ON THE DETECTION OF GROSS AND SYSTEMATIC ERRORS IN COMBINED ADJUSTMENT OF TERRESTRIAL AND PHOTOGRAMMETRIC DATA

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Abstract

A special bundle adjustment program which accepts terrestrial and photogrammetric data has been developed with self-calibration capability and a built-in gross-error detector with "data snooping". The program computes the redundancy numbers as well as the external reliability factor for each adjusted image point. Using actual and simulated data, in the form of terrestrial observations between object points, the effect of additional constraints on the ability of a photogrammetric system to detect gross and systematic errors has been studied. In the combined adjustment, the detection of gross errors was improved significantly, particularly in areas where the intersection of rays is geometrically weak. The detection of systematic errors did not improve, but their effect on the adjusted object coordinates (external reliability) was greatly reduced.

Introduction

Simultaneous adjustment of terrestrial and photogrammetric observations has been explored already for more than a decade (e.g., Wong and Elphingstone, 1971; Kenefick et al, 1978; and El-Hakim and Faig, 1981). The main purpose of these applications has been to allow a reduction in the number of cotrol points especially in areas where available geodetic observations are insufficient for an adjustment of a complete geodetic network of control points for phototriangulation. Instead of using the usually required number of geodetically adjusted control points, therefore only available control points plus some terrestrial observations, replacing the remaining control points, are entered into a simultaneous adjustment with the photogrammetric measurements.

Another benefit from the combined adjustment, discussed in the present paper, is an improvement in the ability of the photogrammetric system to detect gross and systematic errors. The terrestrial observations enforce certain relationships between the ground coordinates. Points connected by such observations have less freedom to move. Thus, if an error exists in an image coordinate it will appear, depending on the type of terrestrial observation, mainly in the image residual rather than in the ground coordinates, which means a higher reliability for these points. An earlier study [El-Hakim, 1981b] showed that distance observations between points of low reliability, such as edge points, increase the reliability substantially (redundancy numbers for x increased from zero to about 0.8) when adjusted simultaneously with the photogrammetric data. Only two distances at each point are needed. The study is here expanded to include two types of systematic error: radial lens distortion and affine film deformation. Also included, in addition to spatial distances between points, are observed height differences as terrestrial data. The program GEBAT (El-Hakim and Faig 1981), used in the following tests, has been extended to compute parameters such as redundancy numbers and external reliability factors. Three different types of data have been employed: a simulated block with relatively dense netwrok of points and regular flight arrangement, a large-scale actual block, and a small close-range convergent photography block. The bulk of the research has been performed on the simulated block since it provides more flexibility and unlimited variation in its parameters. The two actual blocks have only been used to confirm some findings. In all these studies, the effect of different types of error on the image residuals and the adjusted object coordinates has been computed for the case where (a) only photogrammetric data were used and for the case when (b) the combined adjustment was applied. Before presenting the test results some theoretical investigations are presented.

Error Distribution - Theoretical Study

Errors in observations (vector $\bar{L}$) will affect the adjusted unknowns (vector X) and the corrections to the observations, the residuals (vector V). The ratio by which the error affects each of these variables depends largely on the geometry of the system. This error distribution can be computed by the variance covariance matrix of the adjusted observations and of the residuals.

After the adjustment, the weight-cofactor matrix of the observations can be computed by applying the covariance law on the function

$$\bar{L} = F(X) \tag{1}$$

as follows:

$$Q_{\bar{L}} = \left[\frac{\sigma F}{\sigma X}\right] Q_X \left[\frac{\sigma F}{\sigma X}\right]^T \quad \text{or}$$

$$Q_{\bar{L}} = A . N^{-1} . A^T$$

where N is the matrix of the normal equations. Partitioning the the unknowns into orientation parameters X_1 and object coordinates X_2, equation (3) becomes:

$$Q_{\bar{L}} = [A_2 \quad A_1] \begin{bmatrix} N_{22} & N_{21} \\ N_{12} & N_{11} \end{bmatrix}^{-1} \begin{bmatrix} A_2^T \\ A_1^T \end{bmatrix}$$

