INTERACTIVE BUNDLE ADJUSTMENT WITH METRIC AND NON-METRIC IMAGES INCLUDING TERRESTRIAL OBSERVATIONS AND CONDITIONS

H. Fuchs
Technical University Graz and
Graz Research Center
Wastiangasse 6, A-8010 Graz, Austria
Commission III

1. ABSTRACT:

The program system CRISP for the Kern analytical plotter DSR-1 includes a bundle adjustment technique with data management and an interactive dialogue. The mathematical model and applications are described of an unconventional multi-overlapping block with metric and non-metric images. The system enables one to use additional terrestrial observations and conditions, for example the condition that some points are on a straight line or on a plane.

2. INTRODUCTION:

The combination of a bundle adjustment with an analytical stereo plotter offers the application of conventional photogrammetric methods in a new way. One of the most important possibilities which can be used is the interactive dialogue between operator, computer and analytical stero plotter. The advantages of an interactive bundle adjustment in connection with an analytical stereo plotter can be summarized as follows:

- all measurements are generated interactive
- possibility to edit all measurements (editing = delete, remeasure etc.)
- connection with an analytical plotter allows an automatic transfer of homologue (tie) points
- stepwise collection of data
- stepwise generating of approximations for bundle adjustment
- stepwise elimination of errors (also outlier detection)
- all terrestrial observations (distances, angles, azimuths, differences of heights) can be digitized in a parallax free model
- "measuring" of conditions (points on a plane or on a straight line) is possible.

In the following an overview of the mathematical models and data structure used in CRISP is described.

3. MATHEMATICAL MODELS:

3.1 Determination of approximations:

a.) Non-metric photographs:

For non-metric photographs initial values for bundle adjustment are generated by a Direct Linear Transformation (DLT, Bopp (1978)). For this a minimum of 6 ground control points is necesarry to calculate values for inner and exterior orientation. The same method is used for the calibration of a photograph once and apply its result as a known camera type for the remaining images.

b.) Metric photographs:

Here for calculation of initial values a conventional inner, relative and absolute orientation is performed. Also a 3-D space resection is available. The relative orientation is the solution of the intersection condition of homologue rays. For absolute orientation a modified algorithm for a 3-D conformal transformation (due to numerical stability and large rotations in close range photogrammetry) is used.

3.2 Bundle adjustment:

a.) Image coordinate measurements of ground control points and homologue points.

All measured image coordinates are included in the bundle adjustment by linearized equations. For every point two observation equations (for x and y image coordinate) can be generated:

$$\begin{aligned} dx &= dx_0 + c_1 dc + u_{11}(dX_0 - dX) + u_{12}(dY_0 - dY) + u_{13}(dZ_0 - dZ) \\ &\quad + v'_{11} d\varphi + v'_{12} d\omega + v'_{13} d\varkappa \\ dy &= dy_0 + c_2 dc + u_{21}(dX_0 - dX) + u_{22}(dY_0 - dY) + u_{23}(dZ_0 - dZ) \\ &\quad + v'_{21} d\varphi + v'_{22} d\omega + v'_{23} d\varkappa \end{aligned} \tag{1}$$

where the coefficients are given in Jordan/Eggert/Kneissl, Handbuch der Vermessungskunde, Band III a/1 Photogrammetrie pages 34, 35, 36 (J.B. Metzlersche Verlagsbuchhandlung, Stuttgart, BRD).
For control points the unknowns dX, dY, dZ have to be set equal to zero. For a partially known control point, one has to set the known coordinates (dX/dY/dZ) equal to zero. For example for a X/Z control point (Y unknown) the unknown dY remains in equation (1). For every homologue point (tie point) all three unknown coordinates dX, dY, dZ have to remain in these equations.

b.) Measured image coordinates of object points of observations and conditions.

This kind of measurements is included in the bundle adjustment with linearized formulas (1) given above. Again if one of the object points is a control point, partially known control point or homologue point the unknown coordinate part (dX and/or dY and/or dZ) has to be included in the same way as explained before. If one of the terrestrial observations or conditions spans over several models, then linearized equations (1) are only generated for the image in which an observation or condition object point is seen. The same is done when a control point is only visible in one photograph.

c.) Terrestrial observations.

For terrestrial observations the following linearized formulas are used.

