DATA SNOOPING USING OBSERVATIONS AND PARAMETERS WITH CONSTRAINTS

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Introduction

Least squares methods are widely used in photogrammetric and geodetic computations. One problem in least squares methods is assigning a priori weights to different observations and parameters. Another is the detection of noises that are the size of the random errors. The author has developed a method of detecting noises and compensating for them in a recursive adjustment method. This method was successfully used in the detection of movement in an Electronic Distance Measurement Instrument (EDMI) calibration. This method has both photogrammetric and geodetic applications. The objective of this paper is to present the theoretical account of this method.

Theoretical Background

The optimum estimate $\hat{x}$ of some value x will be defined as the value of x estimated if it minimizes the function $E\{(x - \hat{x})^T Q(x - \hat{x})|Z^T\}$ where $x - \hat{x}$ is a column matrix, $(x - \hat{x})^T$ is the transpose of $x - \hat{x}$, Q is a symmetric, positive definitive matrix, and $E\{|Z^T|\}$ denotes the conditional mean operator given the available data vector Z_t defined at time t or at the t^{th} iteration.

The optimal estimator, which is the conditional mean, is given by

$$\hat{x} = \int_{\Omega} x \; p(x/Z_t)dx \qquad (1)$$

where

Ω = space of all x

$p(x/Z_t)$ = conditional probability density function of x given the data vector Z_t

Extending the function to include α functions in terms of a joint probability we will have

$$\hat{x} = \int_{\Omega}\int_{A} x \; p\;(x,\alpha/Z_t)d\alpha dx$$

where A = space of all α.

Now $p(x,\alpha/Z_t) = p(x/\alpha,Z_T) \cdot p(\alpha/Z_T)$. Then

$$\hat{x} = \int_{\Omega}\int_{A} x \; p(x/\alpha, Z_T) \cdot p(\alpha/Z_T) d\alpha dx$$

$$= \int_{A}\int_{\Omega} x \; p(x/\alpha, Z_T) dx \cdot p(\alpha/Z_T) d\alpha$$

$$= \int_{A} \hat{x}_\alpha \; p(\alpha/Z_T) d\alpha$$

Now

$$p(\alpha_i/Z_k) = \frac{p(Z_k/\alpha_i)\; P(\alpha_i)}{\sum_{j=1}^{n} p(Z_k/\alpha_j)\; p(\alpha_j)} \qquad (2)$$

In an iterative process, $p(Z_k/\alpha_i)$ and $p(\alpha_i)$ can be computed from the current iteration; then $p(\alpha_i/Z_k)$ can be computed for the next iteration.

As iterations proceed, the weights of x_{α_i} with small probability will tend toward zero and those with large probabilities will tend toward one.

Applications in General Least Squares Adjustment

In least squares adjustment the set of observation equations is given by

$$AX - \ell = V$$

where matrix ℓ is the observations with variance covariance of Σ_ℓ or weight $P_\ell = \Sigma_\ell^{-1}$, matrix X is the parameters, and matrix A is the coefficient matrix.

Since the parameters are unknown, we have variance of ℓ, $\Sigma_\ell = \Sigma_V$, variance of V.

$$\therefore P_v = P_\ell = P$$

The least squares solutions of the parameters are given by

$$\hat{X} = (A^T P A)^{-1} A^T P \ell$$

$$\Sigma_{\hat{X}} = \sigma_0^2 (A^T P A)^{-1}$$

where

$$\sigma_0^2 = \frac{V^T P V}{n - U}$$

n = number of equations

U = number of parameters

Now

$$\bar{V} = A\hat{X} - \ell$$

$$= (A - I) \binom{X}{\ell}$$

$$\therefore \Sigma_{\bar{V}} = (A - I) \begin{pmatrix} \Sigma_{\hat{X}} & \\ & \Sigma_\ell \end{pmatrix} \begin{pmatrix} A^T \\ -I \end{pmatrix}$$

$$= A \Sigma_X A^T + \Sigma_\ell \qquad (3)$$

$$\therefore P_{\bar{V}} = \Sigma_V^{-1} = (A \Sigma_X A^T + \Sigma_\ell)^{-1} \qquad (4)$$

Now Equation (1) could be written as

$$\hat{x} = \Sigma\, p(x/Z)x_i$$

which can be compared with

$$\hat{x} = \frac{\Sigma P_i x_i}{\Sigma P_i}$$

where P_i is weight and $p(x/Z)$ is probability. Thus,

$$\frac{P_i}{\Sigma P_i} = p_i(x/Z)$$

or

$$P_i \propto p_i(x/Z)$$

From

$$v_i = A_i\hat{x} - \ell_i \qquad (5)$$

$$p(v_i) = p(\ell_i)$$

$$\therefore P(v_i) \propto P(\ell_i)$$

The residual V can be assumed to be distributed as a normal distribution with mean zero and variance given by the diagonal term of Σ_V given by Equation (3):

$$\therefore p(\hat{v}_i) = N(0, \sigma^{2}_{\hat{v}_i})$$

$$= \frac{1}{\sqrt{2\pi\,\sigma^2_{v_i}}}\, e^{\left(-v_i^2/2\sigma_i^2\right)}$$

Thus in Equation (2) we could substitute $P(\hat{v}_i)$ for probability $p(\alpha_i)$.

$$\therefore P(\hat{v}_i) = \frac{1}{\sqrt{2\pi\,\sigma^2_{v_i}}}\, e^{\left(-v_i^2/2\sigma_i^2\right)} = p(\alpha_i) \qquad (6)$$

and $P_{\bar{V}_i}$ for the conditional probability $p(Z/\alpha_i)$.

$$\therefore P_{\bar{V}_i} = p(z/\alpha_i) = \Sigma^{-1}_{V_{ii}} \text{ given by Equation (4)} \qquad (7)$$

Data Snooping Procedure

On the basis of arguments given in the two previous paragraphs, a procedure could be developed for reducing the weights for weaker observations in an iterative procedure and thereby obtaining the most likely determination of the parameters.