from which:

$$Q_{\bar{L}} = A_1 \xi^{-1} A_1^T + A_2 N_{22}^{-1} A_2^T + A_2 N_{22}^{-1} N_{21} \xi^{-1} N_{12} N_{22}^{-1} A_2^T - A_2 N_{22}^{-1} N_{21} \xi^{-1} A_1^T - A_1 \xi^{-1} N_{12} N_{22}^{-1} A_2^T \tag{4}$$

where

$$\xi = N_{11} - N_{12} N_{22}^{-1} N_{21} \tag{5}$$

$$N_{ij} = A_i^T P A_j \qquad (i = 1,2 \; ; \; j = 1,2)$$

and P is the weight matrix of the observations.
Each diagonal element of $Q_{\bar{L}}$ represents the geometrical strength at the corresponding observation point. Equation (4) can be rewritten in a diagonal form as:

$$q_{\bar{L}} = e_1 + e_2 + e_{12} \tag{6}$$

where e_1 is the diagonal of $A_1 \xi^{-1} A_1^T$, e_2 is the diagonal of $A_2 N_{22}^{-1} A_2^T$, and e_{12} is the diagonal of the remaining right hand side of equation (4). Factor e_1 represents the part of image error affecting the orientation parameters, and e_2 represents the part affecting the adjusted object coordinates (indication of external reliability), while e_{12} represents the interaction between the two effects.
The part of image error affecting the residuals can be computed from:

$$Q_{vv} = Q_L - Q_{\bar{L}} \tag{7}$$

where Q_L is the a priori (or given) weight-cofactor matrix of the observations. The diagonal elements of Q_{vv} are called the redundancy numbers r_i for observation i and represent the part of the error affecting the residuals. Factors r and e_2 are those of importance to us and will be referred to in the following tests. They are related by the function:

$$r + e_1 + e_2 + e_{12} = 1. \tag{8}$$

It is of course important to reduce the effect of image error on adjusted object coordinates (e_2) and increase the effect on image residual (r) so that it can be easily detected. This can be achieved by improving the geometry, or increasing the number of intersecting rays at object points. Table 1 gives the values of e_2 and r (averaged for all non-control points) for points with different numbers of intersecting rays and for different blocks.

It is clear that improving the geometry, by increasing the number of intersecting rays, leads to the desired increase in r and decrease in e_2 (see also figure 1). In fact, the average of $e_2(x)$ and $e_2(y)$ is always:

$$\frac{1.5}{n} \quad (9)$$

where n is the number of intersecting rays. In any block, the average of $e_2(x)$ and $e_2(y)$ for all the points, each appearing n times in the block, always follows equation (9). This could be due to the fact that 1.5 points (3 observation) results in zero redundancy and the error appears entirely at adjusted coordinates (average $e_2(x)$ and $e_2(y)$ = 1.0).

The above analysis applies when no additional constraints or conditions exist between the object coordinates. Now, is it possible to increase the redundancy number r and decrease the factor e_2 through added constraints rather than improving the geometric strength of intersecting rays? This is the objective of the next sections.

Effect of Additional Constraints on Gross-Error Detection

The constraints used in this test are spatial distances and height differences. These are probably the most useful terrestrial data for inclusion in a combined adjustment and also the easiest to acquire in practice. It is expected, as mentioned in the previous section, that the combined adjustment will increase the effect of the gross errors on the residuals while their effect on the adjusted object coordinates will decrease. This is demonstrated using combined adjustment with distances only and with distances and height differences together. The redundancy numbers are computed for different cases as shown in tables 2 and 4. An error of 100 μm is introduced at each of these cases and the effect on the adjusted object coordinates is computed with and without terrestrial data (tables 2 and 4). Two blocks are used here, the simulated block and the close-range block. All the selected points, distances, and height differences were on the perimeter of the block (figures 2 and 3). This is of course the area where the geometric structure is the weakest, and thus improvement by additional constraints is most needed and more noticeable than anywhere else in the block.

