ANALYTICAL CALIBRATION OF CLOSE-RANGE CAMERAS

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Commission V

ABSTRACT:

A mathematical model based on the collinearity condition is suggested for calibration of close-range cameras. This approach is realized by developing a computer program and tested by an experiment with Zeiss UMK 10/1318 cameras. The influence of number and distribution of the control points necessary on the accuracy of camera parameters is analyzed. The conclusion recomends the optimal number and distribution of these control points.

KEY WORDS: Analytical, Calibration, Camera, Close-range, Terrestrial.

INTRODUCTION

A necessary prerequisite for the precise solution of an analytical photogrammetric problem is the correspondence of the photographs with the central projection and the high precision of their parameters. The accuracy requirements for the parameters are determined according to the accuracy requirements for the final result and the specific object its depth and the geometry of the photos used.

In the case of close-range photogrammetry the precision requirements are usually higher. Mainly focusable metric and nonmetric cameras are used for which the optical laboratory methods of calibration are not suitable (Livingston, 1980). Different analytical calibration methods are used which can be divided into three principle groups:

(i) camera calibration on a test object (pre-calibration);
(ii) calibration simultaneous with the solution of the photogrammetric problem(simultaneous calibration);
(iii)self-calibration - calibration using the information from three or more convergent photographs without control points.

Depending on the accuracy requirement for the results various degrees of sophistication of the internal orientation can be used - only the position of the internal perspective centre is determined or the radial or tangential lens distortion or film deformation are also taken into account.

The authors, who investigated the influence of the analytical calibration on the accuracy of the results were convinced in its effectiveness although there is a difference in their opinion about the degree of the accuracy improvement (from 40% to 250%), about the number and location of the control points for calibration and about the necessity for sophistication of the internal orientation (Livingston, 1980; Wester-Ebinghaus, 1986; Preuss, 1975).

All this accounts for the interest in the theoretical and experimental investigation of the accuracy of the different calibration methods; in the explanation of the influence of the distribution and number of the control points; in the recommendation of a proper distribution of the points for each calibration method depending on the camera used.

TEST CALIBRATION

It is done by single photos or stereopairs taken of a spatial (three dimensional) test object of control points whicht is suitable for the calibration of metric cameras. The internal orientation elements obtained are used as known quantities in the solution of the photogrammetric problem.

In order to obtain stable camera parameters a specific distribution of the points in the test object is required. The accuracy of the determination of the internal orientation elements is influenced by the accuracy with which the external orientation elements of the photographs are determined. In the simutaneous calibration the internal and external orientation elements are determined together in the process of the analytical solution.

Theoretical analysis

The analysis of the points distribution influence on the calibration accuracy is based on the collinearity condition, simplified for the normal case of terrestrial stereophotogrammetry:

$$x - x_o = f \frac{X - X_s}{Y - Y_s}, \quad z - z_o = f \frac{Z - Z_s}{Y - Y_s}, \quad (1)$$

where $\mathbf{p}(x,z)$ is an image point, $\mathbf{P}(X,Y,Z)$ is the respective point in the object space, $o(x_o, z_o)$ is the principal point of the photo, $S(X_s, Y_s, Z_s)$ is the perspective centre, f is the focal length.

For two pairs of points 1,2 and 3,4 located respectively in the two vertical planes **I** and **III**, parallel to the focal plane for the image x-coordinate is obtained :

$$x_1^{I} - x_2^{I} = f \frac{X_1^{I} - X_2^{I}}{Y^{I} - Y_s},$$

$$x_3^{III} - x_4^{III} = f \frac{X_3^{III} - X_4^{III}}{Y^{III} - Y_s}. \tag{2}$$

The relations for the image z-coordinate are analogous.

