# INFLUENCE OF CONTROL POINT ERRORS ON THE ADJUSTED COORDINATES OF PHOTOGRAMMETRIC BLOCKS

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## INTRODUCTION

In photogrammetric block adjustment, ground control points are needed for the transformation of photogrammetric block into the national coordinate system. It is well known that the quality of the adjusted block point coordinates depends mainly on the location and the number of control points [1], [2], [3].

The ground control points are usually the results of previous geodetic surveys. Since a geodetic survey is the realization of only one stochastic experiment, the grounds control points possess the stochastic characters and therefore should be treated in the photogrammetric block adjustment as stochastic, i.e. weighted, variables and be submitted to change in the adjustment whenever necessary. However in most cases, because of the hierarchy, the previously determined ground control points, especially the national triangulation points, must be treated as error-free. It means that their coordinates have to be taken as constant in the photogrammetric block adjustment. (At least for the last run of the adjustment when all blunders in the block are cleared.) Under this circumstance, the tension between control points will act on the photogrammetric block as systematic errors and will affect the accuracy of the adjusted block coordinates. Normally, the exact amount of the tension in ground control network is not known, but based on the stochastic properties, i.e. variance and co-variance, of the control point coordinates, an estimation of it could be made. After that, it should be investigated to what extent the adjusted coordinates are affected by such control point errors.

The purpose of this study is to investigate how the errors of the control points affect the adjusted coordinates of photogrammetric block points. Because of the weakness of the height accuracy and the lack of redundancy in height for normal blocks with 20% side-lap, the tension between the height coordinates of the control points can hardly be detected by the photogrammetric block adjustment. Hence, only the planimetry will be investigated here. Until today, the method of independent models is still being utilized for adjusting most of the blocks in the practice, so that only this method is used in this study.

## RELATIONSHIP BETWEEN CONTROL POINT ERRORS AND ADJUSTED BLOCK COORDINATES

From the basic linearized observation equation,

$$l + v = A \hat{x} \qquad (1)$$

in which
- $l$: the observation vector
- $v$: the residuals
- $A$: the co-efficient matrix
- $\hat{x}$: the estimated value for the unknown vector,

the following relationship between the observations and the unknowns can be derived after the least squares adjustment:

$$\hat{x} = ( A^T \;P\; A )^{-1} \; A^T \; P \; 1 \qquad (2)$$

where P is the weight matrix for the observations. Eq. (2) describes the relationship between the observations 1 and the unknowns x and serves as the basic equation for the study of external reliability. In the photogrammetric block adjustment, the orientation parameters and the adjusted ground coordinates will be treated as unknowns, the photogrammetric measurements and the given coordinates of the ground control points will be treated as observations.

Eq. (2) can be used for calculating the influence of control point errors on the adjusted block points. By introducing the estimated errors of control points into 1 in Eq. (2), the corresponding changes of $\hat{x}$ on the left side of Eq. (2) then represent the influence.

## INFLUENCE SIMULATION

Since the influence can be expressed directly by the change in the adjusted coordinates, any existing block adjustment program can be used to study the influence of control point errors. For any given block parameters, a simulated error-free block can be constructed whose coordinates will serve as the true values of the block points. Then the estimated errors of control point will be introduced into the block and the block will be adjusted. The changes of the coordinates of the block points describe the influence of the control point errors.

In this study, the Stuttgarter PAT-M Program [4] is used for the block adjustment. For simplicity, no tension is simulated. Instead, errors of unit size (1 m on grounds) will always be added to those ground control points which are assumed to be errorneous. The main block parameters considered here are block size, photo scale, and distance between the planimetric control points (only perimeter control pattern is considered here). Simulated blocks always have a square form with 20% side-lap and 60% forward-lap. All models have the six Gruber points together with two projection centers.

## RESULTS

From the simulated blocks, the following results can be derived:

(1) Influence are superposable.

When several control point errors affect the block points simultaneously, their influence equals the sum of influences caused by each control point error separately.