Distance:

$$ds_{ij} = -\frac{1}{s_{ij}} [-(X_j-X_i)dX_i - (Y_j-Y_i)dY_i - (Z_j-Z_i)dZ_i + (X_j-X_i)dX_j + (Y_j-Y_i)dY_j + (Z_j-Z_i)dZ_j] \tag{2}$$

Azimuth:

$$d\nu_{ij} = \frac{1}{s_{ij}^2} [(Y_j-Y_i)dX_i - (X_j-X_i)dY_i - (Y_j-Y_i)dX_j + (X_j-X_i)dY_j] \tag{3}$$

Height-difference:

$$d\Delta H_{ij} = \Delta Z_j - \Delta Z_i \tag{4}$$

Angle in 3-D space (i ... Station point):
non-linear form:

$$\cos\alpha_{i,j,k} = \frac{1}{s_{ij}s_{ik}} [(X_j-X_i)(X_k-X_i) + (Y_j-Y_i)(Y_k-Y_i) + (Z_j-Z_i)(Z_k-Z_i)] \tag{5}$$

linearized form:

$$\begin{aligned} - s_{ij}s_{ik}\sin\alpha\, d\alpha = {} & a_i dX_i + b_i dY_i + c_i dZ_i + \\ & a_j dX_j + b_j dY_j + c_j dZ_j + \\ & a_k dX_k + b_k dY_k + c_k dZ_k + \end{aligned} \qquad \begin{aligned} & a_i = -(X_j+X_k) + 2X_i \\ & a_j = X_k - X_i \\ & a_k = X_j - X_i, \\ & \text{for } b_i,\ b_j,\ b_k \text{ replace X by Y} \\ & \text{for } c_i,\ c_j,\ c_k \text{ replace X by Z} \end{aligned} \tag{6}$$

d.) Conditions for a straight line and a plane:
Straight line:

$$(X_k-X_i) + (X_j-X_i)\frac{(Z_k-Z_i)}{(Z_j-Z_i)} = 0 \tag{7}$$

$$(Y_k-Y_i) + (Y_j-Y_i)\frac{(Z_k-Z_i)}{(Z_j-Z_i)} = 0$$

Plane:

$$\begin{vmatrix} X_i & Y_i & Z_i & 1 \\ X_j & Y_j & Z_j & 1 \\ X_k & Y_k & Z_k & 1 \\ X_l & Y_l & Z_l & 1 \end{vmatrix} = 0 \qquad (8)$$

The non linear conditions (7) have to be linearized with respect to X, Y, Z of points i, j, k for the line. The same has to be done with condition (8) for the plane (linearized with respect to points i, j, k, l). Every additional point on the line gives raise to one more pair of line-conditions (7), every additional point on the plane gives raise to one more condition (8).

Remark:

Notice that observations and conditions are included in two different ways in the adjustment. First image coordinate measurements of every object point lead to equations (1). Secondly observations and conditions itself are included by linearized equations (2), (3), (4), (5), (6), (7), (8). For conditions a relatively high observation weight is chosen to fulfill the condition.

3.3 Solution of adjustment:

For the solution of the least squares adjustment the method of conjugate gradients is used. This method for solving symmetric definite equation systems has been developed by Hestens and Stiefel (1952). Applications in geodesy has been demonstrated by H. R. Schwarz (1970), Gruendig (1980) and Steidler (1980).
This iteration procedure is directly working with the observation equations and therefore saves the memory- and time consuming computation and solution of the normal equations. A special quality of the conjugate gradients method is the quick convergence in a local area.

Besides the simple and quick computation and the limited need for core memory, another advantage occurs, which is due to all gradients methods. For adjustment of observation equations with rank deficiency (for example free networks, self-contained partial blocks of images) a transformation onto the approximate coordinates is automatically executed. For a free adjustment of the network this approach offers a plausible solution without additional expense.

A short overview of the method of conjugate gradients is described below.
Given are the observation equations

$$Ax - l = v \qquad (9)$$

This is leading to a function

$$F(x) = v^T v = x^T A^T A x - 2x^T A^T \ell + \ell^T \ell \tag{10}$$

which shall be minimized. By differentiation with respect to x we get the normal equation system

$$A^T A x - A^T \ell = 0 \tag{11}$$

The solution of this symmetric definite equation system corresponds to the search for the minimum of the quadratic function F(x). The main principle of the relaxation computation is the following. Starting from a trial vector x(0) one chooses a direction vector h. The length of this vector h is determined in such a way, that the quadratic function F(x) decreases. In this way one gets a better approximation x(1) for the vector of the unknowns. These steps are repeated until the minimum of the function F(x) is reached. This is the case, if the system (11) is valid without contradictions. The various methods distinguish by the choice of the direction h (relaxation directions) and the choice of the length of these vectors.

A possible strategy is to choose the directions in such a way, that in the local area of the approximate solutions they point to the direction of the largest descent of the quadratic function F(x).

$$\mathrm{grad}(F(x)) = A^T A x^{(i)} - A^T \ell = r^{(i)} = -h^{(i+1)} \tag{12}$$

In the procedure of conjugate gradients the relaxation direction h form a system of conjugate directions and the vectors of residuals r form an orthogonal system. From a theoretical point of view, this procedure leads to an exact solution after m steps (m = number of unknown parameters).

3.4 Mathematical models for error detection:

Conventional least squares adjustment minimizes the sum of the squares of the residuals. It minimizes the 2-norm:

$$\|v\|_2 = \sum_{i=1}^{n} v_i^2 = \mathrm{Min} \tag{13}$$

subject to certain constraints.