Denoting by l^{I} and l^{III} and by L^{I} and L^{III} the respective distances 1-2 and 3-4 in the image and object planes the following relations for the focal length f and for the projection centre Y_s-coordinate are obtained:

$$f = \frac{l^{I}\, l^{III}\, (Y^{III} - Y^{I})}{l^{I}\, L^{III} - l^{III}\, L^{I}},$$

$$Y_s = \frac{l^{I}\, L^{III}\, Y^{I} - l^{III}\, L^{I}\, Y^{III}}{l^{I}\, L^{III} - l^{III}\, L^{I}}. \tag{3}$$

The deduction of these relations does not take into account the influence of the angular elements of orientation. This influence can be easily analytically evaluated in the case when the points form a square or a rectangle, because by inclining the photo in a plane, perpendicular to two of its sides, the distances between the images of the respective points are changed in different directions.

The relations (3) are simplified in the case when the distances between the points in the object plane or in the image plane are equal.

In the first case, when $L^{I} = L^{III} = L$ (see Fig.1) they are:

$$f = \frac{l^{I}\, l^{III} (Y^{III} - Y^{I})}{(l^{I} - l^{III})\, L},$$

$$Y_s = \frac{l^{I}\, Y^{I} - l^{III}\, Y^{III}}{l^{I} - l^{III}}. \tag{4}$$

Using the formulae (4) the errors in f and Y_s are analytically estimated as functions of the distances between the points and between the planes in which they are located. The following conclusions are drawn from this estimate:

(i) the error in the determination of Y_s decreases with increasing the distance between the points in the image plane and with increasing the ratio of the distance between the two planes with control points and the distance between the object and the camera station;

(ii) the error in the focal lenght f decreases with increasing the distance between the two planes with control points and with increasing the distance between the control points in each plane.

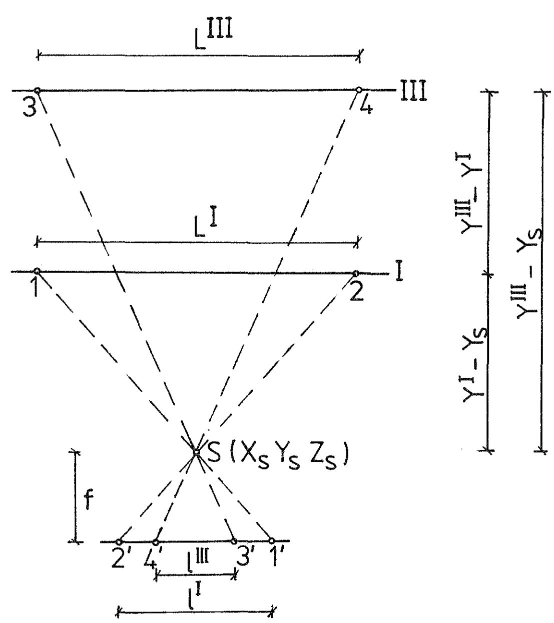

Fig.1

If the points in the two planes of the test object are located at equal distances, one behind the other (for instance at the apices of a parallelopiped), their images will project on different points on the photograph. In this case lens distortion will introduce different deformations of the respective points lying in each of the two planes. If the distortion is not eliminated it will influence negatively the accuracy of the determination of the orientation elements. This requires either a preliminary correction of the image coordinates, using camera certificate data, or data from the camera lens test, or a determination of lens distorsion simultaneously with the elements of the camera internal and external orientation. In the last case the collinearity condition is supplemented with some terms, taking into account the influence of the radial and tangential distortion:

$$x = x_r(k_1 r^2 + k_2 r^4 + k_3 r^6) + p_1(r^2 + 2x_r^2) + 2p_2 x_r z_r \tag{5}$$

$$z = z_r(k_1 r^2 + k_2 r^4 + k_3 r^6) + 2p_1 x_r z_r + p_2(r^2 + 2z_r^2),$$

where

$$r = (x_r^2 - z_r^2)^{1/2}, \quad x_r = x - x_o, \quad z_r = z - z_o .$$

Experimental investigation

Using the advantage of single photographs which allows a better analysis of the influence of different factors on the calibration accuracy, an experimental test calibration is done. The purpose of the investigation is to recommend the distribution and the number of the control points, as well as to check the effectiveness of the sophistication of defining the interior orientation and its influence on the accuracy of the calibration and of the determination of the camera exterior orientation elements.