This property can also be found directly from Eq. (2). When the errors of the control points are not so big as to distort the geometry (co-efficient matrix A) of the block, they will be transformed linearly by the matrix $(A^T PA)^{-1} A^T P$ into $\hat{x}$ independently, so that each will simply be added to the elements of $\hat{x}$.

(2) Influence is independent of photo scale.

For blocks which have different photo scale but same size (number of strips and number of models per stip) and same control pattern, the errors caused by the same control point errors will be the same at the corresponding location in the block. This can be better illustrated by an example.

Fig. 1 shows the error vectors on the adjusted block points at different locations in two blocks caused by the same amount of control point error at point No. 1 (1 m error toward the west). Both blocks (A and B) have five strips and 10 models/strip. Block A has a photo scale of 1:10000, block B, 1:20000. Both have eight planimetric control point (i = 5b). The plotting scale and the vector scale for both blocks in Fig. 1 are the same so that a direct comparison of the two blocks is possible. Although block B covers an area four times as much as block A, at the same location in the block (referred to the stip number and model number), the error vectors are the same in both blocks. From the error vectors in Fig. 1, it can be found that the size of the errors of adjusted block points depends on the distance of the block point from the errorneous control point, which is the error source.

(3) Absolute amount of influence is independent of the direction of the error of the control point.

In Fig. 1, we have on control point 1 an error of 1 m toward west. If the 1 m error is toward another direction, north for example, the absolute values of the errors on all adjusted block points will remain the same. Only the directions are different.

(4) Corner control point error has less influence than side control point.

In Fig. 1, if both control points 1 and 2 have the same amount of error, the errors on the adjusted block points caused by the corner point 2 will be smaller than those caused by the side point 1. But when control points are sparsely distributed this difference will not be apparent.

(5) Influence decreases approximately exponentially with increasing distance from the errorneous control point.

The maximal error of the adjusted block points occurs naturally on the errorneous control point itself. With increasing distance from this point, the error of the adjusted block point decreases. If we plot the size of error as function of distance measured in number of base lengths, we will find that it resembles an exponential function. Figs. 2 and 3 give some examples. In these Figures, the ordinate shows the amount of error on adjusted block points which taking the amount of error source (error of the control point) as unit (one). The abscissa represents the distance (in number of base lengths) of the block point from the error source.

From Figs. 2 and 4, we can find:

(i) Influence is almost independent of block size.

In Fig. 2, the influences of unit error of the control point in the middle of block side for three different block sizes, 5x10, 10x20, and 20x40, with the same control pattern ( i=5b ) are shown. It can be seen that the difference between them is negligible small.

(ii) In blocks with more dense perimeter control pattern, the influence of errorneous point decreases more rapidly as distance increases.

In Fig. 3, influences from the same control point error are shown for the same block size ( 10x20 ) but with different control patterns (curve 1 is i=10b, curve 2 is i=5b, and curve 3 is i=2b).

It can be seen that for i=2b, the influence decreases most rapidly. Generally we can say that at a distance which equals the distance between the control points, e.g. at distance 5b for i=5b control pattern, the error of adjusted block point will only be 20% of the error source.

Hence, the influence depends only on the control pattern and the distance of the block points from the error source. The error on any block point can be estimated by the following exponential approximations:

$$e = \exp\left(-1.5 * \sqrt{d/B}\right) * E \qquad (3)$$

where e : amount of error on adjusted block point caused by error E of control point

d : distance in number of base lengths from the errorneous control point to block point

B : distance between the planimetric control points, also in number of base lengths

Eq. (3) is only for points inside the block. For points on block border, no general approximation can be found, because there are other control points on the border and they will reduce the influence in their neighbourhood.

For the blocks here investigated, ranging from 5x10 to 20x40 models with i=2b to i=10b control pattern, Eq. (3) has an accuracy of within 1/10 of the source error E.

The following illustration will show how to use the equations to estimate the influence of a known control point error.

