ERROR TREATMENT IN PHOTOGRAMMETRIC DIGITAL TECHNIQUES

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INTRODUCTION

A photogrammetric observation may be contaminated by three types of errors - random errors, systematic errors and blunders. This paper deals with the basic principles of the numerical treatment of blunders and systematic errors and their interrelation. Random errors being in practice considered relatively harmless are, in addition to that, supported by extensive theory. In contrast to that, systematic errors and blunders are neither considered harmless nor are they treated theoretically to a sufficient extent. Photogrammetric practitioners have, in the past, developed certain skill in the treatment of those errors based mainly on experience, insight and common sense. None of these prerequisites for successful error treatment is easy to acquire. Due to the successful application of computer, photogrammetry has relatively quickly transferred from basically analogue to digital methods. The old manual treatment of errors has become too slow, too tedious and too unreliable. The speed and power of computers need to be exploited for "slave" work and only those decisions where insight, intuition or additional information are absolutely necessary, should be left to human beings. Our intention is to investigate the practical applicability of methods for the treatment of blunders and systematic errors in the computer environment. However, in order to limit the size of the paper, we shall not discuss any particular algorithms, their implementation or computational efficiency. It must be mentioned, however, that these problems can easily be handled with the present generation of computers.

ERROR MODEL

The philosophy of blunder treatment depends strongly on the mechanism by which the blunders are believed to be generated. We shall, for this purpose, adopt the model proposed by Kubik [9], however, in a slightly modified form. Fig. 1 illustrates the main properties of such error distribution. The random error ε is assumed to be approximately normally distributed within the interval $(-a, +a)$. The blunder may be defined as an error, which does not belong within the interval $(-a, +a)$. It is considered to be a contaminant and does not belong to the same distribution as ε. The assumption that the distribution of blunders is unknown seems to be the only realistic alternative.

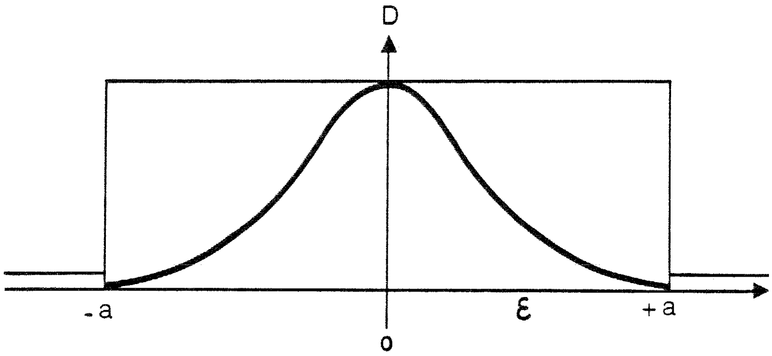

Figure 1

FUNCTIONS OF OBSERVATIONAL ERRORS

The treatment of blunders specifically in adjustment procedures arouses the greatest interest in photogrammetric practice. The basic mathematics in such situations is well known and extensively treated elsewhere (see [10], [15]). The relationship between the vector of residuals $\underline{v}$ computed after adjustment and the "true" observational errors is given by the formula

$$\underline{v} = Q\,\underline{\varepsilon} \qquad Q = \overline{Q}\,W \tag{1}$$

which is valid generally for any type of least squares adjustment. The matrix $\overline{Q}$ represents the cofactor matrix of residuals and W is the weight coefficient matrix of observations. The characteristics of matrix Q are known - it is a singular idempotent matrix (Q = QQ) whose rank is equal to the number of redundant observations and whose elements are all smaller or equal to 1. The functions of observational errors used in blunder detection procedures are:

