

ANALYTICAL PHOTOGRAMMETRIC SURVEY OF THE LEANING TOWER OF PISA
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Commission V

The University of Milan and AGIP SpA repeated the photogrammetric survey of the Leaning Tower of Pisa. The paper relates the procedure followed and the results obtained. The exposures were taken from an elevator at a height of about twenty-three meters from the ground such that all the orders which make up the Tower and the Bell Chamber were clearly visible in the photographs. Sixty-six exposures were taken in all, some in color and some in black and white. The color ones were employed for plotting after their reliability was tested. The aim, as in the previous survey was to determine the shape and the spatial position of the successive orders of the Tower and of the dripstones which limit them. An analytical plotter was employed to determine the coordinates of the points. The collected data were processed using a CAD station and appropriate software in order to solve the geometric problems and to display the results. Finally, the results of the previous survey and the present ones were compared.

Key words: analytical, close-range, photogrammetry, terrestrial

1. INTRODUCTION

The Torre Campanaria della Primaziale di Pisa, or Bell Tower of Pisa cathedral, which we shall simply call the Tower from now on, is world-renowned not only for its artistic beauty but also for its extraordinary "lean". Much has been written about the Tower, which was started in 1174 and was completed shortly after 1370, and details of the dates in its construction are included in an earlier work by the present authors (Baj E. and Bozzolato G., 1989).

To help set in context the purpose of the second, and more exhaustive photogrammetric survey of the exterior of the Tower of Pisa, we feel it useful to concentrate on the construction and control techniques used by the builders of the Tower. Our aim is to show how the way the Tower was built created the very premise for using photogrammetry to achieve our aim. This technique is in fact one of the few available that can reproduce a geometric configuration as well as provide data that may be integrated to other information collected by means of direct measurements.

2. BACKGROUND TO THE CONSTRUCTION AND CONTROLS

At the time the Tower was built, the technique of leveling had not yet been developed (and was actually first described by M. Thevénot in 1666, that is some four hundred years later). At the time, the constructors had at their disposal only the plumb-line and other simple devices. Rudimentary as they were, these instruments were able to provide accuracy of the horizontal to within \pm one centimeter across the entire eleven-to fifteen-metre width of the Tower.

The unstable nature of the ground in the city of Pisa was considered a typical feature of the area. The frail upper surface initially sank under the weight of a heavy body but, over a period of time, increased its consistency and resistance.

It was common practice for a stock of elements, such as the hewn blocks of stone, the columns, and the arches to be prepared before the building work was to start. This enabled the marble workers and the masons to keep pace with the rest of the building processes. This was particularly so in the case of modular structures such as the Tower

of Pisa. Given the instability of the land, it is likely that the master builder of the Tower of Pisa arranged for each "storey" to be put up as quickly as possible and for there to be a "waiting period" while the ground settled, after which the next "storey" could be started.

Measurements taken on certain exposed areas of the stone at different heights of the Tower have revealed that the stones were trimmed to accommodate settlement. This shows that subsidence in more than one direction must have taken place during the construction of the Tower, the north-south shift being the most evident and the one that took precedence after completion (Lumini, 1967).

While it is clear that this subsidence was noticeable, the phenomenon would not have appeared to have concerned the architects overmuch, used as they were to the fact that many of the city's buildings were not quite perpendicular. A document published in 1298, in other words while construction was still underway discusses the first checks on the trueness of the Tower. While describing the approach used in detail, it makes no reference to any settlement. One thing is certain, though. In about 1384, Antonio Veneziano painted a fresco of the Camposanto in Pisa which shows the completed Tower leaning.

Over the years, the issue of the Tower's propensity to lean has attracted considerable attention. Among others, Vasari wrote about it in 1562 while De La Condamine examined the issue in 1755 and Soufflot three years later. De La Lande returned to the subject in 1790 as did Da Morrona and Gherardesca in 1812 and 1838 respectively. Relatively reliable measurements were taken by Cresy and Taylor in 1817 and by De Fleury in 1859.

It was only this century, though, that techniques and instruments that might provide more accurate readings were developed. Using the latest techniques, the Institute of Geodesy, Topography, and Photogrammetry in the Faculty of Engineering of the University of Pisa set in train a program of annual testing of the degree of inclination based on specific points on the Tower (Cicconetti, 1927, 1931, 1948; Ballarin, 1953, 1959; Geri and Palla, 1988).

