THE SPATIAL ANBLOCK METHOD

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Commission III/3

The Anblock method, developed in 1960 [2], is a model triangulation method restricted to planimetry only. This means that the models must be pre-levelled to avoid disturbing the planimetric accuracy. The procedure is non-iterative--the two-dimensional similarity transformation being a linear problem.

The spatial Anblock method is an extension of the planimetric Anblock method. It starts to level (using equations) all models in a block simultaneously.

Absolute and relative or independent ground control points (distances, slopes, lake points) are used to level the models with respect to the object photographed. Relative ground control points, of course, can be determined easier than their absolute coordinates.

Two levelling procedures will be described. The first is based on distances and heights and the other on only slopes.

Finally, because the coefficient matrix of the partially reduced normal equations of the Anblock approach has a very special structure, an efficient algorithm for the solution of the system of normal equations is described.

Bundle models.

A bundle model is used as a computational unit [3]. The bundle model is defined by the points of intersection of the projecting rays from (measured) corresponding image points in a triplet of consecutive and relatively oriented photographs and the perspective centre of the central photograph. The relative orientation of a triplet is determined by eleven independent elements of orientation. An efficient relative orientation method was developed by Molenaar [4, 5]. The rotation elements are determined independent of the base elements.

If independent models are measured, bundle models are formed by connecting pairs of consecutive independent models (spatial similarity transformation) [1].

Bundle models are used because they have the same lateral overlap as the photographs. This means that the number of the bundle models in a strip is two less than the number of photographs. All points can be taken into account in the adjustment, apart from the points in the first part of the first and in the last part of the last model in a strip. The perspective centres cannot be used to determine the tilt of the models. They are used only if auxiliary data are available, i.e., APR and/or statoscope measurements.

We suppose that all bundle models in a block are approximately similar to the area of the object (terrain) photographed. In that case, the triangulation (adjustment included) involves the application of spatial similarity transformation to each model in a block, taking into account all tie and ground control conditions.

The spatial similarity transformation.

Let x_{ij}, y_{ij}, z_{ij} be the coordinates of an object point j measured in the orthogonal model coordinate system x_i, y_i, z_i of an arbitrary model (i) and $\bar{u}_{ij}$, $\bar{v}_{ij}$, $\bar{w}_{ij}$, u_{ij}, v_{ij}, w_{ij} and r_{ij}, s_{ij}, t_{ij} the transformed model coordinates after rotations ω^i, ϕ^i and κ^i about the X-, Y- and Z-axes of the orthogonal object coordinate system XYZ, respectively.
We get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega^i & -\sin\omega^i \\ 0 & \sin\omega^i & \cos\omega^i \end{bmatrix} \begin{bmatrix} x_{ij} \\ y_{ij} \\ z_{ij} \end{bmatrix} = \begin{bmatrix} \bar{U}_{ij} \\ \bar{V}_{ij} \\ \bar{W}_{ij} \end{bmatrix} \qquad (1a)$$

$$\begin{bmatrix} \cos\phi^i & 0 & -\sin\phi^i \\ 0 & 1 & 0 \\ \sin\phi^i & 0 & \cos\phi^i \end{bmatrix} \begin{bmatrix} \bar{U}_{ij} \\ \bar{V}_{ij} \\ \bar{W}_{ij} \end{bmatrix} = \begin{bmatrix} u_{ij} \\ v_{ij} \\ w_{ij} \end{bmatrix} \qquad (1b)$$

$$\begin{bmatrix} \cos\kappa^i & -\sin\kappa^i & 0 \\ \sin\kappa^i & \cos\kappa^i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{ij} \\ v_{ij} \\ w_{ij} \end{bmatrix} = \begin{bmatrix} r_{ij} \\ s_{ij} \\ t_{ij} \end{bmatrix} \qquad (1c)$$

Let X_j, Y_j, Z_j be the object coordinates of the object point j, X_i, Y_i, Z_i the object coordinates of the origin i of the model coordinate system x_i, y_i, z_i and λ_i a scale factor.
We then get

$$\begin{bmatrix} X_j - X_i \\ Y_j - Y_i \\ Z_j - Z_i \end{bmatrix} = \lambda_i \begin{bmatrix} r_{ij} \\ s_{ij} \\ t_{ij} \end{bmatrix} \qquad (1d)$$

