

REALIZATION OF AUTOMATIC ERROR DETECTION IN THE
BLOCK ADJUSTMENT PROGRAM PAT-M43 USING ROBUST ESTIMATORS

Hermann Klein and Wolfgang Förstner
Photogrammetric Institute, Stuttgart University
Germany, Federal Republic
Commission III

Abstract: The detection of outliers can be automated using robust estimators. The principle is to interpret the residuals v_i of the observations after each iteration as errors in order to calculate new weights based on a weight function $p(v_i)$. The new weights $p_i = p(v_i)$ are then used in the following iteration step.

The paper reports on the realization of this error detection strategy in PAT-M43. Main topic is the extension of the method, especially the choice of a proper weight function, the iteration sequence and the stopping rule. The significant facilitation in handling the program is explained.

1. The original program:

The computer program PAT-M43 performs a blockadjustment by independent photogrammetric models. This approach implies a spatial similarity transformation for each model. The adjustment is based on a least squares solution. The nonlinear observational equations are linearized with respect to the orientation parameters. Because of computational economy the program iterates sequential horizontal and vertical adjustments, applying 4-parameter and 3-parameter transformations, respectively. For each iteration the partially reduced normal equations that contain only the unknown orientation parameters are formed directly from the model and control coordinates and are solved by a modified Cholesky method (Ackermann et. al. , 1970). An extension allows the combined adjustment of photogrammetric models with APR and/or statorscope data, including photogrammetric height measurements of shorelines of lakes (Ackermann et. al. 1972).

2. Manual data cleaning:

One of the main problems handling blockadjustment programs is the detection and location of outliers. Dependent on the number and distribution of the observations, errors are shown up only partly by the residuals of the corresponding observations, the other parts falsify the absolute orientation of the photogrammetric models (Förstner, 1978). The mutual interference of outliers, especially of different size, is a further handicap. For that reason several adjustments for a step by step location and elimination of outliers in accordance with the size of the errors and some further adjustments in order to avoid wrong decisions are necessary. Nevertheless the quality of manual data cleaning is sufficiently good and comparable with most of the more sophisticated procedures (Förstner, 1982), but in general it requires a great deal of time by fully qualified persons. Thus the main argument for the development of an automatic procedure has been: to shorten the processing time needed by persons in charge with blockadjustment.

3. From least squares to robust adjustment:

The above mentioned problems arising by adjustment of data with gross errors are not a specific attribute of the manual data cleaning procedure, but a bad point of the method of least squares. Applying a constant weight $p = \text{const}$ for each observation the influence function (first derivative of the minimum function by the residual) shows, that the influence of a defective observation onto the result of the adjustment is directly proportional to the size of the error. Thus as a matter of fact the method of least squares is applicable for errorfree data only and unsuitable for automatic error detection procedures.

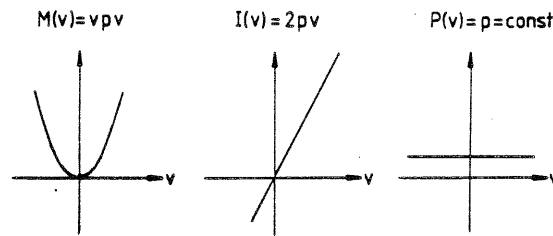


Fig. 1. Least squares: minimum function, influence function and weight function

Logically erroneous observations have to be handled with reduced weights and can not be treated with the same weights as errorfree data. All the observations must be introduced into the adjustment with weights, chosen in correspondence with their errors. The problem of locating gross errors is therefore identical with the determination of proper weights for the observations.

An alternative to least squares is the minimum norm method (Huber, 1981). Thereby the weights of the observations are progressively determined in an iterative process. After each iteration step, new weights for the observations are calculated as a function of the residuals with $P(v) = \frac{1}{|v|}$. The influence function shows, that after convergency of the procedure the influence of all the observations onto the result is equal. Observations with gross errors have the same influence onto the result as errorfree data. This is better than with least squares but still not sufficient.

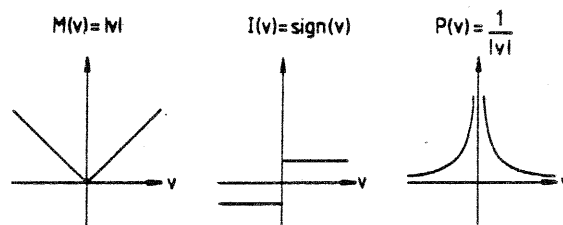


Fig. 2. Minimum Norm: minimum function, influence function and weight function

An adjustment procedure using weight functions eliminating the influence of gross errors completely is the so-called method of robust estimators (robust against the influence of gross errors) (Krarup, 1980; Kubik, 1984). After convergency of the iterative process proper weights are determined for all observations and erroneous data will get weights approximately equal to zero and will have no influence at all onto the result of the adjustment. Their residuals will show up the true errors. The method of robust estimators can be interpreted as an a posteriori estimation of the variances. Many simple weight functions can be found which meet the conditions of robust estimators, but most of them cover only a small range of gross errors and will fail with the variety of gross errors occurring in practical cases. The reason for the failure in these cases is the assumption of linearity by the robust estimators (Huber, 1981).

4. Weight function for PAT-M43:

Thus, a lot of research was necessary to find a weight function and to develop a procedure which covers the wide range of gross errors, their combinations and the different geometry of photo-

grammetric blocks. (Werner, 1984). Because of their effectiveness the following hyperbolic weight function was chosen for the blockadjustment program PAT-M43:

$$\begin{aligned} P &= P_i \cdot F(v_i, \sigma_{v_i}, Q) \\ &= P_i \cdot \frac{1}{1+(\alpha_i \cdot |v_i|)^d} \end{aligned} \quad (1)$$

in which:

$$\alpha_i = \frac{1}{1.4 \cdot \hat{\sigma}_{v_i}} = \frac{\sqrt{P_i}}{1.4 \cdot \sqrt{r_i} \cdot \hat{\sigma}_0} \quad (2)$$

$$d = 3.5 + \frac{82}{81 + Q^4} \quad (3)$$

$$Q = \frac{\hat{\sigma}_0}{\sigma_{\text{a priori}}} \quad (4)$$

v_i = residual of observation i

P_i = a priori weight of observation i

r_i = local redundancy of observation i

$\hat{\sigma}_{v_i}$ = estimated sigma of the residual v_i

$\hat{\sigma}_0$ = estimated sigma-naught

worth mentioning are two attributes of the weight function expanding the range of gross errors locatable with this function.