Table 2 displays the changes in r_i for two different blocks and for different combinations of distances for points with different number of intersecting rays. When two or more measured distances originate from a point, the redundancy number increases to 0.50-0.9 range. One distance only does not improve the reliability (case D), also if the distance is in x direction, the increase in r_y is small (case B).

Table 3 shows the effect of an 100 μm image error, for the cases of table 2, on the adjusted object coordinates, without and with distances. Except for case D (one distance only), the effect on adjusted object coordinates is reduced substantially when distances are used. In cases E to H, the object coordinates are almost unaffected by the error. In cases A and B, where the distances are in X-direction, the improvement is mainly in X, with moderate improvement in Y, and little or no improvement in Z. These two cases are repeated in the next test where height differences and distances are used in the combined adjustment. Table 4 shows the effect of the combined adjustment on the redundancy numbers. There is an additional improvement in r(x) (about 25%) and no change in r(y). However, the improvement in the effect on object coordinates is substantial especially when the error is in x coordinate (case A). In this case the object coordinates are almost unchanged due to the error. When the error is in y (case B), the resulting error in Z is almost eliminated while the errors in X and Y are reduced slightly.

It is now clear that the combined photogrammetric and terrestrial adjustment has a great advantage in improving the reliability both internal and external. All that is needed is the measurement of distances between points (two distances to each point) on the perimeter of the block where the reliability is originally the lowest. Height differences are not needed for cases where the ratio between variation in terrain elevations and camera station height is large enough to cause correlation between planimetric and height coordinates such as in close range photogrammetry. However, in cases of nearly flat terrain, height differences will help at least in improving the external reliability.

Effect of Additional Constraints on Some Systematic Errors

Since systematic errors are much smaller than most gross errors and affect all the points in the block, it is expected that the influence of the combined adjustment will be very different on the two types of error. In the case of systematic errors it will probably depend more on the source of error and the distribution of the terrestrial observations. Since many factors are needed to be studied here, only the simulated block is used in the following tests.

(a) Image Coordinates Contain Radial Lens Distortion:

A generated lens distortion data, using the Wild Aviogon lens distortion curve, have been added to the simulated image coordinates. The following parameters are studied:

a. type of terrestrial observation
b. number and distribution of terrestrial observations
c. number of control points

Various tests have been carried out with the results displayed in table 6 (tests 1-8). The different distance distributions are shown in figure 4. The height differences are at the perimeter of the block. Control point distributions are also shown in that figure. Analyzing these tests, the following comments can be made:

1. The overall effect on the residuals is negligible. The standard error of unit weight has not changed while the residuals at individual points have changed slightly up or down.
2. When no terrestrial data have existed, the control point distribution is critical (compare object coordinate error in tests 1 and 2) while additional control points do not improve the results significantly in the case of combined adjustment (compare cases 3 and 4).
   Comparing test 1, where 20 planimetric and 34 vertical control points have been employed without additional constraints, with test 8, where 8 planimetric and 14 vertical control points have been used with terrestrial observations, it is clear that the terrestrial data not only replace many of the control points but also improve the accuracy.
3. The optimum distance distribution is 28 perimeter distances (test 6). These distances do not form a closed polygon around the block like in test 3 but have few gaps which have not affected the accuracy but, on the other

hand, have reduced the measurement effort. Using 60 distances as shown in figure 4 does not change the results.

4. The accuracy in Z does not change significantly until height differences are introduced (test 8). This is probably because the elevation differences compared to the flying height is small (nearly flat terrain).

Table 6 shows the overall accuracy of the different tests, and it may be useful to look what happens at the individual object points. The points included in table 7 and ~~shown in~~ figure 5 are selected as an example of points with constraints in the block. By examining table 7 and figure 5 comparing test 2 and 3, it is obvious that the error along the distance direction has been removed. For points 68 and 82 the distances are in Y-direction while for points 138, 149 and 165 they are in X-direction. The improvement in the perpendicular direction or in Z-direction is smaller. When height differences are added to the adjustment, the error in Z has almost disappeared. Some increase in the errors has taken place in the perpendicular direction but it is too small to be corrected by the distances.