We know a model can be levelled by a combination of the rotations ω^i and ϕ^i about the X- and Y-axes, respectively. We therefore eliminate the coordinates $\bar{u}_{ij}$, $\bar{v}_{ij}$ and $\bar{w}_{ij}$ from the equations (1a) and (1b). This gives

$$\begin{bmatrix} \cos\phi_i & -\sin\phi^i\sin\omega^i & -\sin\phi^i\cos\omega^i \\ 0 & \cos\omega^i & -\sin\omega^i \\ \sin\phi^i & \cos\phi^i\sin\omega^i & \cos\phi^i\cos\omega^i \end{bmatrix} \begin{bmatrix} x_{ij} \\ y_{ij} \\ z_{ij} \end{bmatrix} = \begin{bmatrix} u_{ij} \\ v_{ij} \\ w_{ij} \end{bmatrix} \qquad (1e)$$

or, simply

$$\begin{bmatrix} a_{11}^{i} & a_{12}^{i} & a_{13}^{i} \\ 0 & a_{22}^{i} & a_{23}^{i} \\ a_{31}^{i} & a_{32}^{i} & a_{33}^{i} \end{bmatrix} \begin{bmatrix} x_{ij} \\ y_{ij} \\ z_{ij} \end{bmatrix} = A^{i} \begin{bmatrix} x_{ij} \\ y_{ij} \\ z_{ij} \end{bmatrix} = \begin{bmatrix} u_{ij} \\ v_{ij} \\ w_{ij} \end{bmatrix} \qquad (1f)$$

The elements a_{31}^{i} and a_{32}^{i} of the matrix A^{i} are chosen as independent elements of rotation and called tilt elements. The remaining elements are now simple functions of the tilt elements

$$\begin{aligned} a_{33}^{i} &= \sqrt{1 - (a_{31}^{i})^2 - (a_{32}^{i})^2} \\ a_{31}^{i} &= \sqrt{1 - (a_{31}^{i})^2} \\ a_{22}^{i} &= a_{11}^{i} \cdot a_{33}^{i} \;/\; (a_{11}^{i})^2 \\ a_{23}^{i} &= -a_{11}^{i} \cdot a_{32}^{i} \;/\; (a_{11}^{i})^2 \\ a_{12}^{i} &= a_{12}^{i} \cdot a_{31}^{i} \\ a_{13}^{i} &= -a_{22}^{i} \cdot a_{31}^{i} \end{aligned} \qquad (1g)$$

We see that if the tilt elements of all models in a block are known, the transformed coordinates (u_{ij}, v_{ij}, w_{ij}) of all measured model points are determined; see: the rel. (1f) and (1g). In other words, the models are levelled. This means that a planimetric triangulation can be carried out next. The two-dimensional transformation equations are obtained by elimination of the coordinates r_{ij}, s_{ij} and t_{ij} from the eq. (1c) and (1d).

$$\begin{bmatrix} X_j - X_i \\ Y_j - Y_i \\ Z_j - Z_i \end{bmatrix} = \lambda_i \begin{bmatrix} \cos\kappa^{i} & -\sin\kappa^{i} & 0 \\ \sin\kappa^{i} & \cos\kappa^{i} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{ij} \\ v_{ij} \\ w_{ij} \end{bmatrix} \qquad (1h)$$

or, simply

$$\begin{bmatrix} X_j - X_i \\ Y_j - Y_i \\ Z_j - Y_i \end{bmatrix} = \lambda_i \begin{bmatrix} b_{11} & -b_{21} & 0 \\ b_{21} & b_{11} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{ij} \\ v_{ij} \\ w_{ij} \end{bmatrix} = \lambda_i B^{i} \begin{bmatrix} u_{ij} \\ v_{ij} \\ w_{ij} \end{bmatrix} \qquad (1i)$$

in which

$$b_{11}^{i} = \sqrt{1 - (b_{21}^{i})^2}$$

According to the last equation of (1f), equation (1i) can also be written in the form:

$$\begin{bmatrix} X_j & 0 & -X_i & 0 & -u_{ij} & v_{ij} \\ 0 & Y_j & 0 & -Y_i & -v_{ij} & u_{ij} \end{bmatrix} \begin{bmatrix} X_j \\ Y_j \\ X_i \\ Y_i \\ \lambda_i b^i_{11} \\ \lambda_i b^i_{21} \end{bmatrix} = 0 \qquad (2)$$

and

$$Z_j - Z_i - (\lambda_i x_{ij})\, a^i_{31} - (\lambda_i y_{ij})\, a^i_{32} - (\lambda_1 z_{ij})\, a^i_{33} = 0 \qquad (3)$$

Finally, we introduce the matrix C^i; see: the rel. (1f) and (1i).

$$B^i A^i = C^i$$

Equation (2) contain four independent planimetric elements of orientation:

X_i, Y_i, $\lambda_i b^i_{11}$ and $\lambda_i b^i_{21}$. They are linear if the transformed model coordinates u_{ij}, v_{ij}, w_{ij} are known. This is the Anblock approach [2]. To determine these coordinates, we consider equations (3).

