SNAP: A SYSTEM FOR NON-METRIC ARCHITECTURAL PHOTOGRAMMETRY

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ABSTRACT

Architectural Photogrammetry can be based on inexpensive machinery like non-metric cameras, small photogrammetric instruments or even digitizers, a small computer and inexpensive output devices. SNAP offers a solution to architects, archaeologists and surveying engineers for the capturing, processing, documentation and archiving of accurate information concerning monuments, sites and structures of special interest.

Data can be input to SNAP either by a small photogrammetric instrument or by a digitizer. The photogrammetric processing is based on photographs captured by amateur cameras, although it can take advantage of the geometrical stability of metric cameras, if they exist. The photogrammetric adjustment is a self-calibrating bundle adjustment with photo-variant additional parameters and the user can optionally use surveying observations of ground control points coordinates, measured distances and angles or enforce constraints like known distances, parallel lines, perpendicular lines, points on circle or arc etc. The output is driven to DGN (Intergraph) or DXF (Autodesk) format for use by popular graphic editors.

KEY WORDS: Non-metric, Architectural Photogrammetry, Photo-variant parameters, Combined adjustment

1. PHOTOGRAMMETRIC DOCUMENTATION OF MONUMENTS AND SITES

Despite the efforts of many years, the recording of the majority of buildings, monuments, structures and sites of our architectural heritage is far behind schedule. Photogrammetry can offer quick, accurate and economical solutions to this urgent demand. The equipment to be used and the procedures to be followed has been very much refined over the past years, so that the required cost and the user experience is kept to minimum levels (Waldhäusl, 1987). In order to achieve this the use of low-cost computer-based photogrammetric instrumentation and sophisticated and universally applicable procedures (have or) are been developed.

1.1 Equipment

Analytical Photogrammetry is not any more bounded by the restrictions characterizing the good old Analog Photogrammetry. As far Architectural Photogrammetry concerns even the requirement of universal analytical plotters is relaxed, giving ground to smaller analytical systems, or even to digitizers and mouses as input devices.

Of course metric cameras have a lot of advantages like interior stability and large format but their cost, discomfort in use and discontinuity in development leads more and more to the use of semi- and non-metric cameras (Waldhäusl et.al., 1988 and 1989).

1.2 Procedures

Since the time that Analytical Photogrammetry replaced the opto-mechanical rods with analytical relations, there are no more restrictions on photo configurations, while at the same time the development of universal software made possible. In the case of non-metric cameras the less rigourous DLT method has been widely used, whereas adjustment of bundles can offer solutions to a wider range of applications including both metric and non-metric photographs.

The error burden introduced by the use of non-metric photographs (especially that due to film distortion) is quite different from that of metric photos. Therefore the use of self-calibration including photo-variant parameters has been looked into more closely.

While it is known that in order to compensate for systematic errors, redundant control and extensive experience is required, the confinement to as little control and experience as possible is highly desirable. This is due to the fact that many disciplines other than surveyors (eg. architects, archaeologists, historians, geographers) are now involved in documenting historical sites. These people know very little about surveying measurements and normally lack surveying equipment.

2. THEORETICAL ASPECTS ON PHOTOGRAMMETRIC DATA PROCESSING - THE SNAP APPROACH

Summarizing the state-of-the-art in equipment, procedures and requirements of Architectural Photogrammetry, we should note that:
- The data are collected with analog metric, semi-metric and non-metric cameras as well as digital cameras and scanned photos.
- The control can be an extensive number of geodetic coordinates or simply a few taped distances, just enough to define a reference frame (Waldhäusl and Peipe, 1990).
- The measurements of photo coordinates are performed with an analytical plotter, a smaller analytical photogrammetric instrument, a digitizer or a mouse.
- The pre-processing of the data includes incorporation of the camera calibration report (if it exists) and correction of lens and film distortion.
- The adjustment of the measurements is based on bundle adjustment with the use of additional parameters for the compensation of the remaining systematic errors. These parameters are either photo-variant (for non-metric photos) or photo-invariant (for metric photos).

- It is desirable that additional surveying observations (distances, angles) and geometric constraints (parallelism, perpendicularity, points on same line, points on same plane, etc.) could be incorporated into the adjustment of the measurements.

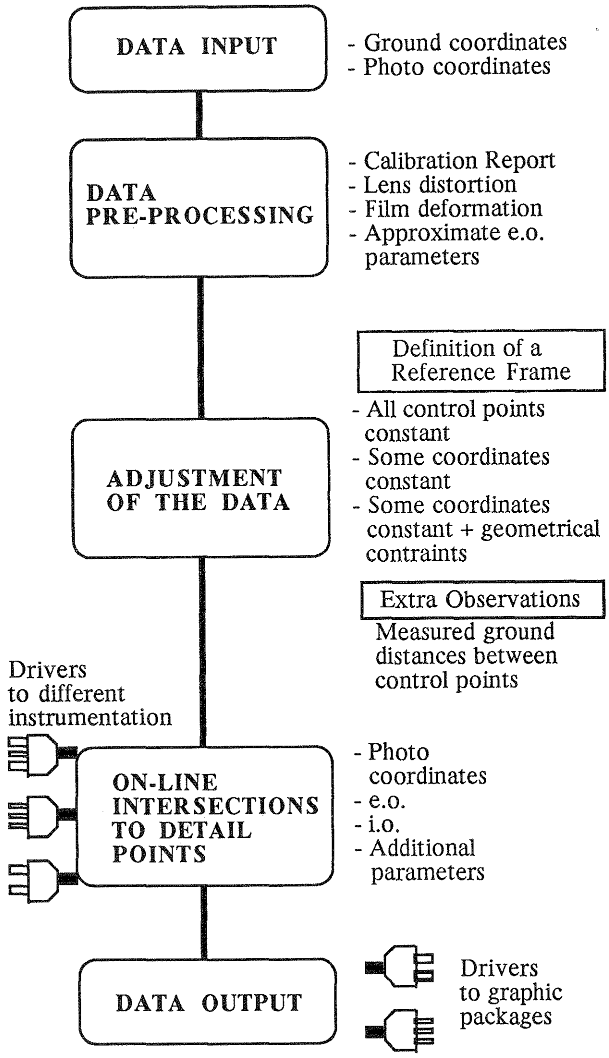

Figure 1. Functions of SNAP's modules.