A test object is created. The points are distributed in such a way that a unique solution for the interrelated elements - the coordinate Ys of the projection centre and the focal length **f** - and for the distortion coefficients is ensured.

The distribution of the points is shown in Fig.2:

17 points in the farthest plane **III**,
9 points in the middle plane **II**,
7 points in the plane nearest to the camera station - plane **I**.

The coordinates of these points are precisely determined by geodetic methods. The photographs are taken from a centrally located camera station using two models of Zeiss Jena focusable universal cameras **UMK 10/1318** in the case of a photoplane, parallel to the planes with control points of the test object. The image coordinates of the photoplates points are measured on the Zeiss Jena Stecometer stereocomparator.

To estimate the influence of the number and distribution of the control points a simultaneous analytical determination of the camera station external orientation elements and of the camera interior orientation elements is done with and without determining the lens distortion. The analytical model used is based on the collinearity condition supplemented with terms accounting for the lens distortion (5). A program for processing the data from single photos is made. The geodetically determined space coordinates of the points are assumed correct and the measured orientation elements are taken as initial approximation.

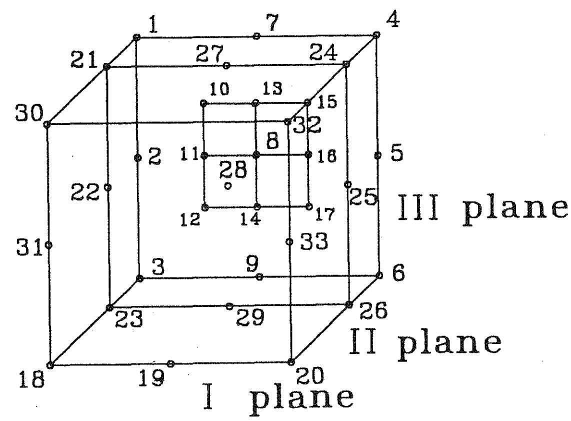

Fig.2

Calculations are made in 7 different cases of distribution and number of points located in two or three of the planes (Fig.2).

Conclusions

The experimental and theoretical results allow the following conclusions:

1. For pre-calibration of the camera by single photos it is recommend to distribut the test object points in two planes which are at a maximum distance one from another but are within the range of the camera depth brightness for the focusing distance used. As an optimum number it is recommended to use 6 points in each of the two planes whose images should lie near the edge of the photo, parallel to the principle line.

2. The determination of the lens distortion requires also radially located points which are evenly distributed in the farthest plane of the test object.

3. The differences observed in the values of the focal length and of the

Y – coordinate of the projection centre indicate that for high-precision problems the pre-calibration of the focusable cameras does not satisfy the accuracy requirements. It is necessary to use the method of simultaneous calibration in which the elements of the internal and external orientation are determined together using photos of the object and of control points suitably distributed in the object space simultaneously with the solution of the photogrammetry problem.

4. The elimination of the lens distortion stabilizes the values of **f** and **Ys** obtained from the different photos and improves the accuracy. Thus before calibration a correction of the image coordinates with a pre-set lens distortion is recommended instead of increasing the number of the control points necessary for the analytical determination of the lens distortion.

SIMULTANEOUS CALIBRATION

The established differences in the analytically determined values of **f** and **Ys**, obtained from different photos motivate the necessity for simultaneous calibration for each photo, taken with a focusable camera. If for the purpose of improving the accuracy of the solution of the photogrammetric problem a double or triple fixing of the object state from each camera station is done, simultaneous calibration and determination of the external orientational elements for each photo is necessary. To avoid averaging of the photogrammetric results from all photos a method for the simultaneous adjustment of the data obtained from the multiple photography of the object from two or more camera stations with a simultaneous calibration and determination of the external orientation elements. The calibration and the determination of the external orientation elements can be done by common or by individual elements for the photos taken from the same station.