1. Residual: v_i
   The relations between residual and observational errors are given in formula (1).
2. Swept residuals: $z_i = v_i/q_{ii}$ (2)
   Such a residual represents the "natural" estimation of observational error ε_i. Assuming that the observation l_i did not take part in the adjustment, but the results of such adjustment allow the computation of a value, which replaces l_i, say $\hat{l}_i$, then the following may be written:
   $r_i = l_i - \hat{l}_i$
3. Variance: $s^2 = (\underline{v}^T W \underline{v})/r$ (3)
4. Standardized residual: $w^2{}_i = v^2{}_i/q_{ii}$ (4)
5. Standardized average residual length: $w^2{}_\beta = \underline{v}^T{}_\beta Q_\beta \underline{v}_\beta/b$
   where w_β is a (b x 1) vector comprising a selected combination of individual residuals, and Q_β is a (b x b) submatrix of Q obtained by elimination of all rows and columns except those corresponding to the selected residuals. Obviously the standardized residual is a specific form of the standardized average residual length when b = 1.
6. Reduced variance: $s^2{}_\beta = (r\,s^2 - bw^2{}_\beta)/(r - b)$, (6)
   which represents the variance, in the case when a group of observations β has been removed from the adjustment. Obviously, s_β

may be easily computed if s and w_β are known. If the group β comprises only one observation we may write:

$$s^2{}_i = (rs^2 - w^2{}_i)/(r - 1) \qquad (6a)$$

THE INFLUENCE OF LINEARIZATION

The above formulae may be easily verified for linear least squares adjustment. However, in cases where observational equations have to be linearized formula (6) is no longer strictly valid. In order to obtain correct values the modifications for the influence of linearizations ε have to be computed (see [16]) and added to (6)

$$s^2{}_\beta = [sr^2 - (b\, w^2{}_\beta + c)]/(r - b) \qquad (6b)$$

TEST STATISTICS

As all are functions of observational errors, the left hand sides of expressions (1) - (6) may be used as test statistics. the tests are usually executed by comparing the statistics with a theoretical value. In order to enable this, the expressions (1) - (6) are scaled by the variance:

$$\hat{v}_i = |v_i|/s, \ \hat{w}_i = w_i/s, \ \hat{w}_\beta = w_\beta/s, \text{ etc.} \qquad (7)$$

The tests are based on the assumption that the original observations are normally distributed. The statistics $\hat{w}_\beta$ and $\hat{s}_\beta$ follow the F-distribution, which is available from published tables. After selecting a level of significance α it is easy to find the critical region t_p and check whether the computed value is smaller than t_p.
The index p indicates so called degrees of freedom. The critical region t_p will be termed tolerance in the following text. The statistics $\hat{v}_i$ and $\hat{z}_i$ do not follow the F-distribution, and consequently they are not so suitable for testing. In spite of this unsuitability the statistic $\hat{v}_i$ was in the past extensively used for that purpose. The test statistic $\hat{w}_i$ (one dimensional variant of $\hat{w}_\beta$) was introduced in survey techniques by Baarda [1] and Pope [11], although in regression techniques it had been known somewhat earlier [14]. The procedure designed by Baarda is based on the forms adapted in statistical hypothesis testing. If $w_i \leqslant t_1$, it is concluded that a blunder is present. The identification of the blunder is, however, not possible without further measures. The formalism of statistical hypothesis testing allows also the control of the probability α_0 of so called type I error (the error of rejecting the true hypothesis) and the probability β_0 of type II error (the error of accepting the false hypothesis). However, the assessment of α_0 and β_0 is based on the assumption that the errors are normally distributed - what is true only in approximation. The approximation tends to be poorest in the extreme tails of the distribution, which are the most significant in blunder detection.

MASKING EFFECTS

The possible difficulties in detection and especially location of blunders may be easily demonstrated with help of expression (1). It shows in the first place that the vector $\underline{\varepsilon}$ is irrevocably lost because Q is a singular matrix. The vector $\underline{v}$ contains the whole information about $\underline{\varepsilon}$, but each element of $\underline{v}$ is a function of, in principle, all elements of $\underline{\varepsilon}$. In all test statistics we use residuals v_i. The combined effect of all members of $\underline{\varepsilon}$ on each v_i is unpredictable, especially in the case when contaminants are present. The effect of this is that in $\underline{v}$ the blunders may be masked. Analysis of the masking effects is the first prerequisite for successful treatment of blunders.