The developed SNAP system is a software package which is based on the current state-of-the-art in Architectural Photogrammetry and is enhanced with a user-friendly interface. The general module setup is illustrated in Figure 1, while some theoretical aspects inherent in its development is explained in the next paragraphs. More specifically, issues concerning the definition of the reference frame, the sequential bundle adjustment on a photo-wise basis and the inclusion of the photo-variant additional parameters are addressed.

2.1 Formalization of observation and normal equations

It is known that the photo coordinates x, y are related to the ground coordinates X, Y, Z through the collinearity conditions:

$$x = x_o - f\frac{R_{11}(X-X_o)+R_{12}(Y-Y_o)+R_{13}(Z-Z_o)}{R_{31}(X-X_o)+R_{32}(Y-Y_o)+R_{33}(Z-Z_o)} + \Delta x$$
$$y = y_o - f\frac{R_{21}(X-X_o)+R_{22}(Y-Y_o)+R_{23}(Z-Z_o)}{R_{31}(X-X_o)+R_{32}(Y-Y_o)+R_{33}(Z-Z_o)} + \Delta y \quad (1)$$

where f is the camera constant, x_o, y_o the photo coordinates of the principal point, X_o, Y_o, Z_o the ground coordinates of the exposure station, and R_{11}, R_{12}, ..., R_{31} are the elements of the rotation matrix $\mathbf{R} = \mathbf{R}(\omega, \varphi, \varkappa)$. Δx and Δy are the corrections to x and y due to remaining systematic errors, which are typically modeled by polynomials of the type $\Delta x = g_1(\mathbf{y}, x, y)$, $\Delta y = g_2(\mathbf{y}, x, y)$ (eg. Murai S. et al, 1984) and $\mathbf{y}$, is the vector of the so-called additional parameters. In SNAP a number of such models can be used.

The linearized observation equations for point j of the i photograph is written as

$$\begin{bmatrix} x^b - x^o \\ y^b - y^o \end{bmatrix}_{ij} = \begin{bmatrix} \frac{\partial x}{\partial X} & \frac{\partial x}{\partial Y} & \frac{\partial x}{\partial Z} \\ \frac{\partial y}{\partial X} & \frac{\partial y}{\partial Y} & \frac{\partial y}{\partial Z} \end{bmatrix}_{ij} \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix}_j +$$

$$+ \begin{bmatrix} \frac{\partial x}{\partial \varkappa} & \frac{\partial x}{\partial \varphi} & \frac{\partial x}{\partial \omega} & \frac{\partial x}{\partial X_o} & \frac{\partial x}{\partial Y_o} & \frac{\partial x}{\partial Z_o} \\ \frac{\partial y}{\partial \varkappa} & \frac{\partial y}{\partial \varphi} & \frac{\partial y}{\partial \omega} & \frac{\partial y}{\partial X_o} & \frac{\partial y}{\partial Y_o} & \frac{\partial y}{\partial Z_o} \end{bmatrix}_{ij} \begin{bmatrix} \delta\varkappa \\ \delta\varphi \\ \delta\omega \\ \delta X_o \\ \delta Y_o \\ \delta Z_o \end{bmatrix}_i + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}_{ij} + \begin{bmatrix} v_x \\ v_y \end{bmatrix}_{ij} \quad (2)$$

or in matrix notation as

$$\mathbf{b}_{ij} = \dot{\mathbf{A}}_{ij}\,\dot{\mathbf{x}}_j + \ddot{\mathbf{A}}_{ij}\,\ddot{\mathbf{x}}_i + \mathbf{D}_{ij}\,\mathbf{y}_i + \mathbf{v}_{ij} \quad (3)$$

The observation equations for all points on the i photo is

$$\mathbf{b}_i = \dot{\mathbf{A}}_i\,\dot{\mathbf{x}} + \ddot{\mathbf{A}}_i\,\ddot{\mathbf{x}}_i + \mathbf{D}_i\,\mathbf{y}_i + \mathbf{v}_i \quad (4)$$

where $\dot{\mathbf{x}}$ is the correction vector to the approximate ground coordinates of the control points, $\ddot{\mathbf{x}}_i$ is the correction vector to the approximate values of the e.o. parameters of the i th photo and $\mathbf{y}_i$ is the vector of the additional parameters of the i th photo (following a general photo-variant approach, eg. Moniwa, 1981). For all m photographs the form of the system is

$$\begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_i \\ \vdots \\ \mathbf{b}_m \end{bmatrix} = \begin{bmatrix} \dot{\mathbf{A}}_1 \\ \dot{\mathbf{A}}_2 \\ \vdots \\ \dot{\mathbf{A}}_i \\ \vdots \\ \dot{\mathbf{A}}_m \end{bmatrix} \dot{\mathbf{x}} + \begin{bmatrix} \ddot{\mathbf{A}}_1 & 0 & \dots & 0 & \dots & 0 \\ 0 & \ddot{\mathbf{A}}_2 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & \ddot{\mathbf{A}}_i & \dots & 0 \\ \vdots & \vdots & \dots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & \dots & \ddot{\mathbf{A}}_m \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{x}}_1 \\ \ddot{\mathbf{x}}_2 \\ \vdots \\ \ddot{\mathbf{x}}_i \\ \vdots \\ \ddot{\mathbf{x}}_m \end{bmatrix} +$$

$$
+\begin{bmatrix} \mathbf{D}_1 & 0 & \dots & 0 & \dots & 0 \\ 0 & \mathbf{D}_2 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & \mathbf{D}_i & \dots & 0 \\ \vdots & \vdots & \dots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & \dots & \mathbf{D}_m \end{bmatrix} \begin{bmatrix} \mathrm{y}_1 \\ \mathrm{y}_2 \\ \vdots \\ \mathrm{y}_i \\ \vdots \\ \mathrm{y}_m \end{bmatrix} + \begin{bmatrix} \mathrm{v}_1 \\ \mathrm{v}_2 \\ \vdots \\ \mathrm{v}_i \\ \vdots \\ \mathrm{v}_m \end{bmatrix} \tag{5}
$$