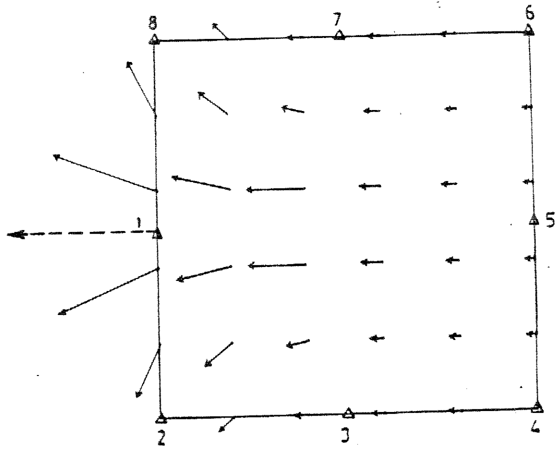
Assuming that we have for a block of any size the i=5b control pattern and that one of the control points on the side of the block has an error of one unit, then at a distance of five base lengths from this errorneous control point, the adjusted control point will have an error of

$$e = \exp(-1.5 * \sqrt{5/5}) = 0.2 \text{ unit}$$

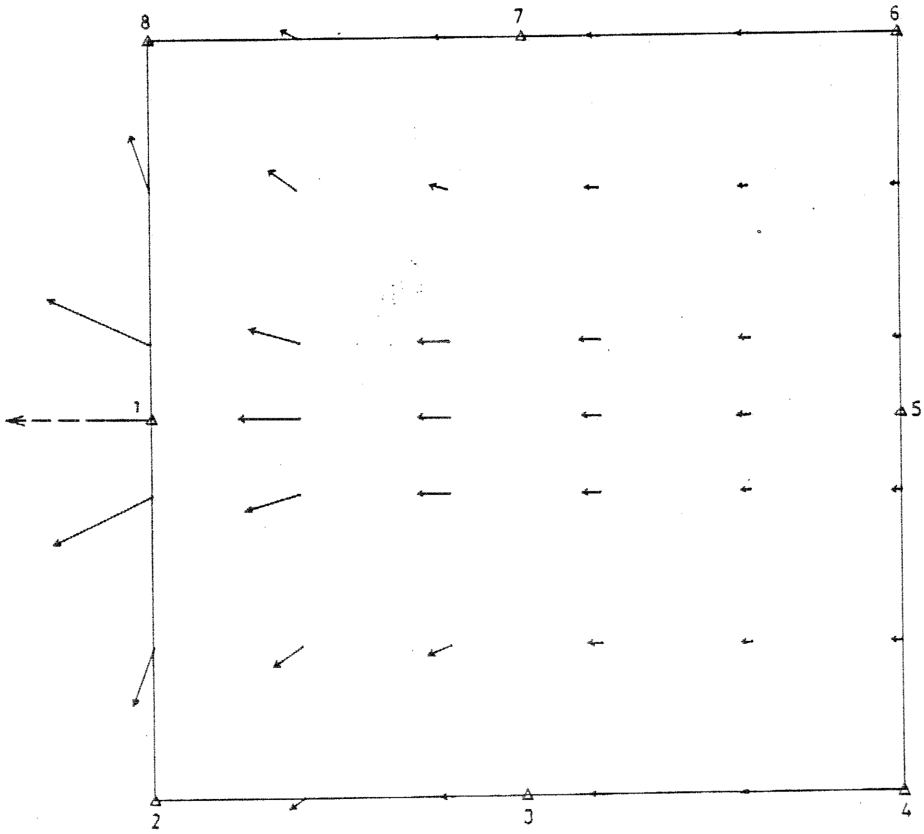
Eq. (3) can only be used for estimating the absolute amount of error. If there are more control points which are errorneous, then according to superimposability, the final error will be the vector sum of errors caused by each control point separately. In this case, Eq. (3) becomes useless. The estimation must be done by simulated block mentioned above.

## CONCLUSION

It happens very often in the practice that the tension between control points, compared to the accuracy that the aerial triangulation can reach, can not be ignored. But if due to any reasons the ground control points have to be fixed and are not allowed to accept any changes, they will be given a very high weight in the block adjustment. In this case, the tension will act as systematic error and will thus affect the accuracy of the adjusted block points. All accuracy estimations and predictions, either according to theoretical deduction or from empirical results of testfields, will be too optimistical. The influence of control point error must be estimated by simulated block in order to get more reasonable estimations of the accuracy of adjusted block points.