Masking effect 1: Contaminated variance

All test statistics treated up till now have been scaled by the variance s^2, which is estimated with the help of a given sample. There is no doubt that s^2, as defined in (3), is an unbiased and sufficient estimator of variance σ^2. However, if $\underline{\varepsilon}$ is contaminated by blunders s^2 tends to be too large, which than affects all ratios involving s^2.
In order to demonstrate this effect we may introduce in (7) the value s^2 as expressed in (6a).

$$\hat{w}_i^2 = r\, w_i^2/[(n-1)s_i^2 + w_i^2] \tag{8}$$

Note that w_i is present in both denominator and numerator. As $\varepsilon_i \rightarrow \infty$ then $w_i \rightarrow \infty$ while s_i does not change. So $\hat{w}_i \rightarrow 1$ and the probability of rejecting w_i approaches zero. The rate with which $\hat{w}_i$ approaches 1 depends on the redundancy. The smaller the redundancy the more severe are the effects of masking. Larger numbers of contaminants (blunders) shall also increase the masking effect.

Masking effect 2: Blank range

The second masking effect becomes apparent when inspecting the expression (1). If any q_{ii} is very small than, because $q_{ii} = \sum q_{ij}^2$ all elements of i^{th} column of Q are also very small. Therefore the influence of ε_i on vector $\underline{v}$ becomes very small and may become negligible. With the help of (1) we may express $\hat{v}_i$, $\hat{w}_i$ and $\hat{z}_i$ as functions of ε_i and neglect ε_j, $j \neq i$:

$$\hat{v}_i = q_{ii}\varepsilon_i/s, \quad \hat{w}_i = \varepsilon_i\sqrt{g_{ii}}/s, \quad \hat{z}_i = \varepsilon_i/s \tag{9}$$

where s is considered constant.
Obviously in $\hat{w}_i$ and especially in $\hat{v}_i$ the influence of error ε_i is substantially reduced and only in $\hat{z}_i$ is it revealed in its full amount. As $q_{ii} \rightarrow 0$ the influence also tends to zero.
But even if all q_{ii} values are acceptably large it may happen what some groups of large observational errors vanish in the results of adjustment. The analysis such as above may be also completed with help of $\hat{w}_\beta$ and (1). The submatrix Q_β plays, in this case, a central role - especially its minimum eigenvalue λ_{min}.

Generalizing we may say that if λ_{min} of a submatrix Q_β approaches zero the influence of the corresponding group of observations on residuals tends to zero.

Masking effect 3: Rank disorder

For the location of blunders generated according to the model given in fig. 1, the rank ordering of observational errors according to the absolute value ($|\varepsilon_1|$ $|\varepsilon_2|\dots$ $|\varepsilon_n|$) is of special interest. Because the observational errors are inaccessible we are interested in how far the test statistics $\hat{v}_i$, $\hat{w}_i$ or $\hat{z}_i$ reflect the rank order of ε_i. Even a simple inspection of expression (1) makes it obvious that only when all diagonal elements of Q are equal to 1, the rank order of original errors is reproduced in all test statistics. In all other cases it may be distorted. Simulation with adjustment of 70 normally distributed observations in 10,000 repetitions has shown that the probability that $|\hat{w}_i|$ and $|\varepsilon_i|$ simultaneously reach maximum values does not exceed 30%. For $|\hat{z}_i|$ and $|\varepsilon_i|$ similar results have been obtained. Even the statistics differ among themselves. Not infrequently does the largest $|\hat{z}_i|$ corrrespond to the smallest $|\hat{v}_i|$. However, as $|\varepsilon_i| \to \infty$ the probability that $|\hat{w}_i|$ shall reach a maximum also increases. Again, as the number of blunders increases the above mentioned probability decreases.

