A STUDY OF ABSOLUTE ORIENTATION METHODS

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1.1 - There are many ways of carrying out absolute orientation in plan, but it is possible to classify them in three main groups:

I - The computational or numerical methods,
II - The semi-numerical methods,
III - The empirical methods.

After relative orientation and model levelling the following steps must be carried out:
a) scaling,
b) finding the best fit between the ground control points (g.c.p.) projected on the sheet, and the respective model points.

Scaling consists of getting a factor m that gives a new base B_n by:

$$B_n = B_o \times m$$

where B_o is the previous base.

Finding the best fit consists of giving a relative shift between model and map, whose components are represented by ΔE and ΔN, as well as a relative rotation defined by the angle θ.

1.2 - If we get the absolute orientation elements m, ΔE, ΔN, θ in a mathematical way, i.e., by computation, we are applying the method I. This is described in detail in chapter 2.

The semi-numerical methods consist of getting the elements m, ΔE, ΔN, and θ, using separate operations and not using all the g.c. ps. available in each operation. The following procedure has been used:
a) the scale factor m was computed by (*):

$$m = \sqrt{\frac{(E_R - E_B)^2 + (N_A - N_B)^2 + (H_A - H_B)^2}{(X_A - X_B)^2 + (Y_A - Y_B)^2 + (Z_A - Z_B)^2}}$$

where X, Y, Z, are machine coordinates of g.c.ps. A, B,...
b) after introduction of new base B_n in the machine, the best coincidence of A and B, and therefore line AB, between model and map is obtained. Doing this, the shift ΔE and ΔN becomes nearly null, as well as rotation θ. If discrepancies in two more points C and D are not acceptable by rotating the sheet through M, the central point of A and B, in such a way that discrepancies in A and B become acceptable. If not we repeat the procedure.

In the empirical methods after adjusting model and sheet at point A and going to point B in the model, two cases can occur:
a) coincidence at B,
b) non coincidence at B.

In case a), scaling is not necessary, and we can check points C,D,.. and get the best fit by trial and error.
In case b), if model distance is shorter than map distance, we increase the base and vice-versa.
The changing of base can be by an entirely trial and error way,based on the experience of operator or getting m by:

$$m = \frac{\text{map distance}}{\text{model distance}}$$

where these distances are measured with a ruler.

When points A and B fall within the accepted accuracy, say 0.1 mm., we visit the other points and by small shifts and rotations, we bring all the points within the accepted accuracy.

(*) In fact, m was a mean of m_{AB} and m_{CD}, where A, B, C, and D, were the four corner points.
This method was suggested by Mr. A.Mroz. It is better than the empirical one, as far as accuracy and speed is concerned, but compared with the simplified computational method, it takes a little more time. However, when the last method is not applicable the semi-computational one gives the best results in accuracy and speed.

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2.1 - This chapter deals with a computational procedure to get absolute orientation in plan. Suppose we have n g.c.p. in the model, whose geodetic coordinates are E_g, N_g. E, N are the coordinates of the same points refered to the centroid and E', N' are the coordinates of the points projected by the machine on the centroid system. The discrepancies (E'- E) and (N' - N) can be measured directly by the telescope of the coordinatograph. We say that absolute orientation is achieved when all discrepancies (E'-E) or (N' - N) are smaller than a given value, as 0.1 mm., for instance, whose value depends on the scale.

The coincidence between model and map points depends on four elements of absolute orientation:
a) The scale factor m.
b) The components ΔE and ΔN of the shift of the centroidal origin.
c) The rotation θ of the sheet through the centroidal origin.

The mathematical expression that relates these elements is:

$$\begin{pmatrix} E' \\ N' \end{pmatrix} = m . R . \begin{pmatrix} E \\ N \end{pmatrix} + \begin{pmatrix} \Delta E \\ \Delta N \end{pmatrix} \qquad 1$$

where R represents a rotation:

$$R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Equation 1 becomes:

$$\begin{pmatrix} E' \\ N' \end{pmatrix} = \begin{pmatrix} m.\cos\theta & -m.\sin\theta \\ m.\sin\theta & m.\cos\theta \end{pmatrix} . \begin{pmatrix} E \\ N \end{pmatrix} + \begin{pmatrix} \Delta E \\ \Delta N \end{pmatrix}$$