(b) Image Coordinates Contain Affine Film Deformation

The affine film deformation, introduced into the image coordinates of the simulated block, produces a very different error pattern in both the residuals and the object coordinates (table 6, tests 9 to 14) from that produced by radial lens distortion. The additional constraints have not improved the results at all. The main reason is that this type of systematic error does not produce significant errors along the coordinate axis that is nearly parallel to the distance directions or in Z. Most of the errors in the object coordinates are in the perpendicular direction where distances have little effect for this size of error. This is clear from table 8, where most of the error in points 68 or 82 is in X (distances are in Y direction, see figure 6) and in Y-direction for points 138, 149 and 165 (distances are in X-direction).

The overall size of image residuals is very small (less than 1 μm), and the additional constraints have little effect on them.

Concluding Remarks

The effectiveness of the combined adjustment as a tool for error detection depends on the following two factors:

1. Error size. Large errors are very effectively detected by the combined adjustment. Points with originally low or no reliability could have a 0.7 or more redundancy number when two or more distances are measured to these points. Systematic errors, due to their small size, could not be detected any better, by the residual, using the combined adjustment. However, the effect on the adjusted object coordinates (external reliability) has, in most cases, been reduced significantly, and thus the overall accuracy of the adjusted coordinates has increased.
2. Error direction. As a rule, terrestrial observations are very effective in eliminating the effect of image errors on the adjusted coordinates in the direction of the observations. If the observations are distances in X-direction, for example, then about 90% of the error in this direction is eliminated compared to only 10-35% in the Y-coordinate. The use of height differences eliminates virtually all errors in Z.

Although more detailed studies are still needed, [using other types of terrestrial observations at more different configurations] it is safe to say that by having such observations in the areas where the intersection of rays is geometrically weak, we can improve significantly the detection of gross errors and the external reliability of blocks containing systematic errors.

References

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Average For All Non-Control Points

# of rays	Simulated block[1]				Sudbury City Block[2]				Industry block[3]				average e2x, e2y (any block)
	r_x	r_y	e_2x	e_2y	r_x	r_y	e_2x	e_2y	r_x	r_y	e_2x	e_2y	
2	0.00	0.40	0.97	0.53	0.00	0.42	1.00	0.50	0.00	0.41	0.95	0.55	0.750
3	0.33	0.43	0.67	0.33	0.24	0.38	0.67	0.33	0.44	0.37	0.43	0.57	0.500
4	0.48	0.55	0.36	0.39	0.44	0.43	0.38	0.37	0.52	0.44	0.31	0.44	0.375
5	0.54	0.56	0.28	0.32	0.45	0.48	0.35	0.25	—	—	—	—	0.300
6	0.55	0.57	0.23	0.27	0.48	0.47	0.29	0.21	—	—	—	—	0.250

1. 52 photographs, dense points.
2. 55 photographs, regular urban large scale block.
3. 4 convergent close-range photographs.

Table 1: Average values for r and e_2 (non control points)

Block/Case	no. of rays	no. of distances	original r_i	r_i
Simulated A	3	2	0.18(X)	0.53
B	3	2	0.43(Y)	0.50
C	4	2	0.56(X)	0.69
Close Range D	2	1	0.00(X)	0.13
E	2	2	0.00(X)	0.71
F	2	3	0.00(X)	0.90
G	2	4	0.00(X)	0.77
H	4	2	0.67(X)	0.90

Table 2: Effect of Distances on Redundancy Numbers

Block/Case	without distances			with distances		
	X	Y	Z	X	Y	Z
Simulated A	162	170	296	14	111	257
B	92	134	41	-12	116	49
C	127	22	-55	18	3	-16
Close Range D	-9	-67	31	-4	-65	32
E	16	-33	-16	2	-2	3
F	-4	-56	29	0	0	-2
G	-22	-50	25	-2	2	-1
H	2	-3	2	0	0	0