TREATMENT OF MASKING EFFECT 1

The masking effect 1 is actually the only one which can be practically and successfully treated at little expense. Instead of the estaimted variance s^2 we may use the external (a priori) variance σ^2, which is, naturally, free of influence of any blunder. Such a variance is, moreover, known for the majority of photogrammetric procedures. Quality control is, anyhow, the main purpose of tests in the production environment. It is worth noting that only a few authors recommend the use of s^2 instead of σ^2 ([2], [11]).
There are also some other possibilities for avoiding contaminated variance, but they are all less effective and more time consuming. For example, one can compute s_β for all possible combinations, with b larger than the largest number of blunders expected, and use the minimum s_β as a scaling factor. The other possibility is the so called backward replacement as discussed later on in the section entitled "Treatment of masking effect 3". In the following we shall always assume that external variance σ^2 is available. This also permits the test on s^2. So we may redefine the test statistics:

$$\bar{v}_i = v_i/\sigma,\ \bar{s} = s/\sigma,\ \bar{w}_i = w_i/\sigma$$

$$\bar{z}_i = z_i/\sigma,\ \bar{w}_\beta = w_\beta/\sigma,\ \bar{s}_\beta = s_\beta/\sigma$$

The $\bar{w}_i$, $\bar{w}_b$, $\bar{s}_\beta$ and $\bar{s}$ follow the following distributions $(\chi_1^2)^{\frac{1}{2}}$, $(\chi_b^2/b)^{\frac{1}{2}}$, $[(\chi_{r-b}^2)/(r-b)]^{\frac{1}{2}}$ and $(\chi_r^2/r)^{\frac{1}{2}}$.

TREATMENT OF MASKING EFFECT 2

As discussed earlier the masking effect 2 depends on the structure of matrix Q. The larger the diagonal elements, the smaller the masking effect. The structure of matrix Q depends on the design and arrangement of observations. This stresses the importance of adequate planning of observations for the treatment of the masking effect 2. Numerous papers deal with this problem (e.g. [4], [5], [10]). However, during the planning of observations we do not have complete freedom and when the adjustment starts we have to deal with the existing situation. The size of the masking effect 2 does not depend only on the structure of the matrix Q, but also on the tolerance accepted for the test in question. For example, for the test $\overline{w}_i < t_1$ we could make the t value arbitrarily small and reduce the masking effect to that arbitrarily small amount. Why we actually do not act in such a way is obvious - as $t \to 0$ the probability that the observations shall be discarded tends to reach certainty, no matter whether the blunders are present or not. The selection of tolerance t becomes, in such a way, a problem of optimization: How does one reduce the masking effect and at the same time minimize the probability of the rejection of good observations? Tolerances may be found in χ^2 tables after the significance level α_0 has been selected. The conventional interpretation of the significance level is as follows: in 100 $(1-\alpha_0)$% of cases the test shall discard an observation although it is not wrong. However, because the error distribution in reality is not exactly normal, statements such as the above should be read with caution. Simulation of random errors according to the model given in fig. 1 shows that the above estimation is conservative if the tails of the distribution are not too heavy. The other factor in optimization of t is traditionally given in the form of β_0 - the probability that blunders stay undetected, so β_0 actually controls the masking effect 2. The selection of β_0 together with α_0 allows for the determination of so called the boundary value $\delta_0/\sqrt{q_{ii}}$ with the following interpretation: the boundary value represents the magnitude of error which can just be detected with the probability β_0. Such a glib formulation is probably of little use to a practitioner, but it is difficult to find anything better. Anyhow, optimization procedures become clear: by increasing t the probability of rejecting good observation decreases, but the masking effect increases and vice versa. For the one dimensional test, t should not be smaller than a as defined in fig. 1. In place of the boundary value the so called blank range (see [15]) may be used where:

$$m_i = (1-\sqrt{1-q_{ii}})\ t_1/\sqrt{q_{ii}}$$

or generalized for the $\overline{w}_\beta$ test where:

$$m_\beta = (1 + \sqrt{1-\lambda_{max}})\ t_p\ /\ \sqrt{\lambda_{min}}$$

The blank range m_i may be interpreted as follows: m_i represents the magnitude of a blunder, starting from which it shall almost certainly be detected provided there are no more blunders. In order to test the blank range, an adjustment was simulated with 70 random observations according to the model in fig. 1.