The degree of incline is only one aspect of the settlement that the Tower has experienced and a complete survey will clearly include a variety of elements and a thorough knowledge of three-dimensional geometry is of primary importance to any such research. Two surveys of the entire structure have been undertaken to date, one in 1908 by Bernieri and the other in 1965, under the leadership of Polvani (published in 1913 and 1971 respectively). These surveys were incomplete for three main reasons:

- i) they used direct measurements and were, by definition, limited to points that were physically accessible to the surveyors
- ii) in such circumstances, a choice of measuring points cannot be planned and the correlation between the points is necessarily random, while the difficulty of access makes measurement difficult and can lead to cumulative errors
- iii) using direct measurements on a monument that is made up of curved forms and a variety of angles of inclination, it is almost impossible to represent the Tower within a system of spatial coordinates that might be sufficiently accurate for use in possible interventions in the future.

If we then consider the fact that the Tower's shifts did, in the past, alter direction then it becomes clear that a knowledge of three-dimensional geometry is insufficient by itself. In order to ascertain the behaviors of the Tower during the two centuries of its construction, it is necessary to conduct a detailed survey of each of the building blocks, storey by storey.

For the reasons stated above, it is clear that only a photogrammetric survey would be sufficient to provide the information to the degree of accuracy required.

A preliminary terrestrial photogrammetric survey was undertaken in 1983. The data gleaned from that partial and experimental survey provided much more information than expected and studies are yet to be completed (Baj and Bozzolato, 1988).

In agreement with the Opera della Primaziale it was decided to undertake a full-scale photogrammetric survey to cover the entire Tower.

3. SETTING UP THE 1990 PHOTOGRAMMETRIC SURVEY

This time around, the AGIP team had at its disposal a helicopter-mounted SER system which was specifically-designed for use in difficult circumstances and has been mounted on an Agusta AB-412 (Bozzolato, 1989). This equipment would have made it possible to provide synchronized stereo images of the entire Tower from the vertical, the horizontal, and from a variety of angles down to a distance of twenty-five meters.

This proved impossible, however, as the government had, in the meantime banned over-flying of the Tower for fear that the added vibrations might further destabilize the structure.

The team therefore was forced to adopt a simpler approach which was an aerial platform mounted on the back of a truck.

During the planning stages it was also decided to mark out the points from which the photographs would be taken, as well as to refurbish the photogrammetric control points used in 1983 and to add some new ones. The spatial coordinates of all the points were also to be determined on the basis of a new triangulation network which would relate to the Italian Cartographical System (Sistema Cartografico Nazionale).

4. THE FIELD WORK

4.1 Pre-targeting

During the preliminary reconnaissance work, we noted that a number of geognostic and geophysical surveys, as well as maintenance work to underground structures over the intervening five years had destroyed a number of the stations of the triangulation network and the photogrammetric control points marked out in 1983 on the paving below the Tower and on the grass adjacent to it.

Apart from a few bolts that had been cemented into the paving, the only undamaged control points were the nails that had been hammered into the masonry of the Tower.

In order to achieve the accuracy required by the project, it was necessary to map out a new, more precise triangulation network (Figure 1).

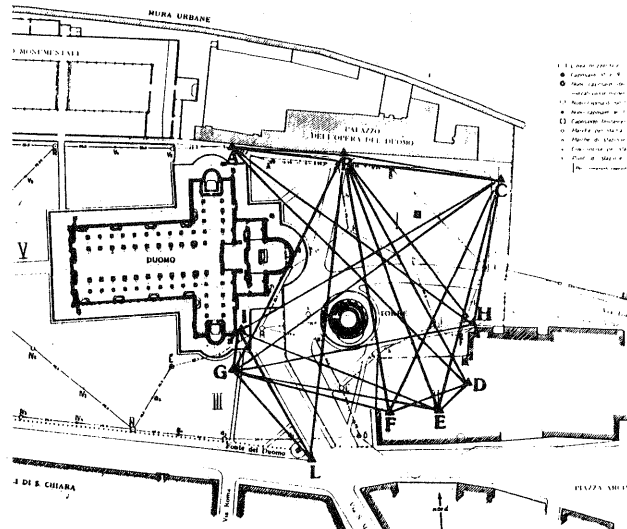


Figure 1
Piazza dei Miracoli, topographic network, 1990

Four concrete-capped pillars were constructed where triangulation stations were fixed. Three other stations were fixed on the parapet of a loggia of the Duomo Museum. All the stations were marked with cemented-in bolts. Three stations were placed on a special stabilized tripod.

The twenty-four 4 x 40 millimeter steel nails (control points) fixed into the interstices of the masonry blocks of the Tower in 1983 were supplemented with a further thirty-two. All of the fifty-six points were marked with a black "Malta cross" on a white ground as well as a number, so as to make them clearly visible from the triangulation stations as well as on the images produced.

The twelve bolts on the ground that had survived following the 1983 experiment were highlighted with paint and were used as supernumerary controls between the earlier findings and the 1990 survey.