Table 3: Effect of 100μm Image Error On Adjusted Object Coordinates (in mm)

Case	No. of Rays	No. of distances	No. of height diff.	original ri	r_i (distance only)	r_i
A	3	2	2	0.18(x)	0.53	0.66
B	3	2	2	0.43(y)	0.50	0.50

Table 4: Effect of Distances and Height Differences on Redundancy Numbers (Simulated Block)

CASE	PHOTO ONLY			WITH DISTANCES			WITH DISTANCES AND HEIGHT DIFF.		
	X	Y	Z	X	Y	Z	X	Y	Z
A	162	170	296	14	111	256	1	1	2
B	92	134	41	-12	116	49	-10	95	2

TABLE 5: Effect of 100μm image error on adjusted object coordinates (mm)(simulated block)

TEST	ERROR	CONSTRAINTS	CONTROL	RESIDUALS (μm)		OBJECT POINT ERROR mm		
				σ_o (x)	σ_o (y)	X	Y	Z
1	Radial Lens	None	20/34*	2.0	2.8	9	9	61
2	Radial Lens	None	8/34	2.0	2.8	24	13	64
3	Radial Lens	D(32)**	8/34	2.0	2.8	8	9	62
4	Radial Lens	D(32)	20/34	2.0	2.8	6	9	61
5	Radial Lens	D(60)	8/34	2.0	2.8	8	8	62
6	Radial Lens	D(28)	8/34	2.0	2.8	8	9	63
7	Radial Lens	D(24)	8/34	2.0	2.8	9	13	63
8	Radial Lens	D(32) + H(32)**	8/14	2.1	2.8	7	10	41
9	Affine Film	None	20/34	0.8	0.8	10	20	2
10	Affine Film	None	8/34	0.3	0.3	12	21	2
11	Affine Film	D(32)	8/34	0.3	0.4	14	19	3
12	Affine Film	D(32)	20/34	1.0	0.9	4	20	2
13	Affine Film	D(60)	8/34	0.6	0.6	13	19	4
14	Affine Film	D(32) + H(32)	8/14	0.4	0.4	14	19	3

* indicates 20 horizontal and 34 vertical control points
** indicates 32 distances
*** indicates 32 height differencrs

TABLE 6: Effect of combined adjustment on <u>overall</u> residuals and object coordinates when systematic errors exist.

POINT #	TEST #2			TEST #3			TEST #8		
	X	Y	Z	X	Y	Z	X	Y	Z
68*	31	34	0	10	1	0	7	1	4
82*	48	21	0	18	3	0	13	2	5
138	16	6	73	1	-5	64	3	-19	3
149	16	20	112	1	3	98	4	-25	2
165	13	22	116	0	7	104	4	-25	2

* Point was vertical control in case 2 and 3

TABLE 7: Object Point Error for Some Points (Figure 3) - Radial Lens Distortion

POINT #	TEST #10			TEST #11			TEST #14		
	X	Y	Z	X	Y	Z	X	Y	Z
68*	30	5	0	25	4	0	26	4	4
82*	39	-1	0	47	7	0	49	7	5
138	-6	-17	1	4	-20	-2	3	-20	3
149	-9	-34	-1	5	-34	-7	5	-31	2
165	-7	-44	-4	6	-38	-4	5	-36	2