A blunder was also simulated having the magnitude of the blank range (for α_0 = 0.001 and q_{ii} = 0.75, m_i = 4.87/0.86). In over 99% of all cases the blunder was detected. As a comparison the boundary value of 4.87/0.86 corresponds approximately to β_0 = 94%, obviously too conservative a value. Anyhow, as already mentioned, during the blunder location procedures we cannot influence the masking effect 2 but only control it by issuing a warning when the magnitude of the blank range becomes too large.

TREATMENT OF MASKING EFFECT 3

The rank disorder is the most disturbing of all masking effects. While the first masking effect can easily be eliminated and the second controlled the third effect is completely unpredictable. Realization of this fact has been why sometimes no attempt is made to locate blunders, but only to detect them. The responsibility for blunder location is than shifted to the operator, who is then forced to use alternative blunder location methods: measurements, scrutiny of field books, insight, hunch, etc.

Classical data snooping developed by Baarda [1] is a typical example of such a method. Here we have to deal only with the first two masking effects and the formalism of statistical hypothesis testing is sufficient. However, although procedures such as this in many situations fulfil requirements, it is obviously desirable to use the power of the computer to locate blunders and relieve the operator from tedious searching. In such a case the above mentioned testing formalism is no longer sufficient. It has to be complemented by some additional rules.

The most obvious idea is to take the largest $\overline{v}_i$, $\overline{w}_i$ or $\overline{z}_i$ to indicate the most suspect observation. Kraus [8] was probably the first to suggest maximum $\overline{w}_i$ as a blunder locator. Later a stepwise procedure based on this idea became quite popular under the name forward elimination. After the observation with the largest $\overline{w}_i$ is eliminated the adjustment is repeated, and the new maximum $\overline{w}_i$ found, until no $\overline{w}_i$ exceeds the tolerance.

The masking effect 3 is the obvious reason why such a procedure is not completely satisfactory. The good observations may easily be eliminated, because w_i does not follow the rank order of ε_i and this is, of course, highly undesirable. Even the hope that, among good observations, all blunders may be eliminated is not justified. In the case of little redundancy it may easily happen that after the elimination of some good observations the blank range for actual blunders assumes such a large value that it becomes undetectable. As a result of the economization on the number of observations the photogrammetric adjustments are usually characterized by small redundancies. So the procedure may end up with the most embarassing result: the elimination of good observations and the acceptance of blunders. The tables 2 and 3 in Appendix illustrate such a situation.

In order to justify the above procedure one could estimate the probability of such an unsuccessful outcome, which as we intuitively feel, would be extremely low. However, on the other hand, it is probably more appropriate to consider the "weakest link" when judging such procedures.

An attempt to avoid the troubles caused by stepwise procedures is the so called Danish method (see [7]), which uses the complete set of observations throughout the whole location procedure.

Actually several adjustment runs are completed and in each run a new set of weights is introduced. The weights are usually proportional to residuals from the preceding run. The procedure is stopped when no further change is significant. Without a thorough analysis it is difficult to clarify whether such a procedure actually eliminates the effect of rank disorder. However, intuitively it seems difficult to believe that any weighting system based on residuals may do that. Obviously, some good observations may easily get weights larger than those assigned to blunders and vice-versa. Practical tests (see [6], [12]) reveal, that failures, indeed happen.

Benciolini et al. [2] propose the method, which may be termed backward replacement, and use s^2 as the scaling factor. At first the forward elimination is completed with fixed number of eliminations. The number of eliminated observations should be larger than what is believed to be the maximum possible number of blunders. It is hoped that in such a way a residual set of observations is obtained which is blunderfree. This fact is exploited to compute the blunderfree variance, s_0^2. Starting from this set the eliminated observations are reinserted in the sequence opposite to their elimination. The procedure is stopped as soon as the value w_i/s_0 exceeds the tolerance. Obviously the procedure does not change anything in the rank order of w_i values. Consequently, it delivers the same results as forward elimination provided σ is used as scaling factor. Therefore the backward replacement may be recommended only in cases when the external σ is not known.