The first is the dependence on Q (see formula 3 and 4). At the start of the iteration process the value of Q is relatively big and it will become smaller with convergency. At the end of the procedure Q will reach approximately the value one. Thus the curve of the weight function is flat at the beginning and will become steeper and steeper with the disappearing influence of the gross errors and the final orientation of the models. This attribute of the weight function allows the correction of wrong decisions caused by false 0-approximations of the residuals at the beginning and makes it easier to distinguish between errorfree and erroneous observations at the end of the iteration process.

The second attribute is the dependence on the estimated standard deviation of the particular residual $\hat{\sigma}_{v_i}$ (see formula 2). Even with the simplification of using the value one as local redundancy for all the observations this feature allows the determination of small gross errors in the critical range of localization.

Without any further modifications the localization of locatable gross errors up to $50 \cdot \sigma_0$ cause no problems, even with geometrically very weak configurations, as long as there are still error-free redundant observations.

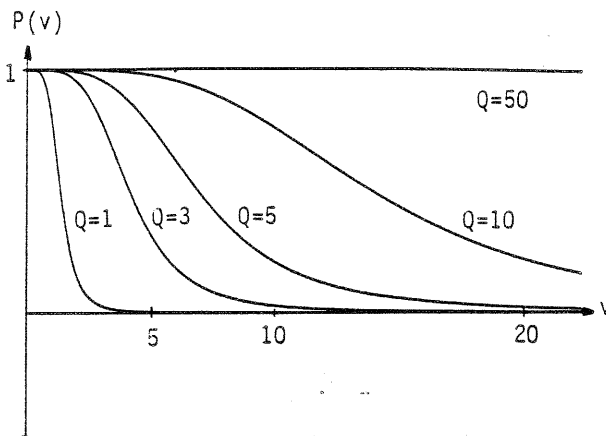


Fig. 3. PAT-M43: weight function

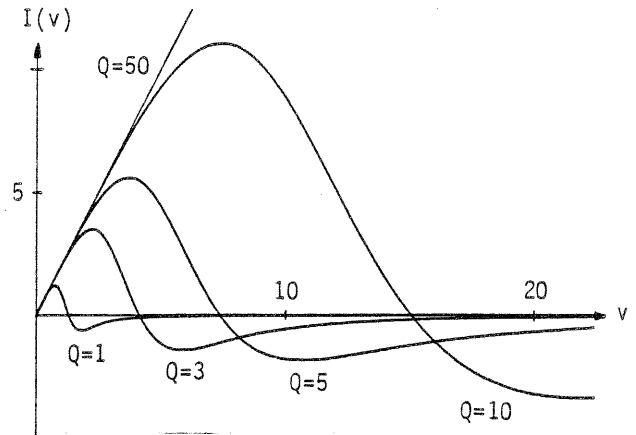


Fig. 4. PAT-M43: influence function

with $P_i=1; r_i=1; \sigma_{a \text{ priori}}=1$ and $Q=1;3;5;10;50$

5. New structure of the program:

If no initial values of the orientation parameters available the program begins with a least squares horizontal adjustment that does not require approximate values. The resulting transformed model coordinates enter into a vertical least squares adjustment using a shift in z only. Thus bigger gross errors in height do not disturb the orientation of the models too much. After this first two iteration steps initial Q -approximations for the residuals are calculated, needed to start consecutive robust iteration steps. Robust estimators can relatively easy be realized using the least squares algorithm and modifying the weights after each iteration step by means of the weight function. Robust iteration steps are repeated until sufficient convergence is reached. The convergence is quite good but highly correlated to the number and the size of gross errors and the geometric stability of the block configuration. Thus the number of iteration steps differs from 6 to 20. If the change of $\hat{\sigma}_0^2$ between two iteration steps becomes less than $2 \cdot \hat{\sigma}(\hat{\sigma}_0^2)$ in planimetry or height after a corresponding iteration step the final elimination of erroneous observations is performed. All observations being used in that iteration step and getting $F(v_i, \hat{\sigma}_{v_i}, Q) < 0.01$ (see formula 1) will be marked as erroneous observations and will get an infinite small weight. The others receive their original a priori weight. Some least squares iteration steps complete the procedure to reach the final result.

Treating errorfree data in a least squares adjustment the favourable sequence of iteration steps is a consecutive alteration between planimetry and height. Handling erroneous data the succession of iteration steps depends on the existing gross errors. That means the succession is data dependent and therefore must be directed by the program itself. The robust iteration steps always begin with horizontal adjustments in order to reduce the influence of the very big gross errors in planimetry because big gross errors in the planimetric coordinates would disturb the levelling of the models completely. Due to the same effect the first robust iteration step in height is using a shift in z only. The sequence of all further iteration steps is chosen properly in order to keep the reduced influence of gross errors in planimetry and height approximately on the same level because of the mutual interference treating erroneous data.

6. Classification of gross errors:

Regarding the different effects of gross errors related to their size we can group them into 3 different classes:

1. small gross errors
2. medium-sized gross errors
3. large gross errors

The classification bounds are not fixed, they depend onto the geometry and may vary for different photogrammetric blocks.

All gross errors greater $4\cdot\sigma$ and less $50\cdot\sigma$ can be designated as small gross errors. They have no significant influence onto the orientation of the models and do not disturb the domain of linearity of the adjustment. Gross errors of the stochastic model and systematic errors are not taken into account but can be considered as small gross errors. Errors less than $4\cdot\sigma$ are integrated within the random errors.

All errors between $50\cdot\sigma$ and 2-3 base lengths belong to the medium-sized gross errors. They have no big influence onto the geometry of the photogrammetric block and don't disturb the convergence of the adjustment but they are not within the range of the linearization and the solution may tend to a different 0-point. Errors bigger than 3 base lengths are named large gross errors. They change the geometry of the block severely and cause worse convergence or even divergence. Especially for blocks with bad geometry the adjustment must be stopped before reaching the point of convergence.

7. Location of small gross errors:

The location of small gross errors poses no problems for the robust adjustment with the chosen weight function. Even small gross errors at the limit of location are detected as long as the observations are sufficiently well distributed within the models.

Only for really worse distributions the consideration of the local redundancy (see formula 2) would improve the effectiveness of the procedure. The check for the inherent limit of localization can be performed only with artificial data. Example 1 shows that the introduced errors greater than the lower limit of 5σ are located without any wrong decision. This lower limit is even better than the theoretical expectation for the statistical test. (R. Schroth, 1980).

