XV Congress of the International Society for
Photogrammetry and Remote Sensing
Rio de Janeiro 1984

Commission III

INVITED PAPER

MODELLING AND CLASSIFYING TERRAIN

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Abstract

Different methods for terrain description and classification are presented. Descriptive classifications are known from geology. These descriptions have also been formalized using the concept of stochastic processes. Other classification methods are based on Fourier-transformation, classifying the terrain types according to their frequencies of undulations. Recent approaches are based on the concept of selfsimilarity, observing the presence or lack of similarity of micro- and macro structures in the terrain. A comparison of these classification models is given and the applicability of these models for interpolation and other deductions is discussed.

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1. DIGITAL TERRAIN MODELS REQUIRE MODELLING

"I should see the garden far better", said Alice to herself, "if I could get to the top of that hill: and here is a path that leads straight to it - at least no, it doesn't do that - but I suppose it will at last. But how curiously it twists! It's more like a cork-skrew than a path! Well this turn goes to the hill, I suppose - no it doesn't. This goes straight back to the house! Well then, I'll try it the other way!"

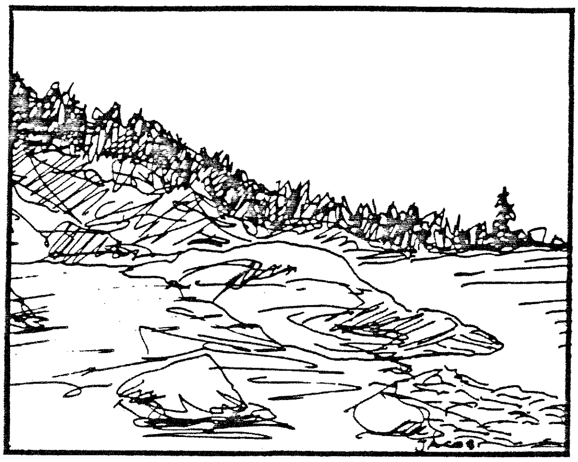
During the last decennium, Digital Terrain Models (DTM) became rather important for various tasks of photogrammetry, such as orthophoto production, high accuracy processing of scanner data and others. The name Digital Terrain Model is however misleading. A DTM is a programme package consisting of routines for data storage, data retrieval, editing, interpolation and contouring. It is often a subsystem of a Land Information System, and is rather an Elevation Information System than a model.

In order to properly understand the behaviour of the terrain forms and their Information Systems, a model of the terrain is necessary. But what is the concept of a model? According to the positivistic school of philosophy the complete understanding of the terrain form and its evolution is impossible for the human mind. We may well describe it with the help of a hypothesis, or a model. Such a model defines observable quantities and relationships between them (Weyl, 1949). The concept of a model allows deductions on the properties of the terrain and on the outcome of new observations. These deductions may refer to the point density necessary for sampling, to the accuracy and quality of interpolation and contouring, or to criteria for the detection of blunders in measurement.

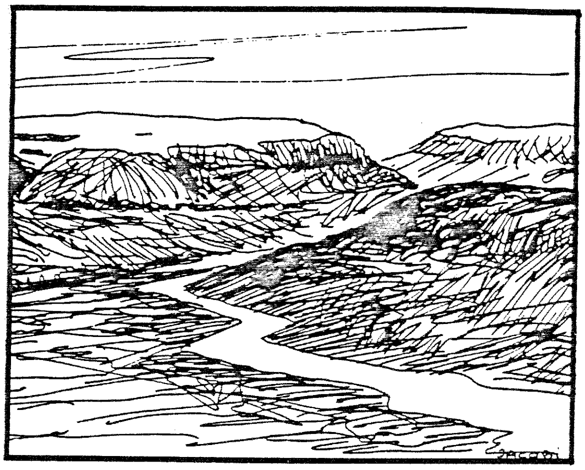
We may define many different models for the terrain. Their value is judged by the criteria of completeness, uniqueness and simplicity. A model should be complete, that is, it should enrich our understanding of the terrain as much as possible and it should not be easily disproveable by results of experiments. The model should be unique in that it allows unique deductions. Finally it should be simple: if two models behave otherwise equally well, the simpler model containing less hypothesis and less quantities is to be preferred.