An attempt to eliminate the masking effect 3 is the procedure proposed in [15], which may be termed selective elimination. Only those observations or groups of them which simultaneously satisfy $\overline{w}_\beta \geq t_p$ and $\overline{s}_\beta \leq t_{r-p}$ may be identified as blunders. The rank order does not play any role in such a definition. One should start by checking each individual observation (b = 1). If no observation satisfies $\overline{s}_i \leq t_{r-1}$ it may be concluded that more than one observation is erroneous. The test should be continued for all combinations without repetition of two observations (b = 2), then for three observations, etc. As soon as a group satisfies the blunder definition the members of the group are excluded from further testing but the test is continued for other groups of the same size and even for groups of larger size. It may happen that several non-overlapping groups satisfy the blunder definition. Then it is not possible to distinguish which of them are erroneous without additional information. The tables 2, 3 and 4 in Appendix illustrate this. This method has the following advantages that it does not rely on the rank order of the statistics and it uses the full observation set in all tests.

PERFORMANCE OF THE PROCEDURES

We may list three measures of performance for blunder detection procedures, which may be of practical importance:

1. β_1 - the probability that test incorrectly concludes that there is a blunder.
2. β_2 - the probability that the test incorrectly concludes that there is no blunder.

3. β_3- the probability that the test incorrectly locates the blunder given that it correctly concludes that there is a blunder.

Although a discussion of those measures would be a very interesting topic, we shall not attempt this in order to shorten the paper.

THE PHILOSOPHY OF BLUNDER LOCATION IN PHOTOGRAMMETRIC PRACTICE

The optimality of blunder location methods in any production environment depends on many factors: The available equipment, the available software, the professional level of personnel, the methods of production, etc. Because these factors vary from place to place an attempt to establish a general recipe for the solution of the blunder problem seems an impossible task. Therefore we shall only discuss the general trends and their possible impacts. One such trend is the application of powerful computers in photogrammetric practice. In such an environment automation of data processing becomes very appealing and procedures such as selective elimination seem especially attractive. Two obstacles to the practical application of these procedures have been mentioned:

1. A large number of combinations have to be tested. This is true especially in the case of multiple blunders and numerous observation. But even in the case of hundreds of thousands of combinations, it is not the number which matters, but the speed with which combinations may be scanned, and this is remarkably high. Moreover, reduction to only one adjustment run saves a lot of computations too.
2. The masking effect 2 for $\overline{s}_\beta$ may reach quite a large magnitude. The simulation shows that for $r = 30$, $\alpha = 0.001$ and $q_{ii} = 0.75$ the blank range amounts to 9σ. However, $\overline{s}_\beta$ is only used to test whether blunders may be suspected in the observations not included in the group β. In addition, however, the observations are also scanned for $\overline{w}_\beta$, which is much more sensitive.

The smaller adjustment problems (such as relative and absolute orientation) can easily be extended to include automatic blunder location procedures, if possible in on-line mode. Aerial triangulation deserves, however, special attention. It is generally accepted, that the blunder tests have to be executed not only after the final adjustment but also prior to it. For this purpose the whole block may be subdivided into smaller units: bundles, models, triplets, strips, subblocks, and the tests executed in each subunit individually. Inside such subunits one usually deals with a smaller number of blunders, which always improves efficiency, even in the case of automatic location procedures. In such a way the blunders may be detected earlier in the process and the whole triangulation procedure may be shortened. The fact that the blank range of $\overline{s}_\beta$ increases with the redundancy causes additional problems in large systems. The consequence of that may be that for too many groups of observations only a tentative location of blunders may be reached.