4.2 Geodetic survey

The following approach was adopted:

- i) all the angles between the directions to the triangulation stations were measured. Given that these were all less than 150 meters apart, three measurements were considered sufficient
- ii) as far as the photogrammetric control points were concerned, we decided to measure both the horizontal and the vertical angles twice, given that they were less than one hundred meters apart
- iii) angles were measured using the WILD T3 theodolite
- iv) the distances between the triangulation stations each way were measured using an AGA 140 geodimeter
- v) the difference in height between the photogrammetric control points was calculated using trigonometric leveling with reference to one specific point. The height of this point was determined by leveling using a JENOPTIK NI 007 in relation to a benchmark of the Istituto Geografico Militare (IGM).

Computations and adjustments were made using Professor Bencini's Planasc program in the Computing Laboratory of the Geodetic Division at the IGM. Thus we obtained the plane/altimetric coordinates of all the triangulation stations and of all the control points, their standard deviations as well as error ellipses.

The maximum values of the error ellipses were found to be \pm three millimeters. The standard height deviation for the control points was \pm 2.7 millimeters. These values are in line with the degree of accuracy required for this kind of project.

In order to ensure the best orientation of the stereograms in the restitution, the height and position of each photography point was determined using an electronic theodolite.

4.3 Planning the photography

As stated above, an aerial platform, the so-called cherry-picker was used. This consists of a basket on an adjustable arm which is in turn mounted on the back of a truck. Large enough to hold two people with their equipment, the basket may be lifted with an extraordinary degree of accuracy to the height required. The use of this machinery offered us a considerable degree of freedom of movement around the monument on paved areas as well as up to a height of twenty-three meters.

Given the cylindrical shape of the building, the twelve photogrammetric stations were positioned almost equidistant from each other, and thus they formed a dodecagon. Two sides were missing however as, in the South-East corner, there is a convent which is now the Museum of the Duomo. To compensate for this, two stations were located higher up and further back where a paved road made manoeuvring of the aerial platform possible (Figure 2).

Each photogrammetric station was thus some thirty-five meters from the Tower and about twenty-three meters above ground level. Each side of the polygon was approximately twenty-seven meters in length. The camera was placed as near to horizontal as possible and at right angles to the

sides of the polygon. This meant that two exposures were taken from each of the stations, one normal to the previous and one normal to the next, respectively. A considerable degree of tolerance had to be allowed because of the pronounced lean of the Tower and the movement of the vehicle on which the camera was mounted.

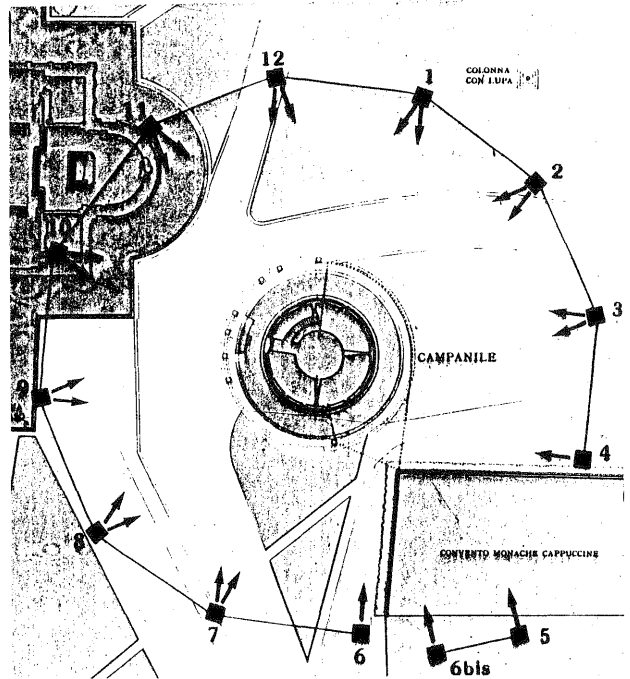


Figure 2.

Plan of the Photogrammetric stations

A total of sixty-six photographs were taken. Of these, forty-four were in black and white on an 15° DIN orthochromatic glass plate. Twenty-two exposures were taken on color transparencies. After a preliminary examination, thirty-three pairs of pictures were selected for possible use in the next stage of the project.

While the format of the exposures was thirteen by eighteen centimeters, with the longer side vertical, the considerable sideways overlap enabled us to survey the entire exterior of the Tower using only six pairs of photographs.

The height that could be achieved by means of the aerial platform enabled us to photograph the entire Tower (Figure 3) with great flexibility and an ideal range of control points.