Models for the terrain have been defined in various sciences. Geography and its subscience Geomorphology supply a vast body of knowledge on terrain forms and - characterization.

Geology and in particular Mathematical Geology, describes the geological processes and also their product, the terrain form. In Applied Mathematics and Statistics terrain is described by various concepts. The following sections give a short review of the major models in use and of their relationship.



Precambrian basement, Norway



Sedimentary rocks, Greenland

Fig. 1. Two types of terrain.

2. TERRAIN MODELS IN GEOMORPHOLOGY

"What's the use of their having names", the Gnat said, "if they won't answer to them".
"No use to them", said Alice; "but it's useful to the people that name them, I suppose. If not; why do things have names at all?"

Geomorphology, a discipline of geography, is concerned with the description of the form of the earth and its genesis. In large scale, geomorphology describes the terrain forms interesting to photogrammetry and their evolution in time.

Terrain forms are classified according to their genesis by folding, erosion, sedimentation and other processes. Terrain forms are also classified by their magnitude and extend, as well as their roughness. These latter entities are related to the genesis of the terrain (Krüger, 1974; King 1966).

Let us, as examples, consider two different types of terrain (Figure 1). Firstly, sedimentary rock landscape in Washington Land, Greenland (G) and secondly precambrian basement landscape south-east of Oslo, Norway (N). The sedimentary rock landscape shows large, relatively smooth land forms, while the Norwegian terrain shows the typical rugged structure of basement rock. Although the sedimentary rocks show considerable larger undulations, both in elevation and extend, they appear relatively more smooth than the Norwegian basement rock.

A considerable number of parameters have been proposed in order to describe the spatial variations of the terrain. The three most important are:

Relief, which describes the vertical dimension of the terrain and which is defined using the extreme values of the elevations. Local Relief is defined as the difference between the highest and lowest elevation with-in a specified area of given extension. In order to eliminate the dependence of the definition on this reference area, the concept of Relative Relief is introduced, by dividing the Local Relief by the extend of the reference area (its diameter or perimeter). Relative Relief is a dimensionless quantity; its dependence on the extend of the reference area and its frequency distribution allows classification of terrain according to roughness and genesis.

Slope is possibly the most important parameter of terrain forms, because it controls the gravitational forces available for geomorphic work (Evans, 1972). The slope is the first derivative of the elevation in arbitrary direction, or in the direction of steepest descent. On a macroscopic scale, slope is defineable at any point of the terrain, except at break lines. Slope SL is either measured as angle or in percent of inclination of triangular facets in a regular grid, or it is measured by derived quantities like the roughness factor RF = 100(1-cos SL), Mark (1975).

Wavelength or extend of the terrain form is the third important parameter here mentioned. The wavelength in a terrain profile is defined as the average distance between successive (local) maxima or minima. This wavelength is measured in length units (meters) and may be studied in various characteristic directions. Its relationship to the magnitude of the terrain forms can be ideally studied by Fourier Transformation (see section 3).

The terrain forms may be classified with respect to these three parameters - relief, slope and wavelength, to their magnitude and their fluctuations and with respect to their mutual relationships. These parameters will be our basic reference when studying other models of the terrain.

3. TERRAIN MODELS FROM FOURIER SPECTRA

"If any of them can explain it", said Alice, "I'll give him sixpence. I don't believe there is an atom of meaning in it". "If there is no meaning in it", said the King, "that saves a world of trouble, you know, as we needn't try to find any".

The terrain may be described in form of a Fourier series, and many of the above parameters may be directly related to the coefficients of the series (Ayeni, 1976; Tempfli and Makarovic, 1979; Frederiksen et al, 1978; Frederiksen, 1981; Tempfli, 1982). Any continuous and continuously differentiable function $Z(x)$, $0 \leq x < L$, can be expanded into a uniformly convergent Fourier series

$$Z(x) = a_o + \Sigma\, a_f \cdot \cos(\frac{2\Pi}{L} \cdot x \cdot f) + \Sigma\, b_f \cdot \sin(\frac{2\Pi}{L} \cdot x \cdot f) \qquad (1)$$

where a_f and b_f are parameters and f is a relative frequency related to the length L of the profile.