Sometimes it is recommended that the photogrammetric observations be separated from the ground control in order to facilitate the blunder detection, although there is no justification for that.

It is anyhow never possible to discover whether the error is in photogrammetric observation or in ground control. Moreover, as increased ground control results in increased diagonal elements in the Q matrix, this militates against such a measure.
Another trend seems to be the application of a very simple treatment of blunders during processing of the subunits (e.g. a check of the residuals), while more sophisticated methods are used in the final adjustment only (see [3], [12]). Such a procedure may only be justified in the case where blunder treatment in subunits has been developed in the past and there is little possibility of changing it. Otherwise it seems difficult to see why simplified blunder detection methods are efficient in small adjustment units and not in larger ones.
At this point we would also like to warn against any approximations in computational procedure. The examples in the Appendix make it clear that even small errors may have fatal consequences. Up till now we have discussed only the location of blunders in adjustment problems. There are, of course, also other possibilities. Examples are checks on double measurements of the same point, checks on consistency of point coding, checks on data format, checks on whether the measured coordinates are inside the model or the photo limits, etc.

INFLUENCE OF SYSTEMATIC ERRORS

Systematic errors are somewhat difficult to define. However, they may become apparent in one of two ways: either in the results of calibrations of measuring instruments; or, in the results of adjustment (i.e., residuals). The calibration of instruments and the further processing of the results of calibration is a special subject which shall not be treated here. When the residuals after adjustment show some regularity it is concluded that systematic errors are present. They may be caused either by insufficient calibration or by errors in the functional model. In aerial triangulation the method of selfcalibration (additional parameters) is considered to be efficient in eliminating both types of systematic errors. The results of tests confirm this. However, the interrelation between systematic errors and blunders has hardly been investigated. Formula (1) makes it obvious that a blunder or a group of blunders may produce residuals which may create an impression of presence of systematic errors. Their correction by means of additional parameters is then, of course, very wrong.
There is also another effect of additional parameters, which is usually neglected. The inclusion of additional parameters in the adjustment causes a reduction of redundancy. Smaller redundancy means smaller diagonal elements of the Q matrix. So, obviously, the additional parameters reduce the power of blunder detection. Genuine systematic errors, on the other hand, hamper the location of blunders. They may cause excessively large variance estimates.

APPENDIX

In Table 1 coordinates measured in a photogrammetric model are listed (X, Y, Z). They are used in absolute orientation adjustment. Although a three dimensional absolute orientation has been executed only ground control heights are presented Z_g. The residuals are given in microns as units. The following control point types are distinguished: 3 - full control, 2 - height control, 1 - planimetric control. The positions of points are given in fig. 2. The estimated standard deviation is s = 12μ. Obviously no blunder is present.

Point	Type	x	y	z	Z_g	v_x	v_y	v_z	$\sqrt{q_{ii}}$
1	3	472.023	917.023	49.237	114.490	-16	6	9	0.65
2	3	535.633	752.674	42.414	112.800	5	12	4	0.90
3	3	625.711	520.263	45.770	123.500	- 3	18	4	0.85
4	3	646.689	466.572	74.579	154.180	2	-16	11	0.78
5	3	429.803	613.915	40.274	112.500	3	5	- 6	0.71
6	3	701.412	814.963	43.999	115.410	4	-18	- 5	0.81
7	3	766.225	647.668	45.469	122.160	- 5	3	- 5	0.82
8	3	590.653	661.710	56.387	130.010	18	-12	-19	0.94
9	3	733.853	731.385	49.929	124.020	- 8	2	6	0.84

Table 1

Table 2 shows the results of adjustment of the same points as in Table 1, only the points 6 and 7 have been declared planimetric points only and the height of point 5 has been distorted by 100μ. the tests are executed with σ = 15μ and α = 0.005.