or

$$\begin{pmatrix} E' \\ N' \end{pmatrix} = \begin{pmatrix} b & -a \\ a & b \end{pmatrix} . \begin{pmatrix} E \\ N \end{pmatrix} + \begin{pmatrix} \Delta E \\ \Delta N \end{pmatrix} \qquad \underline{2}$$

where $\begin{cases} b = m \cos\theta \\ a = m \sin\theta \end{cases}$

and

$$m = \sqrt{a^2 + b^2} \quad \text{and} \quad \tan\theta = \frac{a}{b}$$

From 2 : $\begin{cases} - N.a + E.b + \Delta E = E' \\ + E.a + N.b + \Delta N = N' \end{cases}$

Subtracting respectively by E and N we get:

$$\underline{3} \quad \begin{cases} -N.a + E.(b - 1) + \Delta E = R_E \\ E.a + N.(b - 1) + \Delta N = R_N \end{cases}$$

where $R_E = (E' - E)$ and $R_N = (N' - N)$ are the measured discrepancies. If we have 6 g.c.p. we get 12 equations like 3 to obtain 4 unknowns: a, b, ΔE and ΔN. This leads us to a least square solution of a system of observation equations. The table of observation equations is:

a	(b-1)	ΔE	ΔN	R
-N	E	1	0	R_E
E	N	0	1	R_N

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and the normal equations are:

$$\begin{array}{cccc|c}
[N \cdot N] + [E \cdot E] & 0 & [-N] & [E] & -[N \cdot R_E] + [E \cdot R_N] \\
 & [E \cdot E] + [N \cdot N] & [E] & [N] & [E \cdot R_E] + [N \cdot R_N] \\
 & & n & 0 & [R_E] \\
 & & & n & [R_N]
\end{array} \qquad \underline{5}$$

As E and N are coordinates refered to the centroid:

$$[E] = 0$$

$$[N] = 0$$

and the matrix of system 5 is a diagonal one, then:

$$a = \frac{[E \cdot R_N] - [N \cdot R_E]}{[E \cdot E] + [N \cdot N]}$$

$$b = \frac{[E \cdot R_E] + [N \cdot R_N]}{[E \cdot E] + [N \cdot N]} + 1$$

$$\Delta E = \frac{[R_E]}{n} \qquad \underline{6}$$

$$\Delta N = \frac{[R_N]}{n}$$

From equations 6 we get, finally:

a) scale factor $m = \sqrt{a^2 + b^2}$

b) shift ΔE and ΔN

c) rotation θ given by: $\tan \theta = \frac{a}{b}$

2 . 2 - To develop a simple computational procedure we applied form 1 to three models in a complete least square solution. For each model 3 sets of discrepancies were read.
After that I made some assumptions in order to save time with computations, that is:

a) to consider a smaller number of points,
b) to consider that the g.c.ps. are in standard positions, as is the case with the points given by aerial triangulation.

2.3 - I have developed the following practical method of applying the corrections m, ΔE, ΔN, and θ , whose main steps are:

a) introduction in machine of new base B_n , by

$$B_n = m \times B_o$$

b) with the coordinatograph telescope centred on the centroidal origin C and an auxiliar telescope, using for instance the special arm device, on chosen point P, I give to the sheet one shift represented by ΔE and ΔN, measured on both telescopes.
c) now fixing C, I give to P the tangential displacement d, measured with the respective telescope and found by:

$$d = \tan \theta \cdot \overline{CP} = \frac{a}{b} \cdot \overline{CP}$$

where $\frac{a}{b}$ is known and the distance $\overline{CP}$ is measured with a ruler.

After doing this I visit all points and record the discrepancies, if any. If all are smaller than 0.1 mm., the absolute orientation is considered achieved.

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3.1 - Having developed the method under chapter 2, this was applied to the models whose diapositives were loaned by the Ordnance Survey, and numbered: 089, 090, 091 and 092.

These diapositives at an approximate scale of 1/6.000 belong to a strip whose control points were marked as points on the glass print, thus  and whose coordinates were determined by aerial triangulation.