This model may be used to describe terrain profiles in function of their sine - and cosine components. Equally, twodimensional terrain may be transformed, using twodimensional Fourier series expansions.

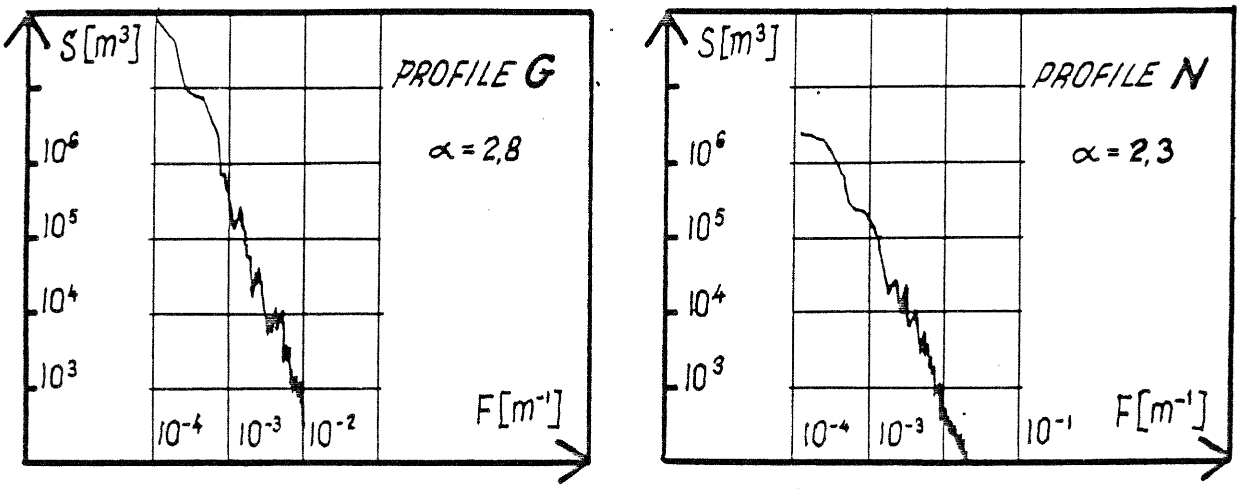


Fig. 2. Fourier-spectra from profiles in Greenland and Norway.

The relationship between a_f, b_f, f and L can be studied by means of the power spectrum S. In order to compensate for the influence of the varying length L of the profile, the spectrum can be written as function of the absolute frequency F = f/L

$$S(F) = S(\frac{f}{L}) = L \cdot (a_f^2 + b_f^2) \quad (2)$$

or as a function of the wavelength $\lambda = \frac{L}{f} = \frac{1}{F}$.

The representation of the terrain in the frequency domain greatly simplifies the separation of various surface forms.

The following model proved to be valid for a large domain of F:

$$S(F) = E \cdot F^{-\alpha} \quad (3)$$

where α and E are characteristic parameters for the terrain (Jacobi, 1980).

The relationship is experimentally verified for our two terrain examples (Fig. 1) and the result is shown in Fig. 2. The average spectrum was computed for a large number of profiles in both areas. Relationship (3) proves valid for wavelengths 1/F ranging from 50 to 10.000 meters. On a double logaritmic scale log S is linearly related to log F. The slope α of this line is significantly larger for the Greenland terrain than for Norway (2.8 versus 2.3). In general, if the slope α of the spectrum is larger than 2.5, the landscape is smooth due to the absence of high amplitudes at high frequencies. On the other hand, a slope less than 2.0 indicates a rough surface with relatively large variations of high frequencies.

Relationship (3) implies that the surface characteristics are independent of the scale of observation. In particular, for α = 2 the landscape looks the same independent of the scale at which it is observed, the amplitudes and wavelengths of the surface details are on the average directly proportional.

Based on this model, the suitability of different interpolation methods can be studied and the accuracy of interpolation and its dependency in the sample spacing can be derived. Jacobi (1980), Frederiksen et al (1978) and Tempfli (1982) proved that the accuracy σ of interpolation is largely independent of the method of interpolation; it depends mainly on the spacing D between the sample points and the surface characteristics α: σ^2 prop $D^{\alpha-1}$.