The entire photographic session took just under two days, with the exposures being developed as soon as they were taken. Only one camera was used. While this offered homogeneity in terms of image, the disadvantage was that synchronization was clearly impossible. There was thus a slight variation in shadows in the pairs of pictures.

Figure 4 shows a couple of photographs.

5. RESTITUTION

The Tower of Pisa is made up of eight orders. A round base, is topped with six arcades with a central core surmounted by a bell chamber. The base and bell chamber are decorated with relief arches while each of the order is circumscribed by cornices or dripstones.

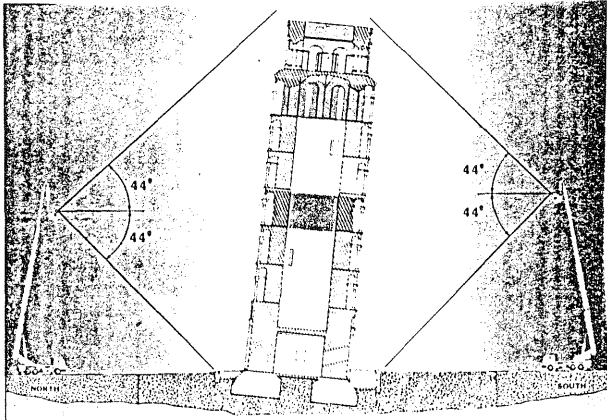


Figure 3.

Altimetric scheme of photogrammetric stations

As confirmed by the first survey, each order should be considered as a separate element limited by the plane of best fitting of the points surveyed on the dripstone.

We used a Wild AC1 for the restitution and, since our aim was to produce a mathematical model of the Tower, we set up a data base into which we filed the information to be processed.

After comparing the data obtained by plotting monochrome and color photographs, we found that the latter gave a more accurate collimation of the points surveyed. After a careful analysis of images available, six pairs of stereo-photograms were chosen for plotting. These were numbers 1 and 12; 2 and 3; 3 and 4; 6 and 7; 8 and 9; 11 and 12 (Figure 2).

The remaining images will be used in a future project that will focus on the details of the Tower.

For the absolute orientation more than seven control points on the Tower were used. The orientation protocol shows that the maximum residual was 0.4 centimeters.

The points surveyed and the way they were codified is shown in Figure 5. The codes LV20 and LV21 refer to the points surveyed on the two edges of the first order dripstone, while LV30 and LV31 relate to the second order, and so on.