* Point was vertical control in case 10 and 11

TABLE 8: Object Point Error for Some Points (Figure 4) - Affine Film Deformation

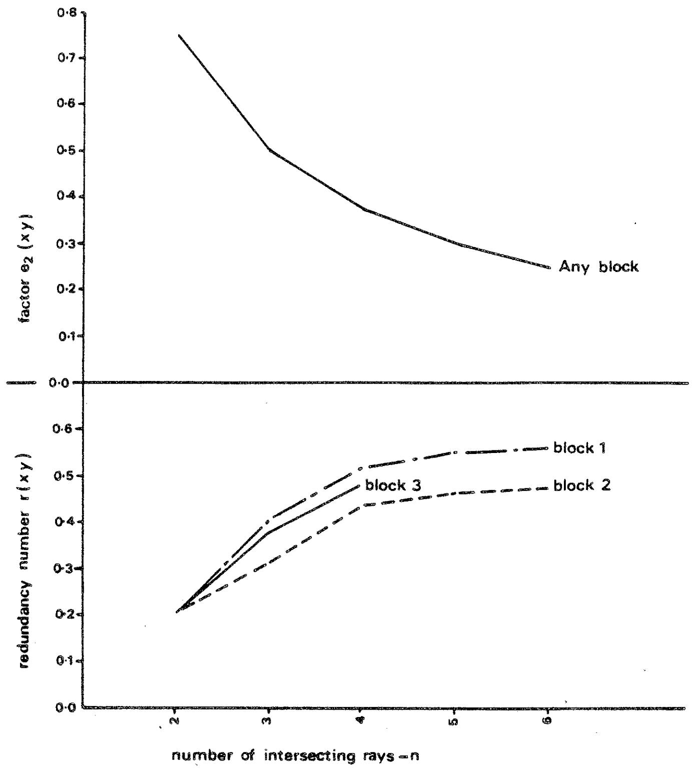


Figure 1: Average Values of r and e_2

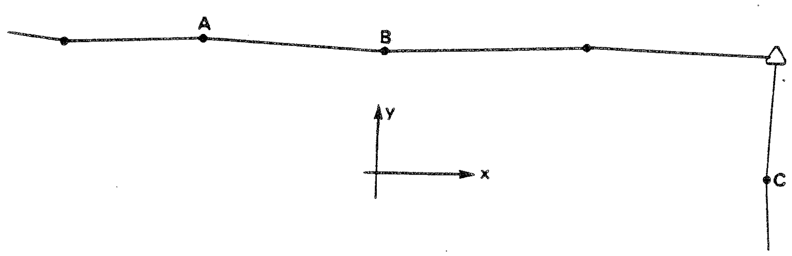


Figure 2: Some Erroneous Points..Simulated Block

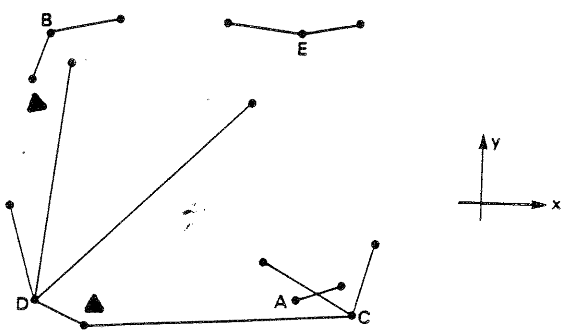
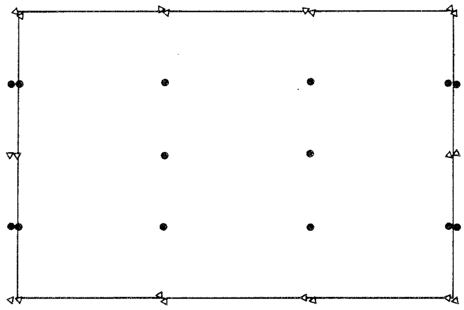
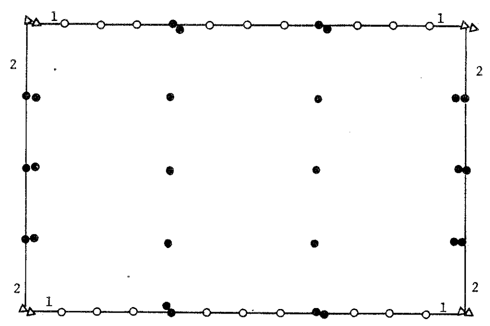


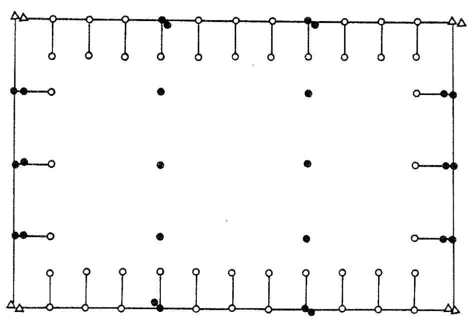
Figure 3: Some Erroneous Points..Close-Range Block



(a) Test 1 in Table 6



(b) Test 3 in Table 6 , Test 6 does not include distances marked by 1 , Test 7 does not include distances marked by 2.



