A FEW REMARKS ON MATHEMATICAL APPROACH TO SELF-CALIBRATION BLOCK ADJUSTMENT

ANTIPOV I.T.
RESEARCH INSTITUTE OF APPLIED GEODESY
USSR
COMMISSION III

The rigorous adjustment of analytical aerial triangulation is mainly based on collinearity conditions of the projection rays leading to the system of observation equations

$$ax + \ell = v, \tag{1}$$

where x - a vector of differential corrections to the approximate values x_0 of elements of the exterior orientation and coordinates of ground points, a - a matrix of the partial derivatives, ℓ - misclosure vector and v - vector of residuals.

System (1) yields normal equations

$$Ax + L = 0, \tag{2}$$

from which

$$x = -A^{-1}L. \tag{3}$$

Assuming for the convenience of consideration that all the measurements are equally accurate, one can write for terms of formulae (2) and (3) that

$$A = a^T a, \quad L = a^T \ell.$$

If the measured x and y - coordinates of the photograph points contain systematic errors, the phototriangulation adjustment may be carried out using self-calibration principle. In this case the system of observation is extended and becomes

$$ax + bS + \ell' = v', \tag{4}$$

where S - corrections for the initial values of self-calibration parameters S_0, and b - a coefficient matrix corresponding to the mathematical model of systematic photogrph distortions employed for adjustment. As a rule, $S_0 = 0$.

For system (4) the solution reliability depends on the distortion model b, which, in principle, can make the system indeterminate. Therefore, to satisfy the least-square method requirements and to retain the system stability, the additional equation imposing restraints on self-calibration parameter values must be introduced simultaneously with the increase of the number of unknowns. This additional equation is $b_1 S = v_S$, in which usually $b_1 = E$, where E - the identity matrix. Thus, in the presence of systematic distortions, the general system of observation equations can be given as

$$\left.\begin{aligned} ax + bS + \ell' &= v' \\ S &= v_S \end{aligned}\right\}. \tag{5}$$

Passing from (5) to normal equations, one shall obtain

$$\left.\begin{aligned} Ax + CS + L_a' &= 0 \\ C^T x + (B+E)S + L_b' &= 0 \end{aligned}\right\}, \qquad (6)$$

where

$$C = a^T b, \quad B = b^T b, \quad L_a' = a^T \ell', \quad L_b' = b^T \ell'.$$

From (6) the main unknowns are

$$x = -A^{-1}CS - A^{-1}L_a'. \qquad (7)$$

Suppose that observation equations (1) and (5) correspond to two phototriangulation blocks which geometric characteristics coincide with the exception of the fact that the systematic errors are present only in the second block. If the initial values x_0 of the unknowns in both blocks are the same, the absolute term vectors will be related as $\ell' = \ell + \delta\ell$, where $\delta\ell$ – is the effect of systematic errors. Consequently, under the predetermined conditions the observation equations (4) may be presented as

$$ax + bS + \ell + \delta\ell = v + \delta v,$$

and system (5) is substituted by two identical systems, one of which coincides perfectly with (1) and the other one is

$$\left.\begin{aligned} bS + \delta\ell &= \delta v \\ S &= v_s \end{aligned}\right\},$$

from which

$$(B+E)S = b^T\delta\ell = 0. \qquad (8)$$

If the systematic distortion model has been established correctly, it is reasonable to require that values (3) and (7) of the unknowns should be the same, that is

$$A^{-1}L = A^{-1}CS + A^{-1}L_a'. \qquad (9)$$

But in accordance with the above used symbols

$$L_a' = a^T\ell' = a^T(\ell + \delta\ell) = L + a^T\delta\ell,$$

therefore, equation (9) will hold if

$$A^{-1}CS + A^{-1}a^T\delta\ell = 0,$$

or

$$a^T bS + a^T\delta\ell = 0.$$

Premultiplying the last expression by pseudoinverse matrix a^{T+}, one can apparently obtain the condition of single-valued solution

$$bS + \delta\ell = 0$$

or

$$bS + b^T\delta\ell = 0. \qquad (10)$$

Comparing formulae (8) and (10), one can come to the conclusion that observation equations (5) fail to yield

SORRY

MISSING PAGE

SORRY

MISSING PAGE