Block A: photoscale 1:10000, ground coverage 9 km x 9 km




Block B: photoscale 1:20000, ground coverage 18 km x 18 km

Fig. 1 Errors on adjusted points in blocks with 5 strips and 10 models/strip, caused by an error of one unit of control point 1. Both blocks have eight control points (i=5b).

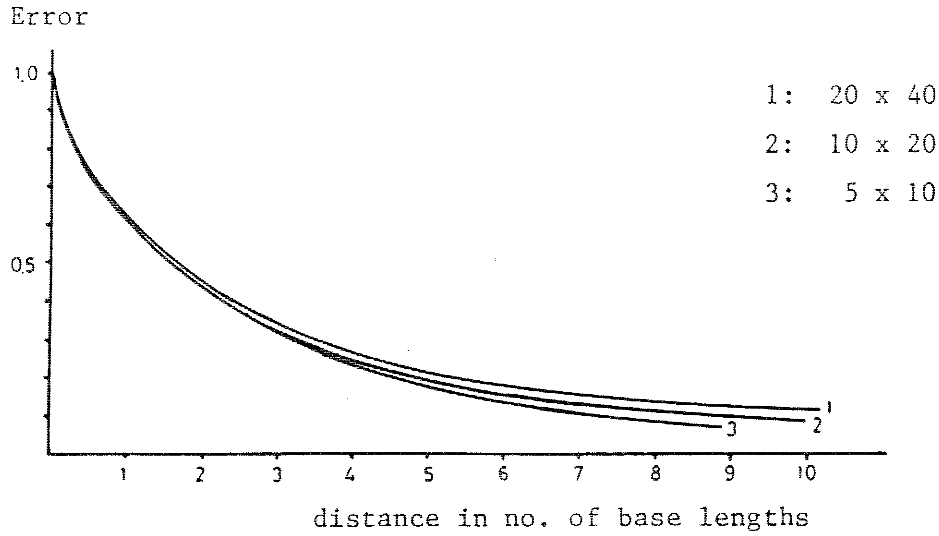



Fig. 2 Size of errors of adjusted block points relating to the distance of the block point from the errorneous control point for 3 blocks of different sizes but with the same (i=5b) control pattern.

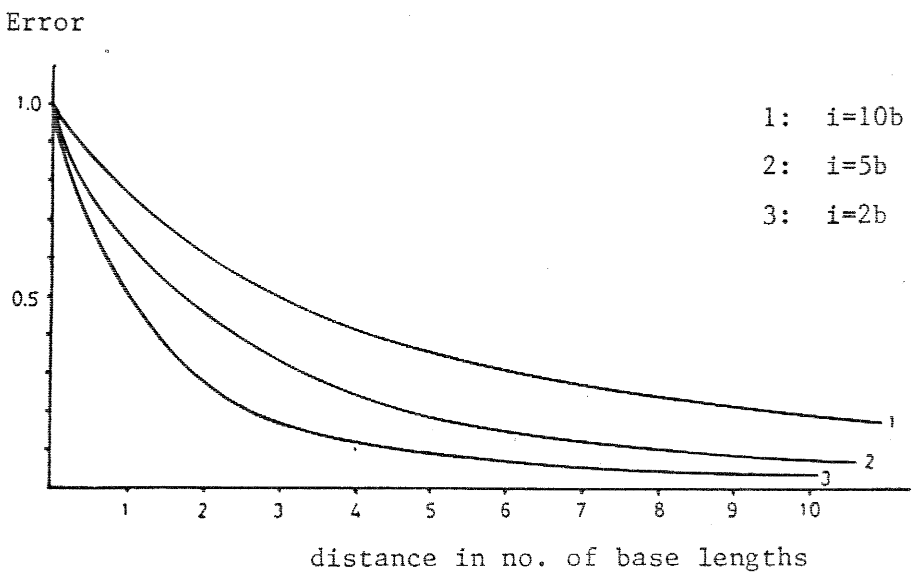



Fig. 3 Errors of adjusted block points caused by the same control point error in 3 blocks with the same size (10x20) but different control patterns.

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