Example 2 shows a practical photogrammetric block and is demonstrating the effectiveness of the robust estimators. At first data cleaning has been performed manually and the cleaned data have been submitted to the automatic procedure. Although the residuals after the manual procedure did not indicate remaining gross errors, the automatic procedure located further ones.

8. Modifications of the procedure with respect to medium-sized and large gross errors:

Medium-sized gross errors and all the more large gross errors do not belong to any normal distribution of observations, they are independent from the a priori weights introduced into the adjustment. Thus as long as bigger gross errors have still an influence onto the adjustment all photogrammetric observations are treated with the starting weight 1, used as a priori weight in the weight function. This starting weight tends to the introduced specific a priori weight in dependency on the value of Q by a weight function. The same is true for all non-photogrammetric observations, but for them the starting weight $1/100$ is used. The weight function is as follows:

Weight function for modified a priori weights:

$$P = SW + (P_i - SW) \cdot \frac{37}{36 + (Q-1)^2}$$

in which:

SW = starting weight

P_i = a priori weight

P = modified a priori weight

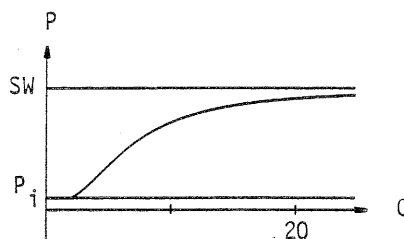


Fig. 5. Modification of a priori weights:
with SW=1; $P_i=0.1$

The ratio of the two starting weights has the effect to reduce the influence of gross errors of photogrammetric observations always a little bit earlier than for control. Thus supports the location of gross errors of control in case of weak control point distributions.

As long as there is no too worse accumulation of medium-sized gross errors in relation to the geometry their location is no problem. But there are two effects to be avoided.

The bigger the gross errors the more falsified are the 0-approximations of the residuals. Sometimes this results in so-called "swimming" models. By means of false 0-approximations the weights for all the observations of a model will be reduced too much in spite of a flat weight function and the model will not be able to get oriented. Nevertheless the calculated weights point to the biggest gross errors.

An other effect is, that after location of the medium-sized gross errors the adjustment approaches a different 0-point and the location of the small gross errors will not be correct. The pre-elimination of large and medium-sized gross errors will solve these problems. As soon as the value of $F(v_i, \hat{\sigma}_{v_i}, Q)$ reaches a certain lower limit the corresponding observation will get the minimum weight for elimination, all other observations receive their a priori weights to start a new robust adjustment. The lower limit of the weight function for preelimination starts with 10^{-18} and is increased for each iteration step by a factor 10 up to the value 10^{-9} . This modification results in a step by step elimination of all larger errors down to gross errors of approximately $50 \cdot \sigma$. Thus "swimming" models will be reincluded into the block and linearity for the final elimination of small gross errors is provided.

Large gross errors may disturb the geometry of the block completely. Already the 0-approximations of the residuals after the starting least squares iteration are such false the point of convergence will not be reached. Therefore large gross errors have to be introduced with already reduced weights into the starting least squares iteration step. The problem can be solved by calculating a center point for each model and the distances to this point for all observations. The ratio of distance and mean distance is used in order to reduce weights by a weight function.

The coordinates of the center point are the arithmetic mean of the coordinates of observations, as long as there are more than 5 observations, otherwise the median is used. The same is true for the mean distance, but the median is used already for 20 and less observations. The calculations are done separately for planimetry and height. The weight functions used are as follows:

$$\text{planimetry: } P = P_i \cdot \frac{256}{256 + R_i^2}$$

$$\text{height: } P = P_i \cdot \frac{81}{81 + R_i^4}$$

in which:

P_i = a priori weight of observation i

$R_i = D_i/D$

D_i = distance of observation i from center point

D = mean distance

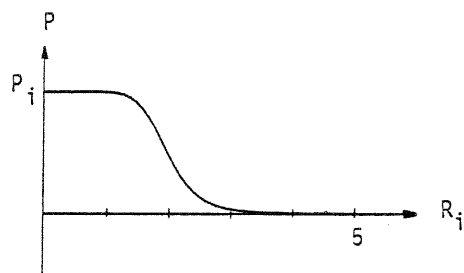


Fig. 6. Modification of weights for the starting least squares planimetric iteration step

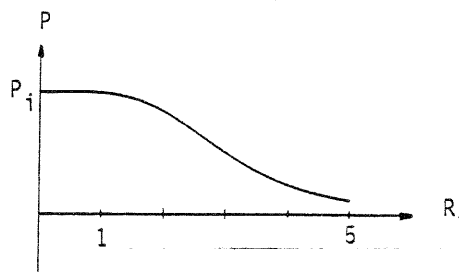


Fig. 7. Modification of weights for the starting least squares height iteration step

The effectiveness of the modifications related to medium sized and large gross errors is shown in Example 3. With the relatively bad geometry the gross error of three base lengths at point 10201 would not be locatable in planimetry without the reduction of the weight in the first least squares iteration step.

9. Reinsertion of observations:

In two cases it is required to reinsert already eliminated observations. Due to falsified 0-approximations it may happen that an observation is wrongly eliminated. After orientation of the model the residuals of this observation will become small and a reinsertion is advisable.

Secondly the result of a least squares adjustment differs from the result of an adjustment with robust estimators in the range of $1-2\sigma$. After the final elimination of the small gross errors at the end of the robust adjustment some iteration steps with least squares are performed and small gross errors just at the limit of localization will tend to the class of random errors in the least squares adjustment and also should be reinserted.

Therefore the weight function (formula 1) is used in the final least squares iteration steps to check for reinsertion of eliminated observations.

During the whole procedure of adjustment, as soon as the value of $F(v_i, \hat{\sigma}_{v_i}, Q)$ becomes bigger than the value 0.01, used for elimination, an already eliminated observation will be reinserted in order to stabilize the geometry of the block respectively to be closer to a final result of adjustment.

10. Conclusion:

The above described procedure of automatic gross errors localization is a specific development for the blockadjustment by the method of independent models and is not transferable to other problems without modifications.

The procedure covers the full range of occurring gross errors from the small ones, just at the limit of localization, up to the big ones with several base lengths and shows the power of robust estimators.