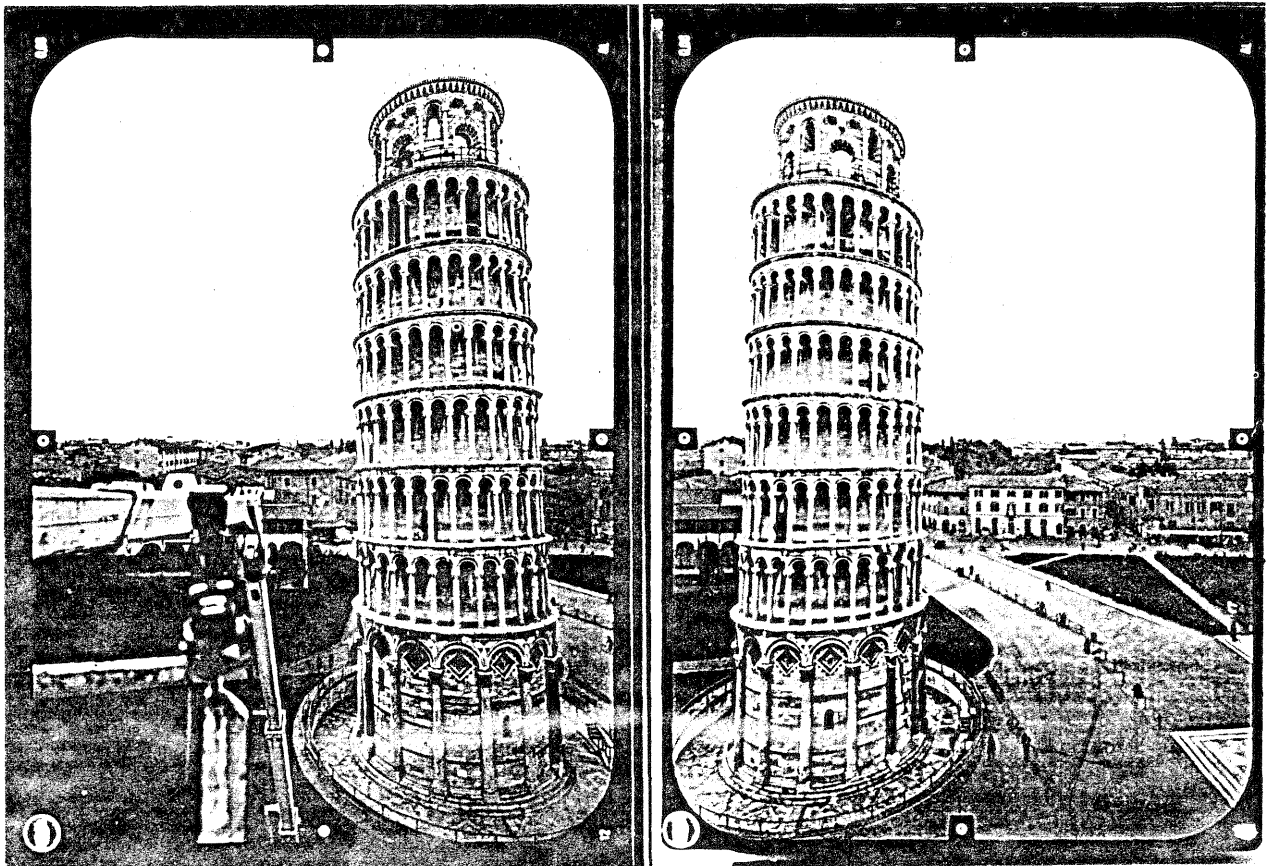
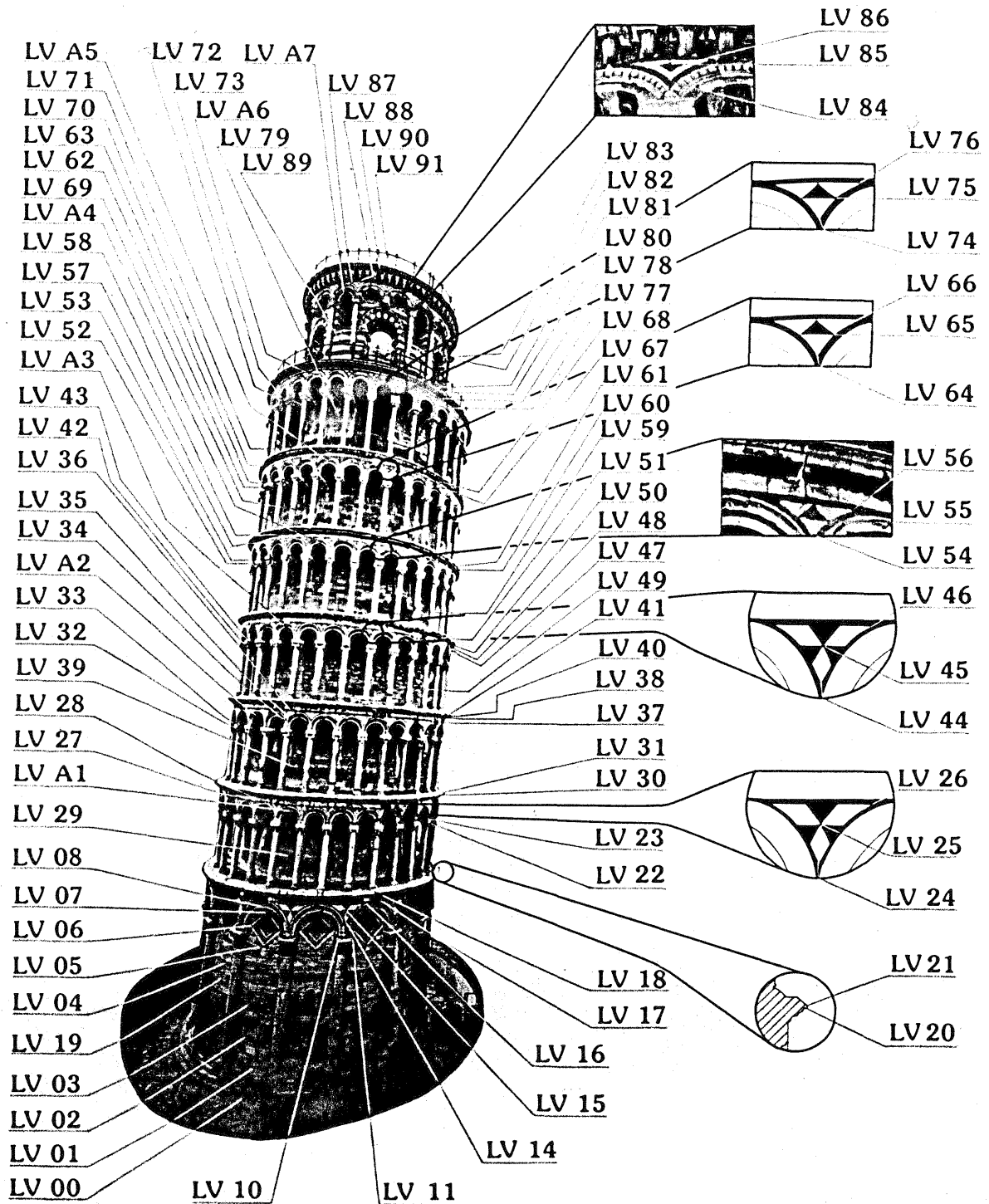


Figure 4.