The characteristics of the plotting are:

a) scale of diapositives:	1/ 6.000
b) model scale:	1/ 3.750
c) principal distance:	210 mm.
d) mean altitude Z:	330 mm.
e) gear ration:	1/3
f) map scale:	1/ 1250
g) approximate base:	90 mm.

The position of the 6 control points in the model, similar in all models, is shown in fig. 1, and it can be considered for future simplification that this position is the standard one.

The numbering of control points is shown in figure 2.

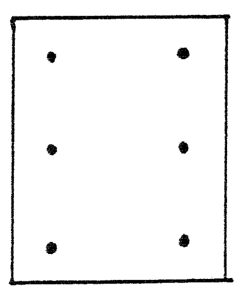
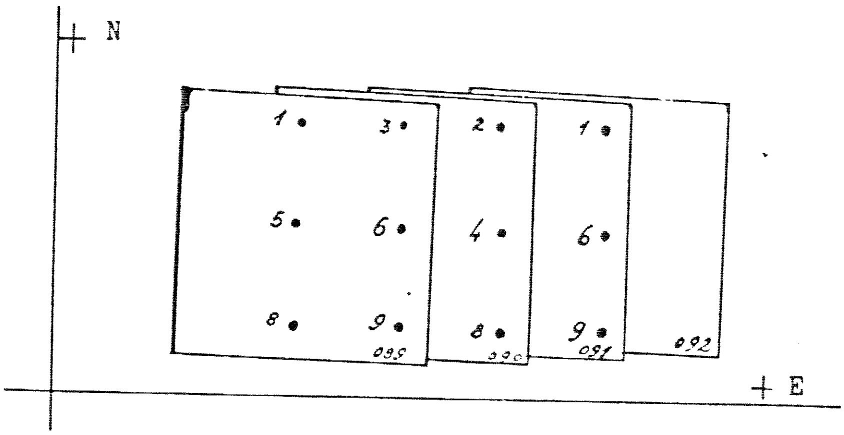


Fig. 1

Figure 2



The geodetic coordinates of these points were refered to the origin :

E = 462 500.00 N = 521 600.00

to enable work with smaller values. These computations are shown in table :

Points	Geodetic coordinates		Origin E_o, N_o		Map coordinates		Heights
	E	N	E	N	E	N	H
1	462 578.39	522 541.52	78.39	941.52	62.71	753.22	20.02
5	" 586.91	" 135.81	86.91	535.81	69.53	428.65	22.15
8	" 585.82	521 705.43	85.82	105.43	68.66	84.34	28.52
3	" 911.04	522 553.97	411.04	953.97	328.83	763.18	16.10
6	" 921.26	" 141.12	421.26	541.12	337.01	432.90	21.14
9	" 987.32	521 736.09	487.32	136.09	389.86	108.87	32.71
2	463 223.06	522 556.40	723.06	956.40	578.45	765.12	17.73
4	" 235.01	" 150.61	735.01	550.61	588.01	440.49	21.31
8	" 274.61	521 749.22	774.61	149.22	619.69	119.38	33.86
1	" 630.01	522 561.45	1130.01	961.45	904.01	769.16	17.78
6	" 636.98	" 121.22	1136.98	521.22	909.58	416.98	24.03
9	" 687.67	521 748.90	1187.67	148.90	950.14	119.12	34.05

Where the 6th. and 7th. columns are transformations into map coordi_ nates considering the map scale as 1/ 1250.

In form 1, next page, these coordinates were transformed into centroidal coordinates, relative to each model. The last ones were plotted on the map, as well as the centroidal origin. However, we must assume, if the computational method is to be applied in production work, that the map goes to the plotting table with the centroidal origin plotted on it. To avoid the computations of the previous table and some of form 1, to the Photogrammetric Dept. must be given, with the coordinates of g.c.ps. or air points, the coordinates of the centroidal origin for each model.

For a Photogrammetric Organization this is an easy thing to do, and if this is done, the computational method as we will see, becomes advisable and not so time consuming as one could expect.

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4.1 - This chapter refers to observations and computations. The observations, listed in the 6th and 7th columns of the respective form and called discrepancies R_E and R_N, consist of a set of 12 readings. Each of these readings represents the component, parallel to the E and N axes, of the displacement between the map and respective model point, measured with the table telescope, after relative orientation, levelling, and rough scaling on two points. Using these observations, a complete least square solution was worked out.