When the worst comes to the worst the procedure results in the elimination of a complete model or in the elimination of observations up to the point no redundancy is remaining in a model and the user has to analyse the observations of the specific model.

In most cases the result of the procedure will be only a proposal, but a very good one, and the person in charge with the project has to judge the proposal and to decide about the final corrections of the gross errors.

The program is in an operational stage and the automatic error detection procedure is easy to handle. No parameters with respect to the procedure have to be changed by the operator, its just the decision whether the adjustment shall be performed with or without automatic error detection.

References

- Ackermann, Ebner/Klein (1970): Ein Programmpaket für die Aerotriangulation mit unabhängigen Modellen, Bildmessung und Luftbildwesen 4/1970.
- Ackermann, Ebner/Klein (1972): Combined Block Adjustment of APR Data and Independent Photogrammetric Models. The canadian surveyor-Vol 26, pp. 384-396.
- Förstner, W. (1978): Die Suche nach groben Fehlern in photogrammetrischen Lageblöcken DGK-C-240, München, 1978.
- Förstner, W. (1982): Results of Test I on Gross Error Detection of ISP WG III/1 and OEEPE. Proceedings of ISPRS-Symposium (Commission III), Helsinki 1982.
- Hampel, F.R.(1973): "Robust Estimation: A condensed Partial Survey". Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete, 1973.
- Huber, P.J. (1981): "Robust Statistics", New York, 1981.
- Krarup/Juhl/Kubik (1980): "Götterdämmerung over Least Squares Adjustment" IAP-Vol. 23-33, Hamburg 1980, pp. 369-378.
- Kubik, K. (1984): Gross Data Errors and Robust Estimation. IAP-Vol. 24-III, Rio de Janeiro 1984.
- Schroth, R. (1980): Innere Zuverlässigkeit von photogrammetrischen Blöcken. Schriftenreihe des Instituts für Photogrammetrie der Universität Stuttgart, Heft 7, pp. 41-58.
- Werner, H. (1984): Automatic Gross Error Detection by Robust Estimators, IAP-Vol. 25-III, Rio de Janeiro 1984.

EXAMPLE 1: EXTRACT FROM PRINTOUT
 ARTIFICIAL BLOCK WITH 32 MODELS
 4 STRIPSS 25 POINTS PER MODEL
 20% SIDELAP
 SCALE= 1/10000
 SIGMA= 10 MICRON
 MODELS IN MICRON
 CONTROL IN METER

SUPERPOSED GROSS ERRORS IN MICRON RESP. METER:

MODEL-NO.	POINT-NO.	DX	DY	DZ
101	20203	80.000	0.000	-80.000
102	10203	0.000	-100.000	100.000
107	30403	30.000	0.000	30.000
304	50503	0.000	-30.000	-30.000
402	80203	-70.000	70.000	100.000
404	80503	-40.000	0.000	-40.000
407	20703	0.000	50.000	50.000
407	70303	30.000	0.000	30.000
HC	10103	-1.000	0.000	
HC	50103	-0.300	0.000	
HC	50903	0.000	-0.500	
HC	90903	0.500	0.500	
VC	10103			-1.000
VC	50103			-0.300
VC	50903			0.500
VC	90903			0.500

END OF ERROR DETECTION IN ELEVATION
 SIGMA REACHED = 3.6602

MODEL	POINT	POINT-NO.	TYPE	VZ	STATUS
MODEL	101	POINT	10103	HV 1	VZ= 0.955 ELIMINATED IN HEIGHT
MODEL	401	POINT	30203	TP 2	VZ= 0.469 ELIMINATED IN HEIGHT
MODEL	102	POINT	10203	TP 2	VZ= 0.799 ELIMINATED IN HEIGHT
MODEL	102	POINT	20203	TP 2	VZ= 0.392 ELIMINATED IN HEIGHT
MODEL	207	POINT	30701	TP 4	VZ= 0.617 ELIMINATED IN HEIGHT
MODEL	208	POINT	50903	HV 2	VZ= 0.467 ELIMINATED IN HEIGHT
MODEL	308	POINT	50903	HV 2	VZ= 0.451 ELIMINATED IN HEIGHT
MODEL	408	POINT	90903	HV 1	VZ= 0.521 ELIMINATED IN HEIGHT

END OF ERROR DETECTION IN PLANIMETRY
 SIGMA REACHED = 3.3794

MODEL	POINT	POINT-NO.	TYPE	VXY	STATUS
MODEL	101	POINT	10103	HV 1	VXY= 1.052 ELIMINATED IN PLANIMETRY
MODEL	401	POINT	30203	TP 2	VXY= 0.353 ELIMINATED IN PLANIMETRY
MODEL	102	POINT	10203	TP 2	VXY= 0.900 ELIMINATED IN PLANIMETRY
MODEL	102	POINT	20203	TP 2	VXY= 0.586 ELIMINATED IN PLANIMETRY
MODEL	207	POINT	30701	TP 4	VZ= 0.421 RE-INSERTED IN HEIGHT
MODEL	208	POINT	50903	HV 2	VZ= 0.470 RE-INSERTED IN HEIGHT
MODEL	308	POINT	50903	HV 2	VZ= 0.454 RE-INSERTED IN HEIGHT
MODEL	408	POINT	90903	HV 1	VXY= 0.702 ELIMINATED IN PLANIMETRY

TRANSFORMED PHOTOGRAMMETRIC MODEL COORDINATES AND RESIDUALS
 (IN UNITS OF THE TERRAIN SYSTEM)