Pair of stereo-photograms



DRAWN BY M. GUIDARELLI

Figure 5.

Details surveyed and codification

Collimating the dripstone points was somewhat difficult because of the uneven wear and positioning of the stones. The number of points surveyed on the two edges of the dripstone, are different for each level. A maximum number of points was surveyed at level 50 at the height of the camera. At the other levels, the ascending and descending perspectives made it difficult to collimate a similar number of points.

The following number of points were surveyed at each level:

level	number
20	164
30	268
40	282
50	343
60	269
70	188
80	179
90	125

The points surveyed in the base area are coded according to their height. Code L29 relates to the points surveyed on the core of the second order, while L39 refers to the third, and so forth.

The number of points surveyed on the base and on the surface of the cylindrical core are as follows:

order	number
first	1548
second	246
third	289
fourth	287
fifth	320
sixth	294
seventh	212
eighth	264

In point of fact only the points surveyed on the two edges of the dripstones and the cylindrical cores were processed.

6. DATA PROCESSING

6.1 Recording and filing

The points surveyed were filed as follows: ID, X, Y, Z, where ID is the identifier of each point, and X, Y, and Z are the coordinates of the absolute reference system.

The data were then subdivided into files containing readings on similar elements, that is that coded the same. Figure 5 illustrates this.

The data was then processed using the VDIM program on an IBM RS/6000 system.

6.2 The VDIM program

This program is used extensively by AGIP in the analysis of simple curved surfaces and lines in space, starting from a random selection of coordinates. The shapes that are best suited to this treatment are:

- circular cylinders
- elliptical cylinders
- truncated cones
- three-dimensional ellipses and spheres
- two-dimensional ellipses and circles

Determining the optimum parameters of a cylinder or a cone using points which have been defined by means of photogrammetric techniques is no easy task.

It soon becomes clear that traditional iterative processes are going to be of little use. Newtonian methods and their variations are not practical because of the amount of algebraic calculations required.

Briefly, the program works in the following fashion:

- the "general quadric" that describes the points observed is defined using the "least" square method
- the generic quadric is reduced to a central quadric (degenerate). The directions of the axes and the center are calculated from this.
- if the sum of the points observed is reasonable, one of the three axes is very close to the axis of the cylinder or the cone that we are aiming to define. Thus for each of the axes, we define a cylinder or a cone

and then select the one of the three for which the distance between the surface and the points surveyed are on average the smallest.

Developed in Fortran 77, this program has proved highly-effective in tests on an extensive range of samples.

The files we selected for treatment with the VDIM program were those which included measurements taken at the base, on the cores, and on the dripstones. As a result, it was possible to plot the approximate geometric figures derived from the points (which is to say, a cylinder for the base and the cores of the Tower and a circumference for the dripstones as in fact was done in the previous survey). The diameters of the dripstones and of the direction cosines of the perpendicular to the dripstones were also calculated as well as the direction cosines of axes of the cores and their diameters. The accuracy of these data was checked by means of a statistical survey.

6.3 The CATIA program

The data were then processed using CATIA (Computer-aided three-dimensional interactive application). Produced by Dassault Systemes of France, this modeling program enables the user to create correlation between two- and three-dimensional areas quickly and easily. All the views can be correlated and each modification in one view is reflected in the model and, as a result, on all of the other views.

A modular program, inasmuch as there is a base unit onto which one can load separate units for different applications, CATIA is used within the AGIP company on an IBM RS/6000 in an X-Windows environment, or using the graphics applications available on the IBM 5080 system.

First of all the data were separated into layers, using an interface program. One layer consisted of the points surveyed and the other of the geometric figures obtained via the VDIM program. The two layers could, of course, be superimposed as well as being see-through, and it was thus possible to detect the most salient anomalies.