4. TERRAIN MODELS FROM RANDOM FUNCTIONS

> *"This conversation is going on a little too fast; lets go back to the last remark but one".*

Another class of terrain models is based on random functions. We shall define a random function as a function $Z(x)$, the value of which for any value x of its argument is a random variable. The argument x is considered a non-random argument. This concept may be used to describe the elevations of a terrain profile as a function of the profile length x.

In order to characterize the random function, the knowledge of the density distribution pr(Z) of the values Z for different arguments x is necessary. The random function can be considered to be defined if all multi-dimensional density distributions are given for any values $x_1, x_2, \ldots, x_n$. Although this often can be done, this method is not always convenient. Hence, in the majority of cases we limit ourselves to specify selected parameters of these density distributions. One can choose various quantities as such parameters, however the most convenient are:

- The expectation or mean

$$m(x) = E\{Z(x)\} = \int_{-\infty}^{+\infty} Z \cdot pr(Z|x)\,dZ \qquad (4a)$$

- The variance

$$\sigma^2(x) = Var(Z(x)) = E\{(Z(x) - m(x))^2\} \qquad (4b)$$

and

- The correlation(covariance) function

$$K(x_1, x_2) = E\{(Z(x_1) - m(X_1))(Z(x_2) - m(x_2))\} \qquad (4c)$$

where $E\{\cdot\}$ denotes the mathematical expectation of the argument. The correlation function describes the correlation between the two random variables $Z(x_1)$ and $Z(x_2)$.

A very important property of a random function is the dependence or independence of its distribution function on the origin of x. In accordance with this, stationary and non-stationary random functions are distinguished. For stationary random functions, the mathematical expectation and variance are constant and the covariance function depends only on the difference of the coordinates x_2-x_1, which is also called lag d. A second attribute which is also used as the basis of a classification of random functions is the form of the density distribution function of Z. The distribution law most frequently assumed is the normal law, although this in many cases is not a proper model for elevations.

For a stationary random function, the Fourier Transform of the correlation function K(d) is equal to the power spectrum S(F), (2)(cf. Champeny, 1973). For the model (3) for terrain spectra the correlation function K(d) follows from the inverse Fourier Transform approximately to

$$K(d) = \sigma^2 - c \cdot d^{\beta}; \quad \beta = (\alpha - 1), \tag{5}$$

This model is a suitable model for terrain profiles, in particular if σ^2 is chosen sufficiently large.

Modelling the terrain by the correlation function is in principle possible, but estimation of the variance and the correlation is difficult in praxis. This has two reasons: Firstly, the elevations are not normally distributed; and individual extreme elevations or local flats have undue influence on the estimates. Robust estimation of the variance and correlation is necessary. The fluctuation of the elevations is estimated from their median (Crüger et al, 1984), correlation is estimated by rank correlation methods (Kendall, 1948).

Secondly, long trends or semi-systematic fluctuations seriously distort the estimates. A proper manner to tackle this problem is to estimate the correlation function from visual inspection of the terrain profiles, and to describe large regional terrain forms by an appropriately large correlation length (which may equal or exceed the profile length) and by a correspondingly large variance (cf. Kubik, 1975). Once these parameters are chosen large enough, the deductions in the model become independent of the exact numerical values of these parameters.

The above pitfalls in estimating the correlation function may also be overcome by relaxing either the condition of stationarity or the condition of normal distribution. The notorious long tailedness of most empirical histogrammes may in many cases best be interpreted by accepting the possibility of an infinite variance *). This means in everyday language, that the variance of height fluctuations in the terrain profile increases with the profile length.