MODEL NUMBER	101	SC=	99.99699					
10100	-0.210	399.974	1500.073	PC 1				0
10101	0.083	-0.092	-0.040	HV 1	-0.041	0.023	0.037	0
10103	0.033	-0.092	-0.040	HV 1 -> HV 0/ 0	-1.352*	0.029*	-0.953*	30
10200	900.252	399.988	1500.079	PC 2	-0.037	-0.017	0.024	0
10201	900.042	-0.023	0.230	TP 2	0.022	0.026	0.008	0
10203	900.042	-0.023	0.230	TP 2 -> SP 1/ 1				0
20101	0.044	399.990	0.038	SP 1				0
20201	900.184	900.180	0.236	TP 2	0.051	-0.097	-0.109	0
20203	900.933	900.180	-0.514	TP 2 -> SP 1/ 1				0
30101	-0.057	1300.031	-0.027	VE 2	0.040	-0.011	-0.022	0
30201	900.113	1799.923	0.076	TP 4	-0.071	0.058	0.061	0
MODEL NUMBER	401	SC=	100.00412					
40100	-0.165	6300.204	1499.734	PC 1				0
40200	399.350	6299.356	1499.353	PC 2	-0.034	0.144	0.091	0
70101	-0.137	5400.061	0.059	VE 2	-0.054	0.030	-0.005	0
70201	399.356	5400.133	-0.156	TP 4	0.056	0.021	-0.003	0
30101	-0.246	6299.994	-0.137	SP 1				0
30201	399.326	6300.090	0.027	TP 2	-0.017	-0.034	-0.135	0
30203	399.326	6300.090	0.027	TP 2 -> SP 1/ 1	-0.735*	0.593*	0.750*	30
90101	-0.029	7200.133	-0.054	HV 1	0.007	-0.042	0.019	0
90201	399.322	7200.103	-0.234	TP 2	0.003	0.045	0.033	0
MODEL NUMBER	102	SC=	100.00261					
10200	900.178	399.954	1500.127	PC 2	0.037	0.017	-0.024	0
10201	900.087	0.024	0.246	TP 2	-0.022	-0.026	-0.007	0
10203	900.087	-0.076	1.246	TP 2 -> SP 1/ 1	-0.045*	0.948*	-1.017*	30
10300	1300.161	900.033	1500.126	PC 2	-0.077	-0.093	-0.006	0
10301	1300.117	0.103	-0.022	TP 2	0.033	-0.006	0.077	0
20201	900.289	399.987	0.069	TP 2	-0.051	0.096	0.107	0
20203	900.289	399.987	0.069	TP 2 -> SP 1/ 1	0.699*	0.193*	-0.583*	30
20301	1300.147	399.334	0.412	TP 2	0.007	0.023	-0.187	0
30201	900.023	1300.073	0.196	TP 4	0.013	-0.092	-0.057	0
30301	1300.119	1799.971	0.034	TP 4	0.019	0.000	0.093	0
MODEL NUMBER	403	SC=	99.99940					
40300	6299.539	6299.904	1499.837	PC 2	0.142	0.147	-0.020	0
40300	7199.497	6299.310	1500.097	PC 1				0
70301	6300.100	5399.757	-0.070	TP 4	-0.013	0.036	-0.052	0
70303	6300.100	5399.757	-0.070	TP 4	0.062	0.036	0.017	0
70901	7200.106	5399.373	0.023	VE 2	0.013	-0.065	0.020	0
30301	6300.189	6299.996	-0.238	TP 2	-0.063	-0.022	-0.019	0
30901	7200.172	6299.611	0.050	SP 1				0
90301	6300.195	7199.889	-0.429	TP 2	0.027	0.008	0.021	0
90901	7200.136	7199.983	-0.071	HV 1	-0.022	0.007	0.029	0
90903	7200.136	7199.983	-0.071	HV 1 -> HV 0/ 0	0.473*	0.509*	0.537*	30

EXAMPLE 2: EXTRACT FROM PRINTOUT:

 PRACTICAL BLOCK WITH 32 MODELS
 4 STRIPS; *18 POINTS PER LAP
 SCALE= 1/28000; 20% SIDELAP
 SIGMA PLANIMETRY = 5.6 MICRON
 SIGMA HEIGHT = 9.3 MICRON
 MODELS IN 1/100 MM
 CONTROL IN METER

ADJUSTMENT WITH AUTOMATIC ERROR DETECTION AFTER MANUAL DATA CLEANING

 ITERATION STEP 9.....VERTICAL ADJUSTMENT

ITERATION STEP FOR ERROR DETECTION

END OF ERROR DETECTION IN ELEVATION
 SIGMA REACHED = 0.5511

VERTICAL CONTROL POINT	15401	HV 2	VZ=	0.554	ELIMINATED
VERTICAL CONTROL POINT	16202	HV 2	VZ=	0.405	ELIMINATED
VERTICAL CONTROL POINT	21101	HV 1	VZ=	0.544	ELIMINATED
VERTICAL CONTROL POINT	21201	HV 2	VZ=	1.494	ELIMINATED
VERTICAL CONTROL POINT	31201	HV 4	VZ=	0.435	ELIMINATED
VERTICAL CONTROL POINT	36201	HV 2	VZ=	0.921	ELIMINATED
MODEL 112111 POINT 6211 TP 2			VZ=	0.381	ELIMINATED IN HEIGHT
MODEL 212211 POINT 16111 TP 4			VZ=	0.352	ELIMINATED IN HEIGHT
MODEL 212211 POINT 26111 TP 4			VZ=	0.799	ELIMINATED IN HEIGHT
MODEL 212211 POINT 26112 TP 4			VZ=	1.008	ELIMINATED IN HEIGHT
MODEL 212211 POINT 26211 TP 4			VZ=	0.344	ELIMINATED IN HEIGHT
MODEL 212211 POINT 26212 TP 4			VZ=	1.074	ELIMINATED IN HEIGHT
MODEL 312311 POINT 36212 TP 4			VZ=	0.397	ELIMINATED IN HEIGHT
MODEL 211210 POINT 21001 HV 2			VZ=	0.363	ELIMINATED IN HEIGHT
MODEL 411410 POINT 41000 PC 2 VXY= 1.365					ELIMINATED IN PLANIMETRY
MODEL 411410 POINT 46101 HO 2			VZ=	0.945	ELIMINATED IN HEIGHT
MODEL 310309 POINT 26011 TP 4			VZ=	0.350	ELIMINATED IN HEIGHT
MODEL 410409 POINT 30802 HV 4			VZ=	0.382	ELIMINATED IN HEIGHT
MODEL 109108 POINT 10300 PC 2 VXY= 1.709					ELIMINATED IN PLANIMETRY
MODEL 109108 POINT 10900 PC 2 VXY= 1.458					ELIMINATED IN PLANIMETRY
MODEL 207206 POINT 20612 TP 2			VZ=	0.786	ELIMINATED IN HEIGHT
MODEL 306305 POINT 35611 TP 4			VZ=	0.905	ELIMINATED IN HEIGHT