The structure of the $\mathbf{D}$ matrix reflects the photo-variation of the additional parameters. In case we use photo-variant additional parameters then $\mathbf{D} = \mathbf{D}_i$ and $\mathbf{y} = \mathbf{y}_i$. Therefore the switching from photo-variant (non-metric photos) to photo-invariant (metric photos) is an algorithmically trivial matter.

The least squares criterion is

$$
\sum_{i=1}^{m} \mathbf{v}_i^T \mathbf{P}_i \mathbf{v}_i = \min. \tag{6}
$$

where $\mathbf{P}_i = \mathbf{Q}_i^{-1}$ is the weight matrix pertaining to observations on the i th photograph

The system of normals is then

$$
\begin{bmatrix} \dot{\mathbf{N}} & \tilde{\mathbf{N}} & \dot{\mathbf{N}}_y \\ \tilde{\mathbf{N}}^T & \ddot{\mathbf{N}} & \ddot{\mathbf{N}}_y \\ \dot{\mathbf{N}}_y^T & \ddot{\mathbf{N}}_y^T & \mathbf{N}_y \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}} \\ \ddot{\mathbf{x}} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \dot{\mathbf{u}} \\ \ddot{\mathbf{u}} \\ \mathbf{u}_y \end{bmatrix} \tag{7}
$$

or in detail

$$
\left[\begin{array}{c|cccc|cccc} \dot{\mathbf{N}} & \tilde{\mathbf{N}}_1 & \tilde{\mathbf{N}}_2 & \cdots & \tilde{\mathbf{N}}_m & \dot{\mathbf{N}}_{y_1} & \dot{\mathbf{N}}_{y_2} & \cdots & \dot{\mathbf{N}}_{y_m} \\ \hline \tilde{\mathbf{N}}_1^T & \ddot{\mathbf{N}}_1 & 0 & \dots & 0 & \ddot{\mathbf{N}}_{y_1} & 0 & \dots & 0 \\ \tilde{\mathbf{N}}_2^T & 0 & \ddot{\mathbf{N}}_2 & \dots & 0 & 0 & \ddot{\mathbf{N}}_{y_2} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{N}}_m^T & 0 & 0 & \dots & \ddot{\mathbf{N}}_m & 0 & 0 & \dots & \ddot{\mathbf{N}}_{y_m} \\ \hline \dot{\mathbf{N}}_{y_1}^T & \ddot{\mathbf{N}}_{y_1}^T & 0 & \dots & 0 & \mathbf{N}_{y_1} & 0 & \dots & 0 \\ \dot{\mathbf{N}}_{y_2}^T & 0 & \ddot{\mathbf{N}}_{y_2}^T & \dots & 0 & 0 & \mathbf{N}_{y_2} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \dot{\mathbf{N}}_{y_m}^T & 0 & 0 & \dots & \ddot{\mathbf{N}}_{y_m}^T & 0 & 0 & \dots & \mathbf{N}_{y_m} \end{array}\right] \left[\begin{array}{c} \dot{\mathbf{x}} \\ \hline \ddot{\mathbf{x}}_1 \\ \ddot{\mathbf{x}}_2 \\ \vdots \\ \ddot{\mathbf{x}}_m \\ \hline \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_m \end{array}\right] = \left[\begin{array}{c} \dot{\mathbf{u}} \\ \hline \ddot{\mathbf{u}}_1 \\ \ddot{\mathbf{u}}_2 \\ \vdots \\ \ddot{\mathbf{u}}_m \\ \hline \mathbf{u}_{y_1} \\ \mathbf{u}_{y_2} \\ \vdots \\ \mathbf{u}_{y_m} \end{array}\right] \tag{8}
$$

where

$$
\dot{\mathbf{N}} = \sum_{i=1}^{m} \dot{\mathbf{N}}_i = \sum_{i=1}^{m} \dot{\mathbf{A}}_i^T \mathbf{P}_i \dot{\mathbf{A}}_i
$$

$$
\dot{\mathbf{u}} = \sum_{i=1}^{m} \dot{\mathbf{u}}_i = \sum_{i=1}^{m} \dot{\mathbf{A}}_i^T \mathbf{P}_i \mathbf{b}_i \tag{9}
$$

$$
\tilde{\mathbf{N}}_i = \dot{\mathbf{A}}_i^T \mathbf{P}_i \ddot{\mathbf{A}}_i \qquad \ddot{\mathbf{N}}_i = \ddot{\mathbf{A}}_i^T \mathbf{P}_i \ddot{\mathbf{A}}_i \tag{10}
$$

$$
\mathbf{N}_{y_i} = \mathbf{D}_i^T \mathbf{P}_i \mathbf{D}_i \quad \dot{\mathbf{N}}_{y_i} = \dot{\mathbf{A}}_i^T \mathbf{P}_i \mathbf{D}_i \quad \ddot{\mathbf{N}}_{y_i} = \ddot{\mathbf{A}}_i^T \mathbf{P}_i \mathbf{D}_i \tag{11}
$$

and

$$
\ddot{\mathbf{u}}_i = \ddot{\mathbf{A}}_i^T \mathbf{P}_i \mathbf{b}_i \qquad \mathbf{u}_{y_i} = \mathbf{D}_i^T \mathbf{P}_i \mathbf{b}_i \ . \tag{12}
$$