The analytical solution is based on the collinearity condition and allows a simultaneous adjustment of the data from multiple photography of the object from two or more camera stations. Each photo participates independently in the adjustment, and the internal and external orientation elements are determined simultaneously with the solution of the photogrammetric problem. The method is analogous to the block phototriangulation by bundle adjustment.

Computer program

On the basis of the method mentioned above a program is created which allows the accomplishment of following:

(i) pre-calibration of cameras by a test object -with or without determination or elimination the lens distortion

(ii) calibration of photographs in the process of solution of the photogrammetric problem:
- with camera calibration elements common to all photos, taken from all stations;
- with orientation elements common to all photos, taken from one station;
- with camera calibration elements different for each photo;

(iii) calibration of all or of only some camera parameters;

(iv) only a correction of the measured values of the camera external orientation elements obtained by a pre-calibration.

Experiments

The method and the program described above were experimented on a test object photographed from two camera station with three photoplates each. The following cases were experimented:

(i) for different number of unknown elements of internal and external orientation:
- **6** elements -the external orientation elements only,
- **7** elements - the same 6 elements plus the focal lenght,
- **9** elements - the same 7 elements plus the prinsiple point coordinates of the photo,
- **10** elements - the same 9 elements plus the first radial distortion coefficent,
- **12** elements - the same 10 elements plus the second and third radial distortion coefficients,

(ii) in the case of simultaneous calibration and determination of the orientation elements, using one, two or three photographs from each station (simultaneous adjustment);

(iii) in the case of different number and distribution of the control points.

The standard errors and **RMS** errors obtained for some of the experimental cases are give in Table 1.

The photogrammetrically obtained spatial coordinates of the points are compared with the geodetically determined ones. The root mean square errors allong each coordinates axis have been computed from their differences and used as criteria of accuracy of the solution. The following accuracy can be considered as realized:

m_x = 1,5 mm , m_y = 5 mm , m_z = 1 mm.

Table 1. RMS errors and Standard errors in milimeters

Data from stereopairs	6 unknowns (f=99.620mm) $m_x/m_y/m_z$ $\sigma_x/\sigma_y/\sigma_z$	6 unknowns (f=99.151mm) $m_x/m_y/m_z$ $\sigma_x/\sigma_y/\sigma_z$	7 unknowns $m_x/m_y/m_z$ $\sigma_x/\sigma_y/\sigma_z$	9 unknowns $m_x/m_y/m_z$ $\sigma_x/\sigma_y/\sigma_z$	10 unknowns $m_x/m_y/m_z$ $\sigma_x/\sigma_y/\sigma_z$	12 unknowns $m_x/m_y/m_z$ $\sigma_x/\sigma_y/\sigma_z$
First				5,7/1,5/1,2 5,8/1,9/1,1		
Second				5,1/1,7/0,6 5,8/2,0/1,2	5,1/2,0/0,6 6,4/2,1/1,2	5,3/2,0/0,5 6,5/2,1/1,2
Third				6,8/1,7/1,2 7,0/2,3/1,3		
All three simultan.	10,3/12,2/4,0 18,6/ 6,1/3,6	10,4/3,5/0,9 4,5/1,5/0,9	4,7/1,3/0,8 3,6/1,2/0,7	4,8/1,3/0,8 3,6/1,2/0,7		

Conclusion

To obtain the maximum accuracy the results of the experimental investigations allow to recommend the use of the following:

1. At last 2 photoplates, taken from each station with suitably distributed control points within the object.

2. Simultaneous calibration of the focal lenght and correction of the measured elements of external orientation (**7** unknowns for each photo). If the distribution of more control points is possible - **9** unknowns (determination of the principal point also), correction of the lens distortion on the basis of preliminary data for the coefficients, or if there are no data, determination of $\mathbf{k_1}$ only by ensuring a sufficient number of control points.

3. On the basis of **4** control points in both front and rear object plane in case when the distorsion is not determined analytically.

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