 ITERATION STEP 10.....HORIZONTAL ADJUSTMENT

ITERATION STEP FOR ERROR DETECTION

END OF ERROR DETECTION IN PLANIMETRY
 SIGMA REACHED = 0.3678

HORIZONTAL CONTROL POINT	46101	HO 2	VXY=	0.911	ELIMINATED
VERTICAL CONTROL POINT	31201	HV 4			RE-INSERTED
MODEL 213212 POINT 16311 TP 2			VXY=	1.511	ELIMINATED IN PLANIMETRY
MODEL 213212 POINT 16312 TP 2			VXY=	1.548	ELIMINATED IN PLANIMETRY
MODEL 112111 POINT 6211 TP 2					RE-INSERTED IN HEIGHT
MODEL 112111 POINT 16101 HV 2			VXY=	0.715	ELIMINATED IN PLANIMETRY
MODEL 212211 POINT 16111 TP 4					RE-INSERTED IN HEIGHT
MODEL 212211 POINT 16201 HV 3			VXY=	0.793	ELIMINATED IN PLANIMETRY
MODEL 212211 POINT 16211 TP 4			VXY=	0.303	ELIMINATED IN PLANIMETRY
MODEL 212211 POINT 16212 TP 4			VXY=	0.308	ELIMINATED IN PLANIMETRY
MODEL 212211 POINT 26111 TP 4					RE-INSERTED IN HEIGHT
MODEL 212211 POINT 26112 TP 4			VZ=	0.942	RE-INSERTED IN HEIGHT
MODEL 212211 POINT 26211 TP 4			VZ=	1.003	RE-INSERTED IN HEIGHT
MODEL 212211 POINT 26212 TP 4			VZ=	0.820	RE-INSERTED IN HEIGHT
MODEL 212211 POINT 26212 TP 4			VZ=	0.962	RE-INSERTED IN HEIGHT
MODEL 212211 POINT 26212 TP 4			VXY=	0.760	ELIMINATED IN PLANIMETRY
MODEL 312311 POINT 36212 TP 4			VZ=	0.399	RE-INSERTED IN HEIGHT
MODEL 211210 POINT 21001 HV 2			VZ=	0.665	RE-INSERTED IN HEIGHT
MODEL 411410 POINT 46101 HO 2			VZ=	0.955	RE-INSERTED IN HEIGHT
MODEL 310309 POINT 26011 TP 4			VZ=	0.363	RE-INSERTED IN HEIGHT
MODEL 310309 POINT 26011 TP 4			VXY=	0.984	ELIMINATED IN PLANIMETRY
MODEL 310309 POINT 26012 TP 4			VXY=	0.362	ELIMINATED IN PLANIMETRY
MODEL 410409 POINT 30802 HV 4					RE-INSERTED IN HEIGHT
MODEL 308307 POINT 25711 TP 4			VXY=	0.342	ELIMINATED IN PLANIMETRY
MODEL 207206 POINT 20612 TP 2					RE-INSERTED IN HEIGHT
MODEL 207206 POINT 25711 TP 4			VXY=	0.922	ELIMINATED IN PLANIMETRY
MODEL 207206 POINT 25712 TP 3			VXY=	0.990	ELIMINATED IN PLANIMETRY
MODEL 306305 POINT 35611 TP 4			VZ=	0.902	RE-INSERTED IN HEIGHT

 ITERATION STEP 11.....VERTICAL ADJUSTMENT

HORIZONTAL CONTROL POINT	46101	HO 2	VXY=	0.933	RE-INSERTED
VERTICAL CONTROL POINT	21101	HV 1			RE-INSERTED
MODEL 112111 POINT 16101 HV 2			VXY=	0.719	RE-INSERTED IN PLANIMETRY
MODEL 212211 POINT 16201 HV 3			VXY=	0.394	RE-INSERTED IN PLANIMETRY
MODEL 212211 POINT 16211 TP 4			VXY=	0.736	RE-INSERTED IN PLANIMETRY
MODEL 212211 POINT 16212 TP 4			VXY=	0.309	RE-INSERTED IN PLANIMETRY
MODEL 212211 POINT 26212 TP 4			VXY=	0.762	RE-INSERTED IN PLANIMETRY
MODEL 308307 POINT 25711 TP 4			VXY=	0.345	RE-INSERTED IN PLANIMETRY

 ITERATION STEP 12.....HORIZONTAL ADJUSTMENT

VERTICAL CONTROL POINT	15401	HV 2	VZ=	0.521	RE-INSERTED
MODEL 207206 POINT 25711 TP 4			VXY=	0.556	RE-INSERTED IN PLANIMETRY
MODEL 207206 POINT 25712 TP 3			VXY=	0.611	RE-INSERTED IN PLANIMETRY
MODEL 310309 POINT 26011 TP 4			VXY=	0.365	RE-INSERTED IN PLANIMETRY
MODEL 310309 POINT 26012 TP 4			VXY=	0.372	RE-INSERTED IN PLANIMETRY

TRANSFORMED PHOTOGRAMMETRIC MODEL COORDINATES AND RESIDUALS
 (IN UNITS OF THE TERRAIN SYSTEM)