Scale - tilt and height - procedure

(a) SCALE DETERMINATION

Considering the scale factor λ_i in equation (2) as an independent parameter, equation (3) contains only three independent elements of orientation:

Z_i, a^i_{31} and a^i_{32} To level the models the scale factors must therefore, be determined first.

Let l_{ijj+1} be the distance between points j and j+1 in the model (i) and l_{jj+1} the distance between these points in the object. It is evident that between these distances and the scale factor λ_i the relation exists

$$\lambda_i \cdot l_{ijj+1} = l_{jj+1} \qquad (4)$$

in which

$$l_{ijj+1} = \sqrt{(x_{ij+1} - x_{ij})^2 + (y_{ij+1} - y_{ij})^2 + (z_{ij+1} - z_{ij})^2}$$

Next, we consider points j and j+1 in the overlap of the models (i) and (i+1). For the points in model (i+1), an equation of type (4) is written.

Eliminating l_{jj+1} from this equation and equation (4) gives:

$$\lambda_i \cdot l_{ijj+1} = \lambda_{i+1} \cdot l_{i+ijj+i} \quad (5)$$

Equations (4) and (5) are linear. This means that no approximate values for the scale factors are required. Equation (5) is evaluated for a pair of common points in each overlap and equation (4) for two points (in one and the same model) of which the distance l_{jj+1} is determined by a geodetic measuring method. Any more distance l_{jj+1} gives a redundancy.

(b) TILT AND HEIGHT DETERMINATION.

If the scale factors are known, they are substituted in the equation of type (3) and next for each measured point an equation of type (3) is evaluated (height control points included). To improve modelling, equations containing only tilt elements can be added to a system of equation (3).

We therefore consider again two points j and j+1 in the model (i). For point j+1, an equation of type (3) is written. Eliminating Z_i and then λ_i from this equation, (3) and (4) give

$$\sin \gamma_{jj+1} = \{a^i_{31} (x_{ij+1} - x_{ij}) + a^i_{32} (y_{ij+1} - y_{ij}) + a_{33} (z_{ij+1} - z_{ij})\}/l_{ijj+1} \quad (6)$$

in which

$$\sin\alpha_{jj+1} = Z_{j+1} - Z_j / l_{jj+1}$$

The angle α_{jj+1} is simply called "slope". It is the angle enclosed by the straight line connecting the measured points j and j+1 in one and the same model and a plane parallel to the XY-plane. The angle α_{jj+1} must be determined by geodetic measuring. This is not necessary, however, if arbitrary pairs of points, for example of the shoreline of a lake (lake points), are measured, because in that case: $\alpha_{jj+1} = 0$.

Equations (3) and (6) are non-linear. If ω^i and ϕ^i are small, however, the tilt elements a^i_{31} and a^i_{32} are also small and the element a^i_{33} is approximately equal to the unity. We therefore put in equations (3) and (6):

$$a^i_{33} = a^{i+1}_{33} = \ldots = 1.$$

If the system of equations (3) and (6) is solved, transformed model coordinates u_{ij}, v_{ij}, w_{ij} (1f) and (1g) are determined next for each measured model point.

The process can be terminated if the following conditions are fulfilled;

$$a_{31}^{i} = a_{32}^{i} = 0$$

For a block containing n models, the scale-tilt and height procedure involves the solution of a system of n linear equations and next a system of 3n non-linear equations (apart from the Z_j-coordinates of the points to be determined). The next procedure involves the solution of a system of 2n non-linear equations and then a system of n linear equations.

Tilt height procedure

(a) TILT-DETERMINATION

Finally, we consider points j and j+1 in the overlap of the models (i) an (i+1). For the points in model (i+1) and equation of type (6) is written. Eliminating $\sin\alpha_{j\,j+1}$ and next the ratio λ_{i+1}/λ_i from the equations, equations (6) and (5) give

$$\{a_{31}^{i}(x_{i\,j+1}-x_{i\,j}) + a_{32}^{i}\,(y_{i\,j+1}-y_{i\,j}) + a_{33}^{i}\,(z_{i\,j+1}-z_{i\,j})\}/1_{i\,j\,j+1} =$$

$$\{a_{31}^{i+1}(x_{i+i\,j+1}-x_{i+i\,j}) + a_{32}^{i+1}(y_{i+1\,j+i}-\ y_{i+i\,j}) \qquad (7)$$

$$+\ a_{33}^{i+1}(z_{i+i\,j+1}-z_{i+i\,j})\}/1_{i+i\,j\,j+1}$$

in which

$$a_{33}^{i} = a_{33}^{i+1} = 1.$$

We see that equations (6) and (7) contain only tilt elements (two per model).