2.2 Definition of the reference frame

Since additional parameters are not related to the reference frame, they can be eliminated and thus the final system is

$$
\begin{bmatrix} \dot{\mathbf{R}} & \tilde{\mathbf{R}} \\ \tilde{\mathbf{R}}^T & \ddot{\mathbf{R}} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}} \\ \ddot{\mathbf{x}} \end{bmatrix} = \begin{bmatrix} \dot{\mathbf{r}} \\ \ddot{\mathbf{r}} \end{bmatrix} \tag{13}
$$

where

$$
\begin{aligned} &\dot{\mathbf{R}} = \dot{\mathbf{N}} - \dot{\mathbf{N}}_y \mathbf{N}_y^{-1} \dot{\mathbf{N}}_y^T && \dot{\mathbf{r}} = \dot{\mathbf{u}} - \dot{\mathbf{N}}_y \mathbf{N}_y^{-1} \mathbf{u}_y \\ &\ddot{\mathbf{R}} = \ddot{\mathbf{N}} - \ddot{\mathbf{N}}_y \mathbf{N}_y^{-1} \ddot{\mathbf{N}}_y^T && \ddot{\mathbf{r}} = \ddot{\mathbf{u}} - \ddot{\mathbf{N}}_y \mathbf{N}_y^{-1} \mathbf{u}_y \\ &\tilde{\mathbf{R}} = \tilde{\mathbf{N}} - \dot{\mathbf{N}}_y \mathbf{N}_y^{-1} \ddot{\mathbf{N}}_y^T \end{aligned} \tag{14}
$$

In order to define the reference frame the minimum required constraints can be introduced with the help of relationships, of the general form

$$
\dot{\mathbf{H}}\,\dot{\mathbf{x}} + \ddot{\mathbf{H}}\,\ddot{\mathbf{x}} = \mathbf{z} \tag{15}
$$

where $\ddot{\mathbf{H}} = [\ddot{\mathbf{H}}_1\ \ddot{\mathbf{H}}_2\ \dots\ \ddot{\mathbf{H}}_m]$. Besides this general form, the constraints may refer only to e.o. parameters $\ddot{\mathbf{H}}\,\ddot{\mathbf{x}} = \mathbf{z}$ ($\dot{\mathbf{H}} = \mathbf{0}$), or refer only to ground control points $\dot{\mathbf{H}}\,\dot{\mathbf{x}} = \mathbf{z}$ ($\ddot{\mathbf{H}} = \mathbf{0}$).

In the most common case, in order to define a reference frame we can simply constraint same coordinates of control points or some e.o. parameters. In this simple case the system of normal equations is easily solved by elimination of the corresponding rows and columns of matrices $\dot{\mathbf{R}}$, $\ddot{\mathbf{R}}$ and $\tilde{\mathbf{R}}$ and the corresponding vectors $\dot{\mathbf{r}}$ and $\ddot{\mathbf{r}}$. This technique is valid both for minimal and redundant number of known parameters (i.e. coordinates of control points, e.o. parameters).

2.3 The sequential photo-wise adjustment

The following matrices can be computed for each photograph using the general form (15) of the minimum constraints relationship

$$
\begin{aligned} &\dot{\mathbf{R}}_i = \dot{\mathbf{N}}_i - \dot{\mathbf{N}}_{y_i} \mathbf{N}_{y_i}^{-1} \dot{\mathbf{N}}_{y_i}^T && \dot{\mathbf{r}}_i = \dot{\mathbf{u}}_i - \dot{\mathbf{N}}_{y_i} \mathbf{N}_{y_i}^{-1} \mathbf{u}_{y_i} \\ &\ddot{\mathbf{R}}_i = \ddot{\mathbf{N}}_i - \ddot{\mathbf{N}}_{y_i} \mathbf{N}_{y_i}^{-1} \ddot{\mathbf{N}}_{y_i}^T + \ddot{\mathbf{H}}_i^T \ddot{\mathbf{H}}_i && \ddot{\mathbf{r}} = \ddot{\mathbf{u}}_i - \ddot{\mathbf{N}}_{y_i} \mathbf{N}_{y_i}^{-1} \mathbf{u}_{y_i} \\ &\tilde{\mathbf{R}}_i = \tilde{\mathbf{N}}_i - \dot{\mathbf{N}}_{y_i} \mathbf{N}_{y_i}^{-1} \ddot{\mathbf{N}}_{y_i}^T + \dot{\mathbf{H}}^T \ddot{\mathbf{H}}_i \end{aligned} \tag{16}
$$

$$
\mathbf{R} = \dot{\mathbf{R}} - \tilde{\mathbf{R}}\,\ddot{\mathbf{R}}^{-1} \tilde{\mathbf{R}}^T = \sum_{i=1}^{m} \{\dot{\mathbf{R}}_i - \tilde{\mathbf{R}}_i \ddot{\mathbf{R}}_i^{-1} \tilde{\mathbf{R}}_i^T\}
$$

$$
\mathbf{r} = \dot{\mathbf{u}} - \tilde{\mathbf{R}}\,\ddot{\mathbf{R}}^{-1} \ddot{\mathbf{u}} = \sum_{i=1}^{m} \{\dot{\mathbf{u}}_i - \tilde{\mathbf{R}}_i \ddot{\mathbf{R}}_i^{-1} \ddot{\mathbf{u}}_i\}
$$