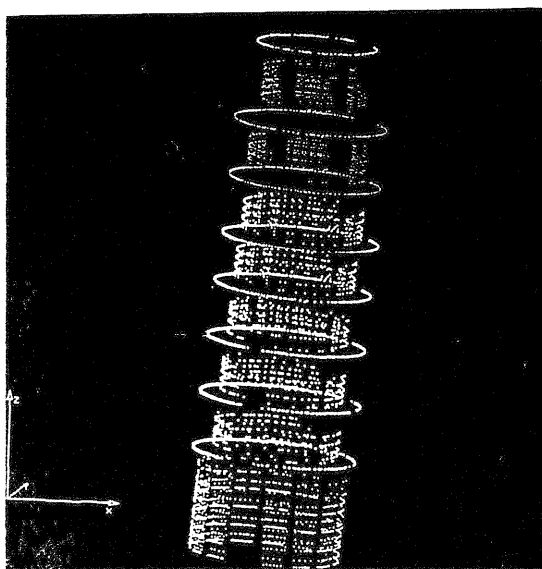


Figure 6
CAD-derived image of point surveyed
on the base, the cores, and the dripstone

6.4 Computer-aided Design processes

A Computer-assisted design program was used for the following reasons:

- i) to detect errors that might have occurred as a result of the processing. This was achieved by overlapping the points onto the geometric figures.
- ii) to complete the geometry of the small part of the cores that could not be seen. In this instance, the cylinders were extended until they intersected the upper and lower dripstone levels
- iii) to provide an effective representation of the work undertaken.

Figure 6 illustrates one outcome of this initiative.

7. RESULTS

Tables 1-4 show the results obtained.

Table 1 relates to the base and to the cylindrical core (henceforth referred to as the cylinder) of each order, and shows:

- a) the coordinates X, Y, and Z of the ends of the core axis assuming that each cylinder is limited at top and bottom by the dripstone plane. The plane limiting the bottom of the base is shown in Figure 5 as level 00.
- b) the diameter of the cylinder and its standard deviation

Table 2 shows the angles between the cylinder axis and the Cartesian axes for each order

Table 3 demonstrates

- a) the X, Y, and Z coordinates of the center of the circumference of the lower edge of the dripstone for every level
- b) the diameter of the lower edge of the dripstone and its standard deviation for every level

Table 4 shows the angles between the circumference perpendicular and the Cartesian axes for each level

The Standard Deviation of all the results depends primarily on two main factors: the precision of the points surveyed and the irregularities of the surfaces surveyed.

7.2 Comparing the 1983 and 1990 results

We would, first of all, like to draw attention to the fact that the data published on the seventh level in the previous survey was unreliable, particularly in terms of the incline of the cylinder axis, inasmuch as only a small part of the core was visible in the photograms. As stated, this value needed to be checked (Baj and Bozzolato, 1988).

Comparing the values from the two surveys, differences of between two and three centimeters between the diameters of the cylinder were noted. Then, comparing the angles of the cylinder axis of each order with the Cartesian Z axis, differences ranging from a few through to fifteen minutes of a sexagesimal degree were noted. The other angles cannot really be compared. Even after having effected a rototranslation of the Cartesian axes in the attempt to make the two frames of reference coincide, a difference remained.

Comparing the value of the dripstone diameters, differences of between two and three centimeters were noted while, comparing the angle of the normal to the dripstone circumference at each level with the Cartesian Z axis, differences of between 1 and 4 minutes of a sexagesimal degree were noted.

The standard deviation for the second survey was slightly larger. In terms of the results obtained for the cylinders, this could be due to the fact that, in the first survey, the photograms were taken from ground level only and so only a part of the core was visible as a result of which the points surveyed were concentrated in a smaller area. It is obviously easier to find a best fitting surface when using a restricted space. Moreover, in the second survey, since the exposures taken at the extremes of the photogrammetric bases were not taken simultaneously, the collimation of the points on the photograms might have been less accurate.

7.3 Future developments

As stated above, our data processing was based on a limited number of points surveyed. The initiative also furnished a considerable number of color and monochrome images which may be used for a variety of other purposes.

As has happened in the cases of restoration projects on other buildings of importance, the stones of the Tower could be numbered and the architectural details recorded. In any event, this comprehensive survey of the Tower constitutes a zero situation for future work.