The coefficients of the tilt elements are formed by x_i - and y_i - coordinate differences. It is therefore recommended to use pairs of points parallel to and perpendicular to the line of flight. We know, that a system of linear equations can only be solved if the coefficient matrix of the system is non-singular. It is also evident that the models must now have a forward overlap of 50% (bundle models).

We consider a block containing only parallel strips and a sidelap of 15 to 20%. To level the models with respect to the object photographed, in each strip a slope $\alpha_{j\,j+i}$ perpendicular and in the first strip a slope parallel to the line of flight must be known. Any extra slope gives redundancy.

The process to determine definite values for the tilt elements is similar to the scale-tilt-height procedure.

(b) HEIGHT-DETERMINATION

After evaluation of the planimetric triangulation, the height triangulation is finally carried out. This triangulation is based on the last equation of (1i)

$$Z_j - Z_i = \lambda_i w_{ij} \tag{8}$$

in which

$$\lambda_i = \sqrt{(\lambda_i b_{11}^i)^2 + (\lambda_i b_{21}^i)^2}$$

is determined by the foregoing planimetric triangulation.

Solution of the planimetric normal equations.

The straightforward approach from equation (2) gives very large systems of equations containing two groups of unknowns: coordinates of the points to be determined and 4 n planimetric elements of orientation. Because of the special structure of the coefficient matrix, one group of unknowns can easily be eliminated; for example, the coordinates.
Let

$$Mx = 1 \tag{9}$$

be the partially reduced system of m normal equations in m unknowns ($m = 4n$), in which M is a coefficient matrix, symmetrical and non-singular, x and 1 being vectors composed of the values of the planimetric parameters and of the constant terms, respectively.

If matrix M is symmetric, the matrix can be resolved into the product of two triangular matrices of which one is the transpose of the other. Thus if S is an upper triangular matrix and S^T its transpose

$$M = S^T S \tag{10}$$

The solution of the system is reduced then to the solution of two triangular systems

$$S^T k = 1 \text{ and } Sx = k \tag{11}$$

because the two systems (11) are equivalent to system (9).
In view of the rule of matrix multiplication, the elements of matrix S are simple functions of the elements of matrix M.

With this square-root method, we have to record only the approximately $\frac{m^2}{2}$ elements of matrix M and of matrix S and the 2 n components of vectors k and l. The number of multiplications, divisions and square roots necessary for finding the elements of triangular matrix S is equal to $\frac{m}{6}$ $(m^2 + 3m + 2m)$.

Matrix M of the partially reduced normal equations has the following structure:

$$M = \begin{bmatrix} M_{11} & 0 & M_{13} & -M_{14} & M_{15} & -M_{16} & - & - \\ - & M_{11} & M_{14} & M_{13} & M_{16} & M_{15} & - & - \\ - & - & M_{33} & 0 & M_{35} & -M_{36} & - & - \\ - & - & - & M_{33} & M_{36} & M_{35} & - & - \\ - & - & - & - & M_{55} & 0 & - & - \\ - & - & - & - & - & M_{55} & - & - \\ - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - \end{bmatrix} \quad (12)$$

Because of this structure, we have now to record only the elements of the odd rows of matrix M: approximately $\frac{m^2}{4}$ elements.

We introduce now a triangular matrix S having the same structure of matrix M.

$$S = \begin{bmatrix} S_{11} & 0 & S_{13} & -S_{14} & S_{15} & -S_{16} & - & - \\ 0 & S_{11} & S_{14} & S_{13} & S_{16} & S_{15} & - & - \\ 0 & 0 & S_{33} & 0 & S_{35} & -S_{36} & - & - \\ 0 & 0 & 0 & S_{33} & S_{36} & S_{35} & - & - \\ 0 & 0 & 0 & 0 & S_{55} & 0 & - & - \\ 0 & 0 & 0 & 0 & 0 & S_{55} & - & - \\ 0 & 0 & 0 & 0 & 0 & 0 & - & - \\ 0 & 0 & 0 & 0 & 0 & 0 & - & - \end{bmatrix} \quad (13)$$

and find that the structure of the product S^TS is similar to the structure of matrix M (12). This means that we may also omit the elements of the odd rows of triangular matrix S. We thus have to record only the approximately $\frac{m^2}{4}$ elements of matrix S and the number of multiplications, divisions and square roots is equal to $\frac{m}{12}$ $(n^2 + 3m + 2m)$. Compared with the conventional square root method, there is reduction by approximately 50% of the number of multiplications.

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ABSTRACTS

Two levelling procedures for bundle models and independent models are described. Absolute and relative (distances, slopes and lake points), ground control is used for the absolute levelling of the models. Finally, an efficient algorithm for the solution of the partially reduced normal equations of the Anblock approach is described.