the aircraft motion is reconsidered with higher order parameters being introduced in equations (6) - (11). Attempting to model aircraft movement during each scan may improve the results, although this might be quite difficult. A more simple solution could be to divide the image into small units. This, however, might lead to inconsistencies at the joining edges.

In summary, results from the time-dependent geometry approach show no significant improvement over the original polynomial solution. The modelling of systematic errors by any method might be extremely difficult if there are irregularities
in the data. Such irregularities might be due to short period movement of the sensor. Further work is envisaged, in particuiar, in the use of the time-dependent geometry approach with other images, especially those with large variations in relief, since the ground height is taken into account by this method.

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