ACKNOWLEDGMENTS

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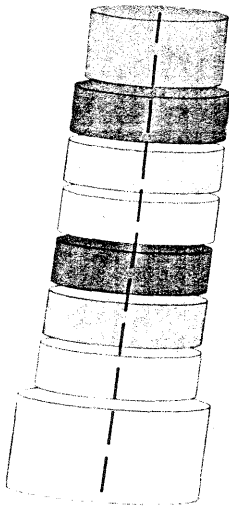
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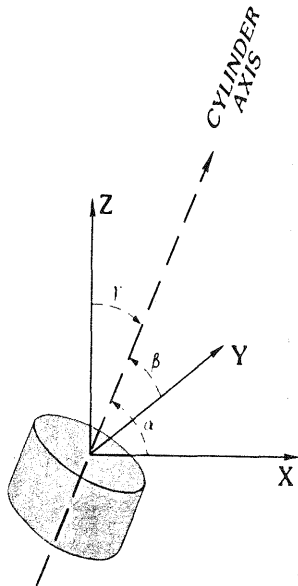
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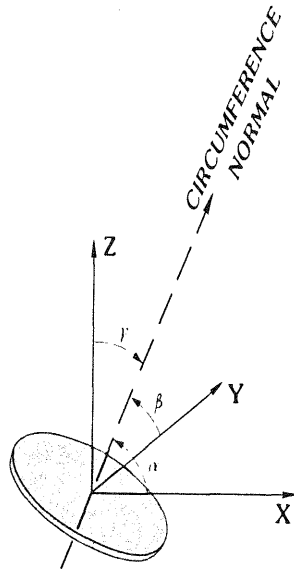
ORDER	1990 END COORDINATES			1990 DIAMETER	STANDARD DEVIATION	1971 MIN. LAV. PUBBLICI DIAMETER
	X	Y	Z			
8th	95573	48402	61927	11185	15.7	
	94943	48391	53434			
7th	94980	48380	53431	12711	11.9	12710
	94444	48360	47242			
6th	94445	48357	47242	12689	14.0	12650
	93949	48299	41602			
5th	93989	48330	41598	12706	12.3	12680
	93444	48321	35849			
4th	93403	48249	35854	12670	10.4	12650
	92830	48213	30075			
3rd	92863	48191	30072	12875	10.4	12860
	92262	48165	24142			
2nd	92277	48136	24140	13059	8.3	13030
	91685	48116	18257			
1st	91677	48094	18258	15522	11.0	15540
	90651	48021	7781			

Table 1 - Cylindrical cores: diameters and axis end coordinates (mm)



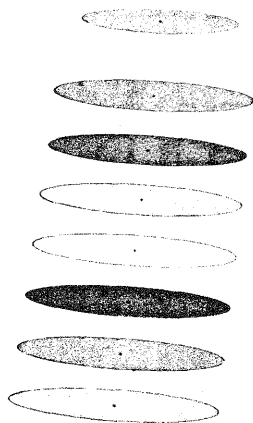
ORDER	1990
8th	α 85°57'04"
	β 89°43'40"
	γ 4°03'30"
7th	α 85°16'15"
	β 89°34'48"
	γ 4°44'52"
6th	α 84°45'11"
	β 89°38'41"
	γ 5°15'32"
5th	α 84°47'32"
	β 89°41'52"
	γ 5°12'59"
4th	α 84°31'26"
	β 89°27'16"
	γ 5°30'12"
3rd	α 84°24'17"
	β 89°33'07"
	γ 5°36'48"
2nd	α 84°26'51"
	β 89°35'46"
	γ 5°34'02"
1st	α 84°29'42"
	β 89°30'35"
	γ 5°31'37"

Table 2 - Cylindrical cores: angle between cylindrical axis and cartesian axes



LEVEL	1990
90	α 86°02'40"
	β 89°37'48"
	γ 3°58'22"
80	α 85°14'14"
	β 89°40'13"
	γ 4°46'27"
70	α 84°54'47"
	β 89°31'43"
	γ 5°06'32"
60	α 84°47'52"
	β 89°35'26"
	γ 5°13'06"
50	α 84°17'04"
	β 89°33'57"
	γ 5°43'56"
40	α 84°27'12"
	β 89°29'15"
	γ 5°34'14"
30	α 84°21'28"
	β 89°35'49"
	γ 5°39'24"
20	α 84°27'49"
	β 89°37'30"
	γ 5°32'56"

Table 4 - Dripstone circumferences: angle between circumference perpendicular and Cartesian axes



LEVEL	1990 COORDINATES OF THE CENTRES			1990 DIAMETER	STANDARD DEVIATION	1971 MIN. LAV. PUBBLICI DIAMETER
	X	Y	Z			
90	X	95468		12755	18.5	
	Y	48451				
	Z	61935				
80	X	94960		16305	12.0	16300
	Y	48396				
	Z	53433				
70	X	94455		16139	16.7	16130
	Y	48358				
	Z	47241				
60	X	93904		16447	16.9	16460
	Y	48311				
	Z	41606				
50	X	93407		16588	12.9	16540
	Y	48262				
	Z	35853				
40	X	92852		16652	16.1	16640
	Y	48216				
	Z	30073				
30	X	92265		16800	9.9	16820
	Y	48160				
	Z	24142				
20	X	91663		17014	14.9	16980
	Y	48115				
	Z	18259				

Table 3 - Dripstone circumferences: coordinates of the centres and the diameters (mm)