In order to avoid this somewhat unmanageable value ∞, we may model the differences of elevations instead of the elevations themselves. De Wijs (1972), Matheron (1971) and Mandelbrot (1982) all proposed the variance of the difference $(Z_i - Z_j)$ for modelling terrain forms, $Var(Z_i - Z_j)$; (6)

This quantity is also called (difference-)variance function, and it depends, for stationary differences, only on $(x_i - x_j) = d$. For a stationary function Z(x), the variance function may be related to the correlation function by

$$Var(d) = 2(\sigma^2 - K(d)). \tag{7}$$

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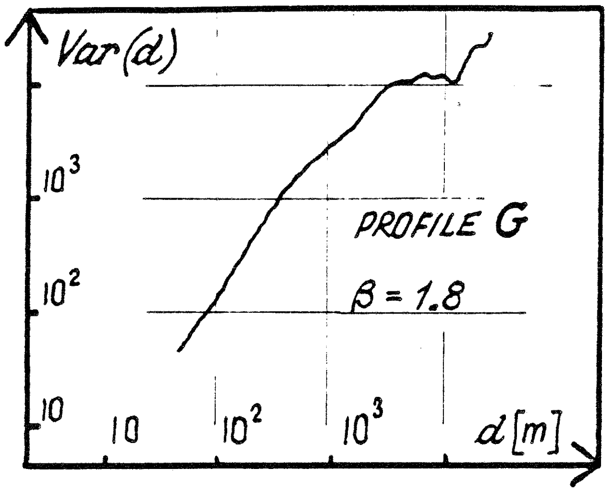
*) Assume, that the distribution function has a tail decreasing slower than $1/Z^2$. Then the integral in (4b) will fail to converge to a finite value. We accepted the idea of a very large variance already before when modelling trends.



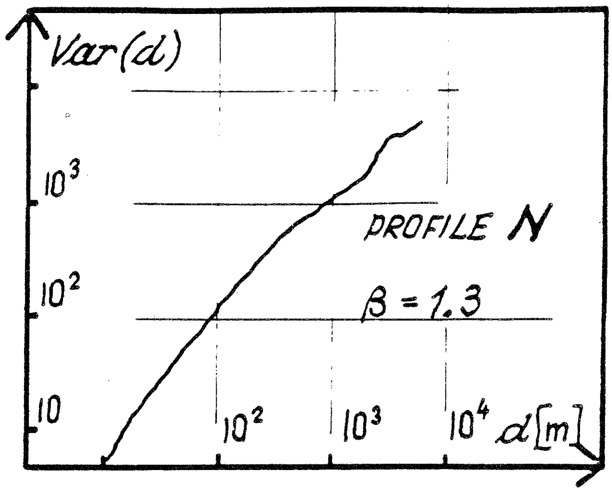


Fig. 3. Variogrammes.

For the terrain model (3), the variance function results to

$$\mathrm{Var}(d) = b \cdot d^{\beta}; \quad b \text{ constant.} \tag{8}$$

Figure 3 shows examples of variogrammes (estimated variance functions) for our terrain example. The variogrammes are plotted on double - logarithmic scale, the slopes of the curves $\beta = 1.3$ and 1.8 are in good agreement with the slopes $\alpha = 2.3$ and 2.8 of the corresponding spectra of figure 2.

The concept of variance function and variogramme is extensively used in the geosciences to model geological, geochemical and geophysical phenomena (de Wijs, 1972; Clark, 1979; David, 1977; Mandelbrot, 1982). In these models it is very important to allow the variance function to tend towards infinity for the lag $d \to \infty$ in order to allow the modelling of trends and large scale forms. This was properly recognized by de Wijs and Mandelbrot, whereas it is forgotten in the other litterature which uses bounded variogramme functions.

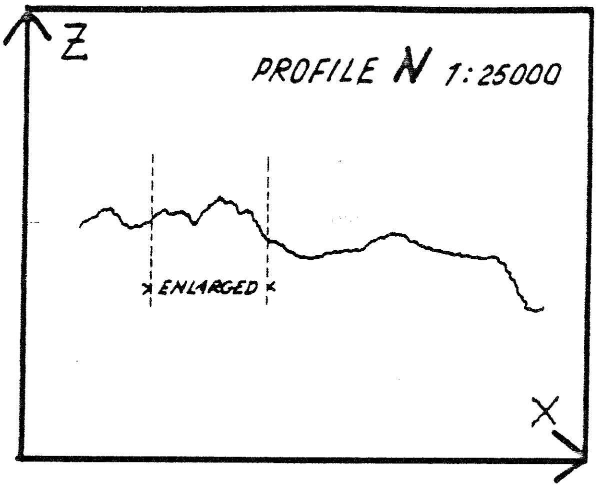
5. SELFSIMILARITY

"It's really dreadful," she muttered to herself, "the way all creatures argue. It's enough to drive one crazy".