MODEL NUMBER	213212				SC=	3.53452		
16201	19985.406	41662.499	657.137	HV 3	-0.001	-0.076	0.295	0
16202	20170.993	43717.534	658.197	HV 2 -> HQ 2/ 2	-0.205	0.001	-0.143	0
16211	20987.462	41513.197	650.274	TP 4	0.005	-0.157	-0.037	0
16212	20957.464	41513.737	650.638	TP 4	0.127	0.037	-0.156	0
16311	18519.591	41480.567	629.558	TP 2 -> TP 1/ 2	-1.079*	0.266*	0.013	20
16312	18469.311	41482.252	632.152	TP 2 -> TP 1/ 2	-1.212*	0.253*	-0.152	20
21200	21100.479	43821.375	4981.772	PC 2	0.309	0.113	-0.002	0
21201	19044.651	46404.112	633.600	HV 2 -> HQ 2/ 2	-0.016	-0.013	-0.112	0
21211	21052.156	43987.740	624.532	TP 2	0.004	0.004	0.194	0
21212	21022.210	43989.129	625.909	TP 2	0.115	0.049	0.360	0
21300	18512.329	43865.191	4982.950	PC 1				0
21311	18457.043	43996.031	650.857	SP 1				0
21312	18427.154	44000.547	650.543	SP 1				0
26211	21062.232	46492.155	600.110	TP 4	-0.156	0.014	0.099	0
26212	21032.230	46492.295	601.323	TP 4	-0.100	-0.025	0.095	0
26311	19507.663	46547.361	621.303	TP 2	0.111	0.055	-0.100	0
26312	18477.640	46549.305	621.239	TP 2	0.217	0.033	-0.003	0
MODEL NUMBER	411410				SC=	3.46565		
31101	23425.562	50695.996	597.317	HQ 4	-0.379	-0.024	0.023	0
36001	25669.497	52144.509	606.611	HV 4	0.065	0.123	-0.083	0
36011	26016.967	51446.019	602.439	TP 4	0.173	0.220	0.083	0
36012	25980.920	51446.310	602.578	TP 4	0.110	0.054	-0.072	0
36101	23350.134	53669.929	570.693	HQ 2	-0.366	-0.102	-0.363	0
36111	23303.741	51350.075	592.963	TP 4	-0.170	0.029	0.120	0
36112	23473.653	51337.030	593.553	TP 4	-0.125	0.057	0.245	0
36113	23570.230	51613.751	598.591	TP 4	-0.218	-0.338	-0.361	0
36114	23559.336	51641.649	598.430	TP 4	-0.153	-0.013	0.045	0
41000	26066.920	53965.705	5040.819	PC 2 -> PC 1/ 2	-0.097*	1.903*	0.025	20
41001	24822.165	56248.214	535.216	HV 1	0.135	-0.080	-0.095	0
41011	26011.589	54037.733	591.393	TP 2	0.024	0.234	0.352	0
41012	25931.642	54033.412	590.659	TP 2	0.037	0.187	0.375	0
41100	23577.763	53996.041	5039.734	PC 2	-0.091	-0.046	0.051	0
41111	23505.941	54014.470	570.090	TP 2	0.043	-0.029	-0.066	0
41112	23477.253	54023.282	570.016	TP 2	-0.020	-0.081	-0.070	0
46011	26043.236	56582.362	547.764	TP 2	0.010	0.024	-0.116	0
46012	26043.799	56612.913	547.332	TP 2	-0.001	-0.062	0.020	0
46101	23037.223	56909.700	608.497	HQ 2	0.119	-0.404	0.411	0
46131	23790.150	56277.211	597.472	TP 2	0.021	0.112	-0.128	0
46132	23709.480	56324.343	601.201	TP 2	0.097	0.093	-0.057	0

MODEL NUMBER	109108				SC=	3.51248		
5701	31761.375	36832.748	583.670	HV 2	0.003	0.252	0.131	0
5702	31224.345	38422.309	581.233	HV 2	-0.333	0.031	-0.119	0
5801	30639.372	36334.185	537.957	HV 2	0.065	0.030	0.179	0
5311	30992.108	36477.780	579.263	TP 2	0.043	-0.024	-0.126	0
5812	30962.101	36479.483	580.450	TP 2	0.369	-0.103	-0.112	0
5911	28582.380	36477.794	538.360	TP 2	-0.133	-0.235	-0.270	0
5912	28553.043	36480.071	539.198	TP 2	-0.367	-0.136	-0.093	0
10800	31120.157	38970.670	4939.630	PC 2 -> PC 1/ 2	0.255*	2.045*	-0.095	20
10801	29397.694	39347.593	592.489	HV 1	0.054	0.096	0.260	0
10811	30977.247	39023.096	589.533	TP 2	0.049	0.169	0.103	0
10900	28607.330	38919.483	4936.889	PC 2 -> PC 1/ 2	0.513*	0.722*	0.011	20
10901	28475.439	40359.040	609.879	HV 2	-0.395	0.249	-0.320	0
10911	28331.137	38379.300	595.718	TP 2	-0.395	0.024	0.261	0
15301	29374.312	41539.693	539.156	HV 2	0.395	-0.122	0.395	0
15311	31031.536	41533.327	561.291	TP 4	-0.253	-0.049	-0.363	0
15312	31001.966	41537.327	561.320	TP 4	-0.340	-0.294	0.329	0
15911	28492.602	41207.073	566.639	TP 4	0.106	-0.037	0.010	0
15912	28495.156	41236.360	567.405	TP 4	0.061	0.079	0.043	0

MODEL NUMBER	109108				SC=	3.51248		
5701	31761.375	36832.748	583.670	HV 2	0.003	0.252	0.131	0
5702	31224.345	38422.309	581.233	HV 2	-0.333	0.031	-0.119	0
5801	30639.372	36334.185	537.957	HV 2	0.065	0.030	0.179	0
5311	30992.108	36477.780	579.263	TP 2	0.043	-0.024	-0.126	0
5812	30962.101	36479.483	580.450	TP 2	0.369	-0.103	-0.112	0
5911	28582.380	36477.794	538.360	TP 2	-0.133	-0.235	-0.270	0
5912	28553.043	36480.071	539.198	TP 2	-0.367	-0.136	-0.093	0
10800	31120.157	38970.670	4939.630	PC 2 -> PC 1/ 2	0.255*	2.045*	-0.095	20
10801	29397.694	39347.593	592.489	HV 1	0.054	0.096	0.260	0
10811	30977.247	39023.096	589.533	TP 2	0.049	0.169	0.103	0
10900	28607.330	38919.483	4936.889	PC 2 -> PC 1/ 2	0.513*	0.722*	0.011	20
10901	28475.439	40359.040	609.879	HV 2	-0.395	0.249	-0.320	0
10911	28331.137	38379.300	595.718	TP 2	-0.395	0.024	0.261	0
15301	29374.312	41539.693	539.156	HV 2	0.395	-0.122	0.395	0
15311	31031.536	41533.327	561.291	TP 4	-0.253	-0.049	-0.363	0
15312	31001.966	41537.327	561.320	TP 4	-0.340	-0.294	0.329	0
15911	28492.602	41207.073	566.639	TP 4	0.106	-0.037	0.010	0
15912	28495.156	41236.360	567.405	TP 4	0.061	0.079	0.043	0