(c) Test 5 in Table 6

Figure 4: Distribution of Distances and Control Points in Simulated Block.

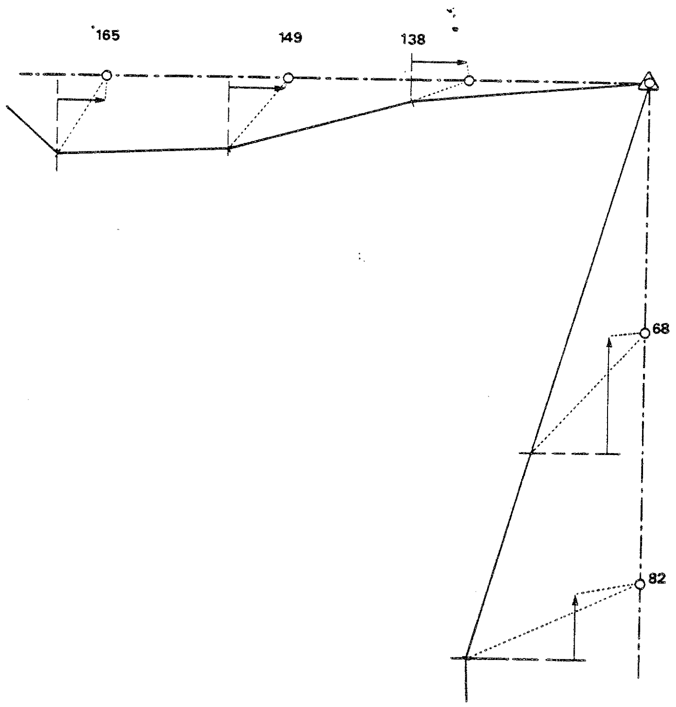


Figure 5: Improvement Component Along Distance Direction..(Radial Lens Distortion)

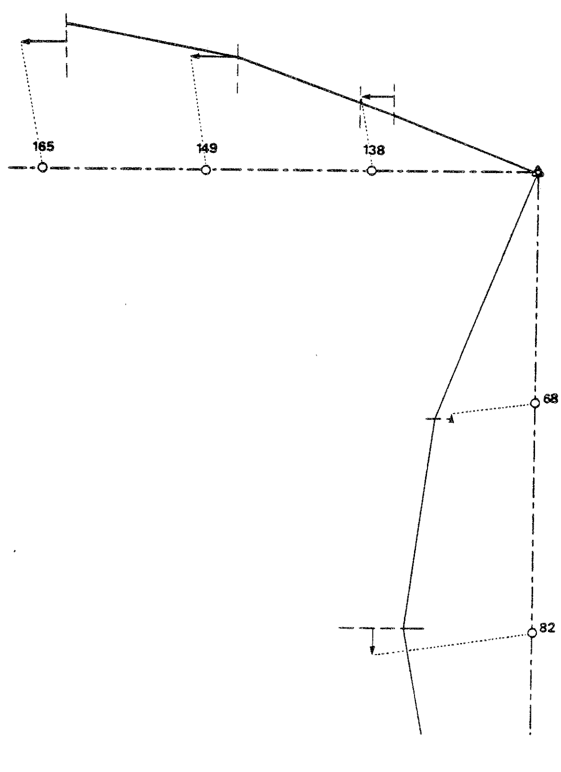


Figure 6: Improvement Component Along Distance Direction..(Affine Film Deformation)