The beforenamed terrain models can in an elegant manner also be derived by the concepts of Selfsimilarity and Fractals, which were introduced by the mathematician Mandelbrot (1968, 1969). By studying various natural phenomena, Mandelbrot observed a form of invariance with respect to changes in scale, and he introduced the concept of selfsimilarity to describe this phenomenon. The increments of a random function are said to be selfsimilar, if the increment $Z(x_1+\Delta x) - Z(x_1)$ is in probability equal to the h times larger increment $Z(x_1+h\cdot\Delta x) - Z(x_1)$ divided by the scaling factor h^{γ},

$$\{Z(x_1+\Delta x) - Z(x_1)\} \stackrel{\Delta}{=} \{h^{-\gamma}(Z(x_1+h\cdot\Delta x) - Z(x_1))\}^{*)}; \quad \gamma \text{ constant} \tag{9}$$

*) The symbol $\stackrel{\Delta}{=}$ denotes, that both sides have equal distribution functions.



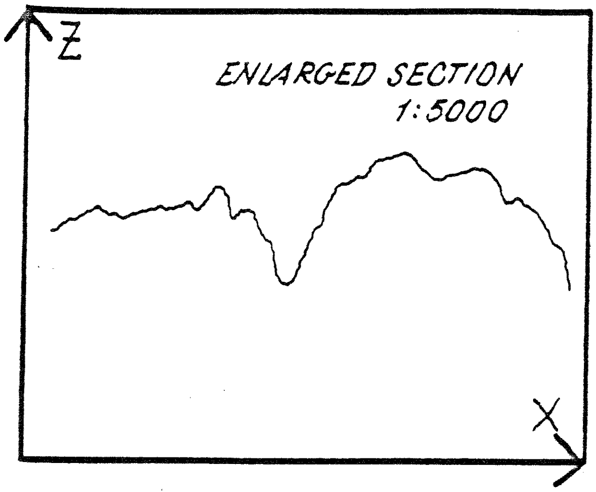


Fig. 4. Selfsimilarity in Norway.

This concept may be illustrated in the Norwegian terrain. Figure 4 shows two drawings of the same profile at different scales. Both these drawings look very similar in their structure and their scale or scale relation cannot be derived by mere inspection of the drawings.

Based on this definition of selfsimilarity, Mandelbrot introduced random functions, called "Fractals", which possess this property and from which - under mild restrictions - the terrain models (3), (5) and (8) can be deduced. These Fractals are in principle deduced by a γ^{th}-summation (integration) of independent random variables. Although we are only familiar with 1^{st}- and 2^{nd} order summations (cf. the error theory of strip triangulation; Vermeir, 1954; Ackermann, 1965), a fractal summation or integration of order γ (γ noninteger) may be defined by extending the classical definition of the n^{th} integral to noninteger values $n = \gamma$ (Holmgreen-Riemann-Liouville fractional integral, cf. Levy, 1953)

$$Z(x) = K(\gamma) \cdot \int_0^x (x-s)^{\gamma-1}\, dB(s) \tag{10}$$

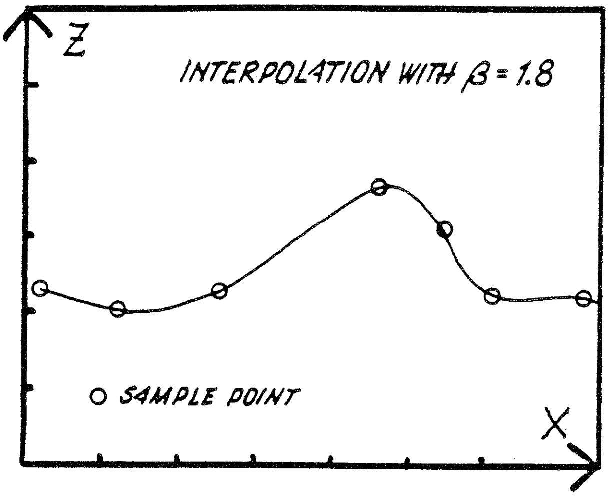
with dB normally distributed, independent and equally accurate increments (white noise) and $K(\gamma)$ a constant depending on γ. The constant γ relates to α and β by $\gamma = \frac{1}{2}(\beta+1) = \frac{1}{2}\,\alpha$. The concept of Fractals is recently applied very intensively to model and classify terrain forms, cf. for example Håkanson, 1978; Shelberg et al 1982; Goodchild 1980, and Mandelbrot 1975. Much more attention for this concept is expected in future.