A quick description of the algorithm according to the preceding formulation is:

a. the matrices $\dot{\mathbf{N}}_i$, $\tilde{\mathbf{N}}_i$, $\ddot{\mathbf{N}}_i$, $\mathbf{N}_{y_i}$, $\dot{\mathbf{N}}_{y_i}$, $\ddot{\mathbf{N}}_{y_i}$ and $\dot{\mathbf{u}}_i$, $\ddot{\mathbf{u}}_i$, $\mathbf{u}_{y_i}$ are formed for each photo as it is introduced into the adjustment,

b. the matrices $\dot{\mathbf{R}}_i$, $\tilde{\mathbf{R}}_i$, $\ddot{\mathbf{R}}_i$ and $\dot{\mathbf{r}}_i$, $\ddot{\mathbf{r}}_i$ are computed, and

c. the contributions ($\dot{\mathbf{R}}_i - \tilde{\mathbf{R}}_i \ddot{\mathbf{R}}_i^{-1} \tilde{\mathbf{R}}_i^T$ and $\dot{\mathbf{r}}_i - \tilde{\mathbf{R}}_i \ddot{\mathbf{R}}_i^{-1} \ddot{\mathbf{r}}_i$) of the i th photograph is added to the system of normal equation.

Then the solution of the normals (in the case of minimum constraints (15)) is given as

$$\dot{\mathbf{x}} = (\mathbf{R} + \dot{\mathbf{H}}^T \dot{\mathbf{H}})^{-1} \mathbf{r} + \dot{\mathbf{E}}^T \mathbf{S}\, \mathbf{z} \tag{17}$$

where $\mathbf{S} = (\dot{\mathbf{H}} \dot{\mathbf{E}}^T + \ddot{\mathbf{H}} \ddot{\mathbf{E}}^T)^{-1}$ and $\dot{\mathbf{E}}$, $\ddot{\mathbf{E}}$ are arbitrary full rank matrices with rank equal to that of the matrices $\dot{\mathbf{H}}$ and $\ddot{\mathbf{H}}$ respectively, so as to fulfil the relations $\dot{\mathbf{A}} \dot{\mathbf{E}}^T = \mathbf{0}$ and $\ddot{\mathbf{A}} \ddot{\mathbf{E}}^T = \mathbf{0}$. A typical choice of $\dot{\mathbf{E}}$, $\ddot{\mathbf{E}}$ is that of inner constraints (see Dermanis, 1991 for analytical representations).

The corrections $\ddot{\mathbf{x}}_i$ as well as the values of the additional parameters $\mathbf{y}_i$, are computed for each photo separately through the relations

$$\ddot{\mathbf{x}}_i = \ddot{\mathbf{R}}_i^{-1} (\ddot{\mathbf{u}}_i - \tilde{\mathbf{R}}_i^T \dot{\mathbf{x}}) \tag{18}$$

$$\hat{\mathbf{y}}_i = \mathbf{N}_{y_i}^{-1} (\mathbf{u}_{y_i} - \dot{\mathbf{N}}_{y_i}^T \dot{\mathbf{x}} - \ddot{\mathbf{N}}_{y_i}^T \ddot{\mathbf{x}}) \quad . \tag{19}$$

After the computation of the vectors $\ddot{\mathbf{x}}_i$ and $\hat{\mathbf{y}}_i$ for each photo, the vector $\hat{\mathbf{v}}_i$ of the estimates for the observation errors can be evaluated as

$$\hat{\mathbf{v}}_i = \mathbf{b}_i - \dot{\mathbf{A}}_i \dot{\mathbf{x}} - \ddot{\mathbf{A}}_i \ddot{\mathbf{x}}_i - \mathbf{D}_i \hat{\mathbf{y}}_i \tag{20}$$

and the quadratic form is,

$$\hat{\varphi}_i = \hat{\mathbf{v}}_i^T \mathbf{P}_i \hat{\mathbf{v}}_i \tag{21}$$

and for all photographs

$$\hat{\varphi} = \sum_{i=1}^{m} \hat{\varphi}_i = \sum_{i=1}^{m} \hat{\mathbf{v}}_i^T \mathbf{P}_i \hat{\mathbf{v}}_i \quad . \tag{22}$$

The a-posteriori variance of unit weight is given by

$$\hat{\sigma}^2 = \frac{\hat{\varphi}}{DF} = \sum_{i=1}^{m} \hat{\mathbf{v}}_i^T \mathbf{P}_i \hat{\mathbf{v}}_i \tag{23}$$

where DF are the degrees of freedom, which can be calculated as

$$DF = n - r + k = \sum_{i=1}^{m} n_i - \left(r_1 + \sum_{i=1}^{m} r_{2i} + \sum_{i=1}^{m} r_{3i} \right) + k \tag{24}$$

where n is the number of observations in all photos, r is the sum of r_1 (total number of coordinates of control points), r_2 (total number of e.o. parameters), r_3 (total number of additional parameters) and k (number of constraints for the definition of the reference frame).

Consequently, the covariance matrices are computed according to

$$\hat{\mathbf{C}}(\dot{\mathbf{x}}) = \hat{\sigma}^2 \dot{\mathbf{Q}} = \hat{\sigma}^2 \left\{ (\mathbf{R} + \dot{\mathbf{H}}^T \dot{\mathbf{H}})^{-1} - \dot{\mathbf{E}}^T \mathbf{S}\, \mathbf{S}^T \dot{\mathbf{E}} \right\} \tag{25}$$