VERTICAL CONTROL POINTS

5701		583.800	HV 2	0.001	2
5702		581.100	HV 2	0.065	2
5801		538.200	HV 2	-0.364	2
5901		602.800	HV 1	0.062	2
10401		548.900	HV 1	0.027	2
10501		592.800	HV 1	-0.062	2
10901		609.600	HV 2	-0.043	2
10902		583.400	HV 2	-0.033	2
15703		587.400	HV 1	-0.015	2
15301		589.300	HV 2	-0.346	2
15901		578.600	HV 4	-0.055	2
16102		623.900	HV 1	0.261	2
16201		657.600	HV 3	-0.113	2
16202		658.600	HV 2 -> HQ 2/ 2	-0.551*	12
20501		572.200	HV 1	0.037	2
20901		592.800	HV 2	0.154	2
21001		588.900	HV 2	-0.003	2
21101		629.800	HV 1	-0.026	2
21201		635.100	HV 2 -> HQ 2/ 2	-1.612*	12
25501		553.900	HV 2	-0.093	2
25801		609.400	HV 4	-0.051	2
35901		537.700	HV 2	0.216	2
36001		606.600	HV 4	-0.072	2
36201		577.300	HV 2 -> HQ 2/ 2	-1.034*	12
40501		541.000	HV 1	-0.323	2
40502		565.900	HV 2	-0.038	2
40601		572.200	HV 2	0.044	2

EXAMPLE 3: EXTRACT FROM PRINTOUT:

 ARTIFICIAL BLOCK WITH 32 MODELS
 4 STRIPS; 6 POINTS PER MODEL
 SCALE= 1/10000; 20% SIDELAP
 SIGMA= 10 MICRON
 MODELS IN MICRON
 CONTROL IN METER

SUPERPOSED GROSS ERRORS IN BASELENGTH:

MODEL-NO.	POINT-NO.	DX	DZ
101	10201	3	3
406	90701	3	3
HVC	10901	3	3

 ITERATION STEP 5.....HORIZONTAL ADJUSTMENT

ITERATION STEP FOR ERROR DETECTION

HORIZONTAL CONTROL POINT	MODEL	POINT	10901	HV 1	VXY= 2699.649	ELIMINATED
	101	POINT	10201	TP 2	VXY= 2704.637	ELIMINATED IN PLANIMETRY
	406	POINT	90701	TP 2	VXY= 2702.266	ELIMINATED IN PLANIMETRY

 ITERATION STEP 6.....VERTICAL ADJUSTMENT

ITERATION STEP FOR ERROR DETECTION

VERTICAL CONTROL POINT	MODEL	POINT	10901	HV 1	VZ= 2699.824	ELIMINATED
	101	POINT	10201	TP 2	VZ= 2695.110	ELIMINATED IN HEIGHT
	406	POINT	90701	TP 2	VZ= 2701.269	ELIMINATED IN HEIGHT

 ITERATION STEP 10.....VERTICAL ADJUSTMENT

ITERATION STEP FOR ERROR DETECTION

END OF ERROR DETECTION IN ELEVATION
 SIGMA REACHED = 9.3370

MODEL	101	POINT	30201	TP 4	VZ= 0.537	ELIMINATED IN HEIGHT
MODEL	102	POINT	10200	PC 2	VZ= 0.632	ELIMINATED IN HEIGHT
MODEL	102	POINT	20201	TP 2	VZ= 3.806	ELIMINATED IN HEIGHT

 ITERATION STEP 12.....VERTICAL ADJUSTMENT

MODEL	101	POINT	30201	TP 4	VZ= 0.346	RE-INSERTED IN HEIGHT
MODEL	102	POINT	10200	PC 2	VZ= 0.405	RE-INSERTED IN HEIGHT
MODEL	102	POINT	20201	TP 2	VZ= 0.286	RE-INSERTED IN HEIGHT

 TRANSFORMED PHOTOGRAMMETRIC MODEL COORDINATES AND RESIDUALS

 (IN UNITS OF THE TERRAIN SYSTEM)

MODEL NUMBER	101					SC= 100.00530		
10100	0.281	899.692	1499.976	PC 1			0.016	0
10101	-0.012	-0.058	-0.031	HV 1	0.036	0.010	0.016	0
10200	900.763	900.162	1499.852	PC 2	-0.043	0.042	0.035	0
10201	2309.769	1909.167	2699.508	TP 2 -> SP 1/ 1	-1909.911*	-1909.243*	-2699.317*	30
20101	0.215	900.059	0.054	SP 1				0
20201	900.054	899.941	-0.126	TP 2	-0.010	-0.008	-0.050	0
30101	0.356	1300.024	-0.052	VE 2	-0.043	0.051	0.016	0
30201	900.113	1800.225	-0.170	TP 4	0.023	-0.054	0.019	0

MODEL NUMBER	406					SC= 100.00953		
40600	4500.055	6299.907	1500.049	PC 2	-0.025	0.159	0.061	0
40700	5399.963	6300.152	1500.020	PC 2	0.031	0.008	0.096	0
70601	4500.007	5400.016	0.144	TP 4	-0.043	0.059	-0.059	0
70701	5399.796	5400.075	0.074	TP 4	0.091	-0.058	-0.006	0
30601	4500.043	6299.955	0.223	TP 2	-0.003	0.014	-0.033	0
80701	5399.800	6299.942	0.239	TP 2	-0.049	0.025	-0.067	0
90601	4499.928	7200.099	0.137	TP 2	0.009	-0.040	0.005	0
90701	7309.120	9109.984	2699.622	TP 2 -> SP 1/ 1	-1909.235*	-1909.934*	-2699.713*	30

 CONTROL POINT COORDINATES AND RESIDUALS

 (IN UNITS OF THE TERRAIN SYSTEM)

HORIZONTAL CONTROL POINTS

10101	0.033	-0.045		HV 1	-0.009	-0.003		2
10501	3600.080	-0.022		HV 2	-0.007	0.026		2
10901	9109.957	1909.005		HV 1 -> SP 1/ 1	-1910.117*	-1908.763*		22
50101	-0.037	3600.061		HV 2	-0.009	-0.031		2
50901	7200.012	3600.000		HV 2	0.010	0.002		2
90101	-0.091	7199.952		HV 1	0.012	0.005		2
90501	3599.979	7200.067		HV 2	-0.001	0.007		2
90901	7200.001	7199.981		HV 1	-0.003	-0.007		2

VERTICAL CONTROL POINTS

10101		-0.009		HV 1		-0.004		2
10501		0.027		HV 2		0.004		2
10901		2700.038		HV 1 -> SP 1/ 1		-2700.153*		12
30101		-0.054		VE 2		0.009		2
30501		0.024		VE 4		-0.001		2
30901		0.003		VE 2		-0.007		2
50101		-0.018		HV 2		0.003		2
50501		-0.072		VE 4		-0.012		2
50901		0.065		HV 2		0.003		2
70101		0.032		VE 2		-0.010		2
70501		0.000		VE 4		0.007		2