It is generally known that in the presence of blunders the result of a least squares adjustment is usually distorted and falsified. Therefore alternatives to least squares adjustment have recently received new attention. One other alternative is obtained by minimizing the norm of residuals in a different way, namely minimizing the 1-norm:

$$\|v\|_1 = \sum_{i=1}^{n} |v_i| = \mathrm{Min} \tag{14}$$

Minimizing this norm is equivalent to minimizing the sum of absolute residuals.

Claerbout, Muir (1973) applied adjustment by 1-norm for blunder detection in geophysical data and Benning (1972) obtained very good results for point intercalation. Barrodale (1968) analysed numerical data and pointed out the advantages of the 1-norm compared to the 2-norm. Adjustment using 1-norm is rather insensitive with regard to outliers among the measurements. It represents a "robust" statistical estimation procedure, confer also Dutter (1980). These robust qualities of adjustment by the 1-norm can be demonstrated easily by considering the case of direct observations. Let us assume a file of observations which contains one outlier:

(2, 2, 2, 2, 100)

We obtain residuals and adjusted values by using the different norms:

a.) 2-norm:

adjusted value: 21,6 (arithmetic mean)
residuals: (19.6, 19.6, 19.6, 19.6 -78.4)

b.) 1-norm:

adjusted value: 2.0 (median value)
residuals: (0, 0, 0, 0, -98)

In case a.) the outlier distorts the adjusted value considerably. In b.) the fifth observation can be identified as an outlier definitely. The adjusted value leads to the so-called median value. In a file of values ordered according to their size, it is the value that is most in the "middle" (in the above example it is the third value). For an even number of measurements two values in the middle (and every value between these two values) qualify as the median. Increasing the fifth value arbitrarily does not affect the median at all, it remains the same.

Now the question arises whether these robust qualities still hold in a more general adjustment model (adjustment of observation equations with more than one parameter) and how to solve it.

Adjustment of observation equations using the 1-norm can be stated as follows:

$$Ax = \ell + v$$
$$\sum_{i=1}^{n} |v_i| = \text{Min} \qquad (15)$$

Problem (15) is not solvable in the conventional way by means of differentiation, the function to be minimized is not a differentiable function. By replacing x and v as differences of two non-negative values a formulation is obtained which leads to a linear program (conf. Fuchs 1982). This linear program has a specific form and can be solved by a simplex method. Barrodale and Roberts (1973) developed an modified simplex algorithm in which essentially only matrix A has to be stored in the computer. It bypasses in one step several steps of the conventional simplex algorithm. A great advantage of the algorithm is that the rank of A can be less than the number of unknown parameters x.

The algorithm searches that combination of observations (out of all observation, number of observation = n) which

a.) is necessary for a non-redundant determination of the unknown parameters x
b.) minimizes the sum of absolute residuals.

In the case of linear independent columns of A (that means, if rank(A) = m, the number of unknown parameters x), exactly m of the observations are necessary for determining the m unknowns x. Consequently this group of observations (called l-NB) has residuals (v-NB) with values equal to zero. The remaining n-m observations (l-B) will receive residuals (v-B), which are generally nonzero. The solution x represents a generalized median.

The robust properties of the median value as described before still hold in the case of m unknown parameters. Any observation (or all observations) out of the group l-B can be changed arbitrarily under the restriction that the signs of the residuals v-B do not change. The solution remains an optimal one and the vector x of unknown parameters will be unchanged. This can be proved with the aid of the theory of linear optimization, confer Fuchs (1982). If such variations of the observations are interpreted as outliers, the robustness of adjustment using 1-norm can be easily seen.

For smaller problems like direct linear transformation or absolute orientation the modified simplex algorithm is directly used. For error detection in the bundle adjustment itself the so-called weight iteration is applied (Krarup 1980). This procedure is very easy to combine with the conjugate gradient method. By the choice of weights

$$\begin{aligned} p &= \frac{1}{|v|}, \quad v * \text{const} > 1 \\ p &= \text{const}, \quad v * \text{const} < 1 \quad \text{resp.} \end{aligned} \tag{16}$$

an adjustment with minimization of the sum of absolute residuals can be obtained (const is a relatively large constant).

4. DATA STRUCTURE:

The program system is designed to run on a 64 k-byte computer. To save important CPU memory all data are stored on an external device (like winchester drive etc.). Only short information and data that needs very fast access are stored in the central memory. Due to that reason a linear linked list is kept in the CPU for important information and the location on the external device. Linked lists are used for point, camera, observation and image data bases. The linked list for the images is also the base for a data structure to handle all image connections for stereo models and multiple overlapping models.

5. CONCLUSIONS:

The combination of a photogrammetric bundle adjustment with an analytical steroplotter offers many new facilities to the user. The most important facility is interactivity. This allowes the operator to measure all his data in an interactive way (delete, remeasure etc.) Checking of results and error detection is possible in every step during the entire procedure and thus reduces the possibility of including errors in the final adjustment.

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