In analogy with fractal integration one may define fractal differentiation,

$$d^{(\gamma)}Z = dB^{(\gamma)}$$

as a unique function of γ passing through the points $d^{(0)}Z,\ d^{(1)}Z,\ d^{(2)}Z\ldots$

In this fashion we model the terrain profile by a (stochastic) differential equation. This differential equation may be used in finite element approximations of the terrain (Kubik, 1971; Bosman et al, 1972; Ebner, 1979). The proper functional to be minimized in the finite element approach is



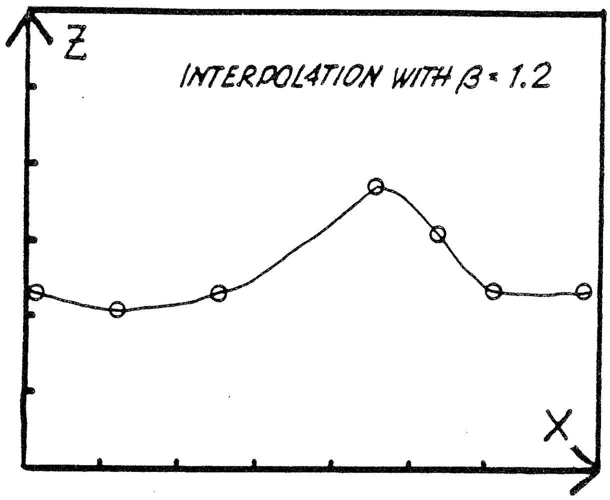


Fig. 5. Interpolation with Fractals.

$$J = \int \left\{ \frac{\partial^{(\gamma)} z}{\partial x^{(\gamma)}} \right\}^2 dx \rightarrow \min \tag{11}$$

The results of the finite element method (11) are identical to the results of the Prediction method using variogramme (8) or the correlation function (5), as it was shown already in 1971 by Kimeldorf and Wabha (see also Dolph and Woodbury, 1952; Kubik, 1973). When interpolating with the prediction method (Wiener, 1949)

$$Z(x) = K(x,x_i) \cdot K(x_i,x_j)^{-1} \cdot Z(x_j) \quad \text{or}$$

$$Z(x) = Var(x-x_i) \cdot Var(x_i-x_j)^{-1} \cdot Z(x_j) \tag{12}$$

(with $Z(x_j)$ vector of sample values) the numerical stability of the computations should be carefully controlled. When using variogrammes, as advocated by few, the smallest elements occur along the diagonal of the matrices involved; the matrices are not positive definite. The accumulation of rounding errors is very serious and meaningless results are obtained when using desk top computers without special precautions. The authors therefore recommend the use of the correlation function (5).

Figure 5 shows some examples of interpolation. For β = 1 we obtain linear interpolation, for β = 3 piecewise 3rd degree interpolation (Spline-interpolation), and for β values between one and three we obtain interpolation forms, which properly model break lines in the terrain while preserving relative smoothness in the other profile sections (cf. Botman and Kubik, 1984).

6. WHICH WAY TO GO?

"Would you tell me please, which way I ought to go from here?" asked Alice. "That depends a good deal on where you want to get to", said the Cat.

Which way do we want to go in modelling and classifying terrain forms? There are many questions still open, which may help us to choose the direction of future research:

- The models and classification methods must be systematically tested on their validity for large scale applications such as in photogrammetry;

- Interpolation methods, which are most adequate in the light of these models, must be elaborated;

- An extention of the theory of fractals to twodimensional terrain forms must be realized.

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