Nr.	Type	z	v_z	$\sqrt{q_{ii}}$	w_i	s_i
1	3	49.237	- 9	.61	16	20
2	3	42.414	-13	.89	15	20
3	3	45.770	-12	.83	14	20
4	3	74.579	- 6	.75	9	19
5	3	40.174	38	.64	59	13
6	1	---	---	---	--	--
7	1	---	---	---	--	--
8	3	56.387	-32	.92	34	17
9	3	49.929	32	.56	63	12
σ.t					42	22

Table 2

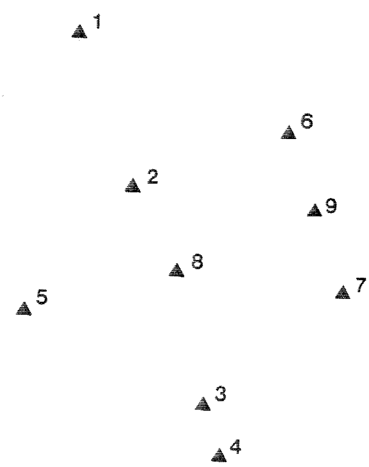

Figure 2

Obviously w_9 is larger than w_5, but because both satisfy the blunder definition according to the selective elimination method both are candidates for elimination. The forward elimination would discard 9 and accept 5.
In Table 3 point 9 has been declared a planimetric point and both heights 6 and 7 have been distorted by 200 μ.

Nr.	Type	Z	v_z	w_i	s_i	Group	w_β	s_β	Group	w_β	s_β	Group	w_β	s_β
1	3	49.237	40	61	31	1,2	70	24	2,3	40	31	4,6	54	29
2	3	42.414	34	38	32	1,3	47	30	2,4	44	31	4,7	45	31
3	3	45.770	33	40	32	1,4	47	30	2,6	42	31	4,8	44	31
4	3	74.579	39	50	32	1,6	47	30	2,7	45	31	6,7	91	12
5	3	40.274	-91	131	12	1,7	62	26	2,8	35	32	6,8	40	32
6	3	44.199	-39	53	32	1,8	52	29	3,4	63	26	7,8	40	32
7	3	45.669	-40	54	32				3,6	48	30			
8	3	56.387	24	26	33				3,7	43	31			
9	1	--	---	---	--				3,8	37	32			
t.σ				42	22		34	22		34	22		34	22

Table 3

The test clearly indicates point 5 as erroneous ($w_i > t\sigma$, $s_i \leq t\sigma$). Further search, however, reveals also that the group consisting of points 6 and 7 is a candidate for rejection. All groups worth testing are given at the right side of table 3 with corresponding w_β and s_β.

In Table 4 again points 6 and 7 are declared to be planimetric points. Points 5 and 9 are both distorted by 200μ. The q_{ii} elements are not excessively small (see table 2) but the blunders, although large, are not apparent and no method will discover them. However, the minimum eigen value for the matrix Q_β for the combination of points 5 and 9 is very small $\lambda_{min} = 0.009$. This explains why the blank range is so large.

Point	Type	z	v_z	w_i
1	3	49.237	16	26
2	3	42.414	11	12
3	3	45.770	12	14
4	3	74.795	18	23
5	3	40.474	-13	21
6	1	---	---	--
7	1	---	---	--
8	3	56.387	-35	38
9	3	49.729	- 8	13
σt			42	22

Table 4

Point	Type	z	v_z	w_i	s_i
1	3	49.237	49	57	35
2	3	42.414	44	35	39
3	3	45.770	44	38	38
4	3	74.597	49	46	36
5	3	40.674	-94	105	19
6	1	---	---	---	--
7	1	---	---	---	--
8	3	56.387	-13	11	40
9	3	49.729	-78	99	23
σt				42	22

Table 5

The situation given in table 4 is also represented in table 5, but this time the height of point 5 is distorted by a larger amount, 400μ. The blunder in point 5 now becomes detactable but not in point 9. Further search will not deliver further results. After the elimination of point 5 the q_{ii} for point 9 will equal the $2\lambda_{min}$ mentioned earlier 0.009. Obviously, the method of selective elimination, although automatic, replicates a photogrammetrist with a lot of insight.

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