The mean time to get B_n, ΔE, ΔN, and d, without simplifications is about 50 minutes.

The forms were computed by electronic computer. The introduction of the absolute orientation elements into the machine and sheet can be done in about 5 minutes, in the way described in chapter 2.

After this the discrepancies on the six points were checked, and the procedure repeated if R_E and R_N proved to be greater than 0.1 mm.

4.2 - In order to simplify the computations, save time and thus produce the most practical computational method, four corner points instead of six were used and it was assumed that these points were symmetrically distributed relative to the centroidal origin.

In that case the computations are much simplified and show the remarkable fact that they achieve the same results. The time taken for computing one form 2 is less than 15 minutes, and the discrepancies obtained after applying the absolute orientation elements fall within the accepted accuracy.

FORM 1

(Example)

MODEL C89/C90

POINTS	MAP E	MAP N	CENTROID E	CENTROID N	DISCREPANCIES R_E	DISCREPANCIES R_N	$E \cdot R_E$	$E \cdot R_N$	$N \cdot R_E$	$N \cdot R_N$
1	62.71	753.22	-146.72	+324.69						
5	69.53	428.65	-139.90	+ 0.12						
8	68.66	84.34	-140.77	-344.19						
3	328.83	763.18	+119.40	+ 4.37						
6	337.01	432.90	+127.58	-319.66						
9	389.86	108.87	+180.43							
			$E \cdot E$	$N \cdot N$						
SUM	1256.16	2571.16								

$$\Delta_E = \frac{[R_E]}{n} = \_\_\_\_ =$$

$$\Delta_N = \frac{[R_N]}{n} = \_\_\_\_ =$$

$$a = \frac{[E \cdot R_N] - [N \cdot R_E]}{[E \cdot E] + [N \cdot N]} = \_\_\_\_\_\_\_\_ =$$

$$b = \frac{[E \cdot R_E] + [N \cdot R_N]}{[E \cdot E] + [N \cdot N]} + 1 = \_\_\_\_\_\_\_\_ + 1 =$$

$m^2 = a^2 + b^2 =$

$m =$

$\frac{a}{b} =$

$\overline{CP} =$

BASE : $B_n = B_. \times m =$

SHIFT : ΔE $\Delta N =$

ROTATION : $d = \frac{a}{b} \overline{CP} =$

1013

FORM 2

MODEL 089/090

POINTS	MAP		CENTROID		DISCREPANCIES		$E \cdot R_E$	$E \cdot R_N$	$N \cdot R_E$	$N \cdot R_N$
	E	N	E	N	R_E	R_N				
1										
—										
8										
3										
—										
9										
—			E·E	N·N						
SUM										

$$\Delta E = \frac{[R_E]}{n} = \_\_\_\_ =$$

$$\Delta N = \frac{[R_N]}{n} = \_\_\_\_ =$$

$$a = \frac{[E \cdot R_N] - [N \cdot R_E]}{[E \cdot E] + [N \cdot N]} = \_\_\_\_\_\_\_\_ =$$

$$b = \frac{[E \cdot R_E] + [N \cdot R_N]}{[E \cdot E] + [N \cdot N]} + 1 = \_\_\_\_\_\_\_\_ + 1 =$$

$$m^2 = a^2 + b^2 =$$

$$m =$$

$$\frac{a}{b} =$$

$$\overline{CP} =$$

BASE : $B_n = B_. \times m =$

SHIFT : ΔE $\Delta N =$

ROTATION : $d = \frac{a}{b}$ $\overline{CF} =$

5.1 - The information obtained from the Ordnance Survey shows that to get the discrepancies smaller than 0.1 mm. at all points, the time consumed by absolute orientation after levelling is about 40 minutes.

With the same kind of diapositives and air points, using the simplified computational method, the same result is achieved in about 30 minutes.

Yet, in favour of the computational procedure it must be emphasized the fact that the last fit between points is more balanced, i.e., errors are more equally distributed over the model, as well as less subjective.

On the other hand, the empirical method has a tendency to produce a good fit on some points and put all the redundancies on the remaining points.

Thus, taking account of all those facts, the simplified computational procedure, that may be applied in most cases, seems to be the best.

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I N D E X