HOW TO INTERPOLATE A SURFACE WITH BREAK LINES ?
A method to modify interpolations for integration of break lines

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Abstract

The introduction of break lines into surface model enhances representation of terrain surfaces. The benefit from break lines is adding a capability to surface model that allows an integration of digital terrain data with different accuracies, specially with precise linear 3-D elevation data, such as routes, rivers, etc. Thus, a more real surface can be represented. Break lines "break" a continuous smooth surface, a new interpolation method is necessary to be developed in order to produce surfaces satisfying the existence of break lines. This paper presents a method to modify existing interpolations on a surface represented by Triangular Irregular Network (TIN).

Background

Before an exist of any break lines, several smooth algorithms such as Akima algorithm and Bezier algorithm, can be used to generate a C and C continuous surface for a TIN surface, and still honor to all given points. Thus these algorithms are popularly used for surface generation from a set of random distributed given points. Their result is a 2.5 dimensional surface.

The C continuity is an inherent property of a terrain surface. There is only one elevation for each point. For any point on the surface, regardless it is a given point or not, the first condition of smooth is continuous. It means that it will be same elevation regardless along which path to approach this point as long as staying on the surface. The C is a more strong condition. It means that there is only one normal direction at each point on the surface. Following the above conditions of C and C, the result surface will be reasonable smooth. Note many literatures use 'spatial first order derivatives' more often than 'normal direction'.

Behavior of Break line

The purpose of introducing break lines into surface modeling is to force surface exactly following break lines wherever they exist. Once break lines are involved, surface has more complex behavior. The surface is not smooth any more at where a break line exists, The surface is forced to take elevation values along the break line. It only keeps C but not C for those points at break lines. It does same at where there is not any break lines. It still keeps C and C at all points beyond break lines. Thus, for each point outside the break lines it has only one elevation and one normal vector. Those points at break lines have one elevation but have uncertain normal vectors.

Interpolation based on triangles

While a surface is represented by Triangular Irregular Network (TIN), the space is covered by many un-overlapping and un-gapping triangles. For simplify the problem and further processing, all break lines only can be on edges of triangles. This simplification shouldn't been thought of a strong additional restriction, because the simplification can be easily achieved by separating a triangle along break lines, if the break lines are inside a triangle. The common case involves two connecting triangles.

First, An elevation at any point beyond the four points, must be interpolated by an algorithm, e.g. Akima, or Bezier. These interpolation algorithms generate a result surface based on TIN structure. They produce a piece of surface independently for each triangle by a polynomial. so the piece of surface inside a triangle is smooth. The result surface is a combination of all these pieces. If their common edge between the two triangles is not a break line, the elevation of those points at the common edge have to be interpolated, because they haven't any elevation value yet. However, those points on the common edges belong to two adjacent triangles. Each interpolation algorithm derives proper boundary conditions according to the elevations at the given points and smooth requirements. The result surfaces generated from these algorithms will be smooth surfaces which pass the given vertexes. At the common edge, all points will follow condition C and C. Only one normal direction exists for each point as well as those points at the edge.

Second, if a common edge is a break line, it means that all points on the edge have been assigned elevation values. Frequently, these elevations on break lines are from more precise data source than other points. For example, break lines can be from engineering or photogrammetry. For simplification, the edge line is assumed as a straight line. Again, this simplification is not a strong additional restriction, because the edge can be separated into smaller segments until that each segment can be seen as a straight line and still satisfy accuracy requirements. Then smaller triangles can be constructed for these segments so that the common break line edge can always be counted as a straight line, and the elevation of all points on the edge can be calculated linearly.

The surface must pass all points at the edge of break lines. Beyond the edge the surface is still smooth. See Figure 1. It shows a profile that crosses the break line edge. It is a continuous curve, but not smooth at the break line point. At any surface point beyond break lines, it has only one normal direction. At a point on break line, its normal direction is uncertain.

Assign ONE normal direction at one point for smooth

At a point on a surface, if its normal direction is always identical, regardless from any path on the surface to approach the point, the surface is smooth at this point. This mathematical condition is essentially used by Akima and Bezier algorithms to force the surface keeping smooth when it crosses boundaries between faceds. For calculating efficiency, the result surface is a mosaic by many small surface pieces which generated from each triangle. It is simple to keep these pieces connect by forcing them sharing common elevation values at their common points. For keeping these pieces connecting smoothly, the strategy used by many interpolation algorithms is to find **one** normal direction at the point on the boundaries. The key point here is assign only one normal direction to any common point of adjacent triangles. "Allowing only one normal direction" is defined to be a condition that the surface pieces at two adjacent triangles must follow. Thus, the result surface is smooth at this point because the adjacent surface pieces have same elevation and same normal direction.

Assign TWO normal directions at point on break lines for "break"

When a common edge between two adjacent triangles is a break line, the traditional interpolations have to be modified. Two different normal directions have to be assigned to one point on the edge for two sides of the edge, separately. It means that one normal direction is selected for the point when the point is approaching from one triangle on one side of the edge. Another normal direction is selected for the same point when the point is approaching from another triangle on the opposite side.

In either Akima or Bezier algorithm, normal directions play crucial role for determining boundary conditions of a polynomial to a triangle. Now, the boundary conditions are determinated by two restraints:

- Same elevation value at all boundary points.
- Two different normal directions at each point on common edge of break line, belongs to two triangle on each side separately.

Two pieces of surface generated from both triangles will connect each other, but no longer be smooth any more at the edge of break line. The result surface then will be broked at the break lines, but will still be connected.

The advantage of this method is that most interpolation algorithms still can be used for each triangle locally. Only difference is that two normal direction has to be found and assigned to those points on the break lines. For example, the Akima and Bezier interpolations can be used with the above modifications.

Find TWO normal directions at a point on break line

There are many ways to find a normal direction at a point on the common edge. A simple method used is to find an 'average' normal direction from given points around. Some methods concerning distance weight from these given point may be better. If the point is on a break line, the surface space is assumed to be cut by the break line. A normal direction at a point on the break line will be calculated for each side from those given points on the side of the break line. Another normal direction is calculated from those given points on another side of the break line. In this way, a point on the break line edge can have more than one normal directions. Each of them behaves corespondently to the surface at one side. The point and one normal direction can be seen as an extension property from one side of the surface.

Conclusions

The method presented in this paper has been successfully implemented in Arc/Info 6.0 above release version. It provides the capability of surface model integrated with break lines. The quality and reality of surface model are enhanced. The performance of the modified interpolation is very good, actually has not been effected obviously, after modified.

References

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