for i = 1, 2, ..., m

$$\begin{aligned}
\hat{\mathbf{C}}(\dot{\mathbf{x}}, \ddot{\mathbf{x}}_i) &= \hat{\sigma}^2 \tilde{\mathbf{Q}}_i = -\hat{\sigma}^2 \left\{ \dot{\mathbf{Q}} \tilde{\mathbf{R}}_i \ddot{\mathbf{R}}_i^{-1} - \dot{\mathbf{E}}^T \mathbf{S}\, \mathbf{S}^T \ddot{\mathbf{E}}_i \right\} \\
\hat{\mathbf{C}}(\ddot{\mathbf{x}}_i) &= \hat{\sigma}^2 \ddot{\mathbf{Q}}_{ii} = \hat{\sigma}^2 \left\{ \ddot{\mathbf{R}}_i^{-1} + \ddot{\mathbf{R}}_i^{-1} \tilde{\mathbf{R}}_i^T \dot{\mathbf{Q}} \tilde{\mathbf{R}}_i \ddot{\mathbf{R}}_i^{-1} - \ddot{\mathbf{E}}_i^T \mathbf{S}\, \mathbf{S}^T \ddot{\mathbf{E}}_i \right\} \\
\hat{\mathbf{C}}(\dot{\mathbf{x}}, \hat{\mathbf{y}}_i) &= -\hat{\sigma}^2 \dot{\mathbf{Q}}_{\hat{y}_i} = -\hat{\sigma}^2 \dot{\mathbf{Q}} \dot{\mathbf{N}}_{y_i} \mathbf{N}_{y_i}^{-1} \\
\hat{\mathbf{C}}(\ddot{\mathbf{x}}_i, \hat{\mathbf{y}}_i) &= \hat{\sigma}^2 \ddot{\mathbf{Q}}_{\hat{y}_i} = \hat{\sigma}^2 \ddot{\mathbf{R}}_i^{-1} \tilde{\mathbf{R}}_i^T \dot{\mathbf{Q}} \dot{\mathbf{N}}_{y_i} \mathbf{N}_{y_i}^{-1} \\
\hat{\mathbf{C}}(\hat{\mathbf{y}}_i) &= \hat{\sigma}^2 \mathbf{Q}_{\hat{y}_i} = \hat{\sigma}^2 \left\{ \mathbf{N}_{y_i}^{-1} + \mathbf{N}_{y_i}^{-1} \dot{\mathbf{N}}_{y_i}^T \dot{\mathbf{Q}} \dot{\mathbf{N}}_{y_i} \mathbf{N}_{y_i}^{-1} \right\}
\end{aligned} \tag{26}$$

and

$$\hat{\mathbf{C}}(\hat{\mathbf{v}}_i) = \hat{\sigma}^2 \mathbf{Q}_{\hat{v}_i}$$

where

$$\mathbf{Q}_{\hat{v}_i} = \mathbf{P}_i^{-1} - \left[\dot{\mathbf{A}}_i \; \ddot{\mathbf{A}}_i \; \mathbf{D}_i \right] \begin{bmatrix} \dot{\mathbf{Q}} & \tilde{\mathbf{Q}}_i & \dot{\mathbf{Q}}_{\hat{y}_i} \\ \tilde{\mathbf{Q}}_i^T & \ddot{\mathbf{Q}}_i & \ddot{\mathbf{Q}}_{\hat{y}_i} \\ \dot{\mathbf{Q}}_{\hat{y}_i}^T & \ddot{\mathbf{Q}}_{\hat{y}_i}^T & \mathbf{Q}_{\hat{y}_i} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{A}}_i^T \\ \ddot{\mathbf{A}}_i^T \\ \mathbf{D}_i^T \end{bmatrix} . \tag{27}$$

2.4 Inclusion of geometrical constraints into the adjustment

Theoretically such geometrical constraints can be imposed either on $\dot{\mathbf{x}}$ or on $\ddot{\mathbf{x}}$. Practically, however, constraints on exterior orientation means that we know the values of these parameters quite accurately, which only rarely (if ever) is the case. On the other hand, geometrical constraints on ground coordinates of points (eg. parallelism, perpendicularity, coplanarity, points on arc, etc.) is very usual. Such constraints can then be used to improve our solution (eg. Ethrog, 1984). Hence,

$$\mathbf{G}\, \dot{\mathbf{x}} = \mathbf{d} \tag{28}$$

Then the solution is given by

$$\begin{aligned}
\dot{\mathbf{x}}^{(G)} &= \dot{\mathbf{x}} - \dot{\mathbf{Q}}\, \mathbf{G}^T (\mathbf{G}\, \dot{\mathbf{Q}}\, \mathbf{G}^T)^{-1} (\mathbf{G} \dot{\mathbf{x}} - \mathbf{d}) \\
\dot{\mathbf{Q}}^{(G)} &= \hat{\sigma}^2 \left\{ \dot{\mathbf{Q}} - \dot{\mathbf{Q}}\, \mathbf{G}^T (\mathbf{G}\, \dot{\mathbf{Q}}\, \mathbf{G}^T)^{-1} \mathbf{G}\, \dot{\mathbf{Q}} \right\} \quad .
\end{aligned} \tag{29}$$

Next the vector $\dot{\mathbf{x}}$ is replaced by $\dot{\mathbf{x}}^{(G)}$ in the relations (18) and (19), and analogously the matrix $\dot{\mathbf{Q}}$ by $\dot{\mathbf{Q}}^{(G)}$ in the relations (26).

The insertion of the constraints can follow a sequential form (Rossikopoulos and Fotiou, 1990) in which case the insertion of the k th constraint

$$\mathbf{g}^T \dot{\mathbf{x}} = d \tag{30}$$

where $\mathbf{g}^T$ is the k th row of the matrix $\mathbf{G}$, is as follows: The quantities $\hat{e}$ and $q^2(\hat{e})$ is computed

$$\hat{e} = \mathbf{g}^T \dot{\mathbf{x}}^{(k-1)} - d = \sum_r \{g_r \dot{x}_r^{(k-1)}\} - d \tag{31}$$

$$q^2(\hat{e}) = \mathbf{g}^T \dot{\mathbf{Q}}^{(k-1)} \mathbf{g} = \sum_r \sum_s g_r g_s (\dot{\mathbf{Q}}^{(k-1)})_{rs} \tag{32}$$

where $\dot{x}_r^{(k-1)}$ is the r element of the vector $\dot{\mathbf{x}}^{(k-1)}$, g_r, g_s are the elements r and s respectively of the vector $\mathbf{g}$ and $(\dot{\mathbf{Q}}^{(k-1)})_{rs}$ is the element of the r th row and the s th column of the matrix $\dot{\mathbf{Q}}^{(k-1)}$. This matrices $\dot{\mathbf{x}}^{(k-1)}$ and $\dot{\mathbf{Q}}^{(k-1)}$ are the solution of the normal equations with the previous k−1 constraints. When k is the first constraint to be introduced then the matrices are the $\dot{\mathbf{x}}$ and $\dot{\mathbf{Q}}$.

Next the coordinate $\dot{x}_i^{(k)}$ in the vector $\dot{\mathbf{x}}^{(k)}$ is computed by

$$\dot{x}_i^{(k)} = \dot{x}_i^{(k-1)} - \delta\dot{x}_i^{(k-1)} = \dot{x}_i^{(k-1)} - \frac{\hat{e}}{q^2(\hat{e})} \sum_r (\dot{\mathbf{Q}}^{(k-1)})_{ir}\, g_r \tag{33}$$

and each elements $(\dot{\mathbf{Q}}^{(k)})_{ij}$ of the matrix $\dot{\mathbf{Q}}^{(k)}$ by

$$(\dot{\mathbf{Q}}^{(k)})_{ij} = (\dot{\mathbf{Q}}^{(k-1)})_{ij} - \frac{1}{q^2(\hat{e})} \sum_r \{g_r (\dot{\mathbf{Q}}^{(k-1)})_{ir} \sum_s g_s (\dot{\mathbf{Q}}^{(k-1)})_{sj}\}\,. \tag{34}$$

The range of all the above summations refer only to those coordinates of the control points which are included in the constraint.

2.5 About combined adjustment

In case we have surveying measurements (angles, distances) we can use them into a combined surveying-photogrammetric adjustment. Generally the inclusion of surveying measurements into the bundle adjustment is straightforward and one has only to consider the updating of the matrices $\dot{\mathbf{N}}$ and $\dot{\mathbf{u}}$ of eq. (9).

A word of caution is however important here concerning the relationship between the weights of measurements of photo coordinates and those of surveying measurements. This is a typical variance component estimation problem well documented in the literature (eg. Kubik, 1967, Förstner, 1979, Schaffrin, 1983, Rao and Kleffe, 1988, Dermanis and Rossikopoulos, 1991, Dermanis and Fotiou, 1992).

3. STATISTICAL TESTING SCENARIOS

During the adjustment of the measurements a number of statistical testing is applied. These tests are applied both for the evaluation of the imposed geometrical constraints and the additional parameters. It is well known that in the first case non-compatible constraints can lead to divergence of the solution, while in the second high correlation between additional parameters or between those and e.o. parameters can lead to ill-conditioned systems of normals.

These tests of course are besides the usual blunder detection module (Dermanis, 1990) which is included in SNAP.

3.1 Test of constraints

Testing of the compatibility of the imposed geometrical constraints can be done both globally and one-by-one as follows:

Global testing of constraints. This test is based on the equation

$$F = \frac{(\mathbf{G}\,\dot{\mathbf{x}} - \mathbf{d})^T (\mathbf{G}\,\dot{\mathbf{Q}}\,\mathbf{G}^T)^{-1} (\mathbf{G}\,\dot{\mathbf{x}} - \mathbf{d})}{q\,\hat{\sigma}^2} \le F^{\alpha}_{q,DF} \tag{35}$$

where q is the number of constraints, and the quantities $\dot{\mathbf{Q}}$ and $\hat{\sigma}^2$ are coming from the solution without constraints. In case this test fails (meaning that at least one constraint is incompatible) one should perform a test for each imposed constraint sequentially. Besides eq. 35, alternative formulas can be used (see eg. Dermanis, 1986).

Sequential testing of constraints. This test follows the general data snooping strategy. That is the testing of the k th constraint $\mathbf{g}^T \dot{\mathbf{x}} = d$ is based on equation

$$F = \frac{\hat{e}^2}{\hat{\sigma}^2 q^2(\hat{e})} \le F^{\alpha_o}_{1,DF} \tag{36}$$

where the quantities $\hat{e} = \mathbf{g}^T \dot{\mathbf{x}}^{(k-1)} - d$, $q^2(\hat{e}) = \mathbf{g}^T \dot{\mathbf{Q}}^{(k-1)} \mathbf{g}$, $\hat{\sigma}^2$ and DF have been computed from the solution with the previous k−1 constraints.

In order for the two tests (eqs. 35, 36) to be equivalent the respective significance levels α and α_o should be chosen appropriately, according to Baarda's reliability theory (Baarda, 1967).

3.2 Test of additional parameters

Let $\mathbf{y}_I$ is the group of additional parameters of the i th photograph, that we are currently testing. These parameters are non-significant if:

$$F = \frac{\hat{\mathbf{y}}_I^T \mathbf{Q}^{-1}_{\hat{\mathbf{y}}_I \hat{\mathbf{y}}_I} \hat{\mathbf{y}}_I}{q\,\hat{\sigma}^2} \le F^{\alpha}_{q,DF} \tag{37}$$

where $\mathbf{Q}_{\hat{\mathbf{y}}_I\hat{\mathbf{y}}_I}$ is the submatrix of $\mathbf{Q}_{\hat{\mathbf{y}}_i}$.

If a single additional parameter y_j of i th photograph is to be tested, then the statistic used is:

$$F = \frac{\hat{y}_j^2}{\hat{\sigma}^2 q^2(\hat{y}_j)} \le F^{\alpha_o}_{1,DF} \tag{38}$$

or

$$t = \frac{\hat{y}_j}{\hat{\sigma}\, q(\hat{y}_j)} \leq \sqrt{F^{\alpha_o}_{1,DF}} = t^{\alpha_o/2}_{DF} \tag{39}$$

where $q^2(\hat{y}_j)$ is the corresponding diagonal element of $\mathbf{Q}_{\hat{y}_i}$.

In case we are testing a set of additional parameters $\mathbf{y}_*$ not included in the available model, with corresponding coefficient matrix $\mathbf{D}_*$, then the statistic used is (Dermanis, 1990):

$$F = \frac{f-q}{q} \frac{\delta\Omega}{DF\,\hat{\sigma}^2 - \delta\Omega} \leq F^{\alpha}_{q,DF-q} \tag{40}$$

where q is the number of the additional parameters $\mathbf{y}_*$, and

$$\delta\Omega = \hat{\mathbf{v}}^T \mathbf{Q}_{\hat{v}}^{-1} \mathbf{D}_* \; [\; \mathbf{D}_*^T \mathbf{Q}_{\hat{v}} \mathbf{D}_* \;]^{-1} \mathbf{D}_*^T \mathbf{Q}_{\hat{v}}^{-1} \hat{\mathbf{v}} \quad . \tag{41}$$

When a single additional parameter y_* with coefficient vector $\mathbf{d}_*$ is to be tested, the test statistic becomes

$$F = r \frac{f-1}{f-r^2} \leq F^{\alpha_o}_{1,DF-1} \tag{42}$$

or

$$t = r \sqrt{\frac{f-1}{f-r^2}} \leq \sqrt{F^{\alpha_o}_{1,DF-1}} = t^{\alpha_o/2}_{DF-1} \tag{43}$$

where

$$r^2 = \frac{[\mathbf{d}_*^T \mathbf{Q}_{\hat{v}}^{-1} \hat{\mathbf{v}}]^2}{\hat{\sigma}^2\, \mathbf{d}_*^T \mathbf{Q}_{\hat{v}} \mathbf{d}_*} \quad . \tag{44}$$

In order to finally choose the appropriate additional parameters two strategies, as the most promising, can be used (Sarjakoski, 1984): the orthogonalization method and the procedure of stepwise regression analysis. The second method is applicable in the case of photo-invariant additional parameters but very cumbersome in the case of photo-variant.

The first method is based on singular value decomposition

$$\mathbf{S} = \mathbf{C}^T \mathbf{N}_y \mathbf{C} \tag{45}$$

where $\mathbf{C}$ is the orthogonal matrix of eigenvectors of $\mathbf{N}_y$ and $\mathbf{S}$ is the diagonal matrix of eigenvalues. Then the new set of additional parameters $\mathbf{a}$ to be estimated is connected to the old set by

$$\mathbf{a} = \mathbf{C}^T \mathbf{y} \quad \text{or} \quad \mathbf{y} = \mathbf{C}\,\mathbf{a} \quad . \tag{46}$$

We are currently experimenting in order to find the most optimum procedure.

4. SNAP - OPERATIONAL ASPECTS

4.1 Hardware considerations

The hardware components of the SNAP system were kept to minimum level. More specifically, a standard IBM-compatible PC is suggested, with Intel 80286 or 80386 microprocessor (math-coprocessors are used when present but not required) with 640Mb of RAM and a hard disk.

4.2 Data acquisition and pre-processing

Today we can distinguish three main types of input devices as far as Architectural Photogrammetry is concerned, namely: small analytical photogrammetric instruments based on comparator measurements (eg. Adams MPS-2, Topcon PA-2000, Galileo-Siscam STEREOBIT), digitizer-based systems (eg. Wild ELCOVISION, Rolleimetric MR2) and small digital stations (eg. DVP, Gagnon et.al., 1990, DIRECT, Patias, 1991).

SNAP can read data (stored in a file) from anyone of the above sources, while at present appropriate drivers for on-line connection to different instruments are been developed. Through these drivers the photo-coordinates of all "control" points involved in reference frame definition, in formation of geometrical constraints and in formation of surveying measurements, are read in a photo-wise manner.

Pre-processing of the data includes incorporation of the camera calibration report, correction of comparator/digitizer/scanner affine errors, correction of radial lens distortion and finally determination of approximate values for the exterior orientation parameters, if they are not available (eg. Hådem, 1990, Zeng and Wang, 1992).

4.3 Data processing and stereo-plotting

The processing of the data includes the following phases:

Definition of the reference frame

In order for a reference frame to be defined the user has the option to fix:

- all control points coordinates
- some of them only
- some control points coordinates and put geometrical constraints, which are recognized on the object (eg. points on same line, or on same distance from camera, parallel or perpendicular lines, etc.).

Formation of normal equations. The user can choose between the photo-variant or photo-invariant approach. In either case the normals are formed sequentially in a "first-come, first-serve" fashion as far the photographs is concerned. There is no restriction on the number of photographs involved. Additionally, the inclusion of the geometrical constraints is sequential and thus the user can check their appropriateness in every step.

Auxiliary observations. Additional surveying measurements can be incorporated into a combined adjustment (currently under development).

Stereo-plotting. After the processing of the data, the i.o. and the e.o. parameters plus the additional parameters pertaining to each photo are computed and kept into a file. The plotting of the detail points is done either on-line or off-line by measuring the photo-coordinates (on as many photos) of the detail points and using the pre-saved information of each photo respectively. The output are ground coordinates of the detail points, which are displayed on the screen simultaneously. SNAP is then relies on popular graphics packages for editing.

5. SNAP'S GROWTH POTENTIAL

SNAP is an operational system based on low-cost equipment and state-of-the-art software. It combines all the characteristics of big bundle-adjustment programs, with coverage of the requirements of Architectural Photogrammetry, concerning non-metric cameras, various input devices, limited ground control and geometrical constraints on the object. While its main body is operational, minor refinements and additions, mainly concerning drivers to different graphic packages are currently under development.

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