The recommended procedure follows:

a. Based on a priori knowledge of the observations, assign weights $P = \Sigma_\ell^{-1}$ and perform a least squares adjustment.

b. Compute $\hat{v}$ and $\Sigma_{\bar{V}}$ using Equations (5) and (3).

c. If $v_i >> 3\,\sigma_{\hat{v}_i}$, reject these observations by assigning $\Sigma_{\ell ii} = \infty$ and repeat steps a and b. This helps eliminate blunders.

d. Now compute $P_{\bar{V}}$ using Equation (4) from which the value for $p(Z/\alpha_i)$ can be obtained (see Equation (7)). Then $p(\alpha_i)$ can be computed from Equation (6).

e. Knowing $p(Z/\alpha_i)$ and $p(\alpha_i)$, the new weights P_{ii} for subsequent adjustments can be computed from Equation (2):

$$P_{ii} = \frac{p(Z/\alpha_i)\; p(\alpha_i)}{\Sigma\; p(Z/\alpha_i)\; p(\alpha_i)}$$

and a new least squares adjustment is performed until

$$F(\alpha,\ r,\ r) > \frac{\sigma^2_{0_{i+1}}}{\sigma^2_{0_i}}$$

where

$\sigma^2_{0_{i+1}}$ is the variance of unit weight after the $(i+1)^{th}$ iteration

$\sigma^2_{0_i}$ is the variance of unit weight after the i^{th} iteration

$F(\alpha, r, r)$ is the F distribution with r degrees of freedom and α confidence level

This process assigns new weights for each observation or constraint parameter depending on the residuals and their variance after adjustment. Any suspicious observation or constrained parameter will be gradually weeded out or will have less effect on the final adjusted parameters. Iteration can be stopped either by using the F test or after the second or third iteration until satisfactory solutions are obtained.

Results

The procedure suggested above was used to detect errors in an EDMI calibration and found to be satisfactory.

In the EDMI calibration baseline, there were 5 monuments with the distances between them (W, X, Y, Z) known with certain precision (Fig. 1). Twenty combinations of the distances between the monuments were then observed by the EDMI for calibrating it, and the errors in constant and scale factor of the EDMI were determined by least squares using a computer program. Table 1 shows the results of calibration. In order to test the validity of the theory and procedure discussed earlier, the program was modified to recompute with different weights for

Table 1

Standard Calibration			Calibration Using Data Snooping Method							
			Iteration 1			Iteration 2		Iteration 3		
Observation	Wt	Residual	Observation	Wt	Residual	Wt	Residual	Wt	Residual	
598.7386	1	0.00003	598.750	1	0.0099	0.003	0.012	0.003	0.012	
748.8924	1	-0.002	748.8924	1	-0.0033	0.048	-0.002	0.056	-0.002	
908.1247	1	-0.0002	908.1247	1	-0.0007	0.062	-0.0005	0.092	-0.0003	
1369.2343	1	-0.0015	1369.2343	1	-0.0020	0.061	-0.001	0.138	0.0004	
598.7419	1	0.0033	598.7419	1	0.0018	0.058	0.004	0.055	0.003	
150.1486	1	-0.005	150.1486	1	-0.005	0.027	-0.005	0.007	-0.007	
309.3820	1	0.002	309.3820	1	-0.002	0.055	-0.003	0.027	-0.004	
770.4949	1	0.002	770.4949	1	-0.00007	0.063	-0.0005	0.077	-0.0007	
748.9003	1	0.005	748.9003	1	0.0048	0.030	0.006	0.036	0.006	
150.1529	1	-0.0007	150.1529	1	-0.0012	0.062	-0.001	0.015	-0.003	
159.2277	1	-0.0004	159.2277	1	-0.0010	0.062	-0.001	0.016	-0.003	
620.3375	1	-0.0017	620.3375	1	-0.0020	0.055	-0.0014	0.055	-0.002	
908.1249	1	0.00004	908.1249	1	-0.0006	0.063	-0.003	0.091	-0.0001	
309.3856	1	0.0015	309.3856	1	0.0014	0.060	0.0004	0.030	-0.001	
159.2243	1	-0.0038	159.2243	1	-0.005	0.035	-0.004	0.091	-0.006	
461.1125	1	0.0037	461.1125	1	0.003	0.049	0.004	0.036	0.003	
461.1130	1	0.0041	461.1130	1	0.003	0.045	0.005	0.033	0.004	
620.3424	1	0.0032	620.3424	1	0.003	0.041	0.003	0.049	0.003	
770.4948	1	-0.00027	770.4948	1	-0.0001	0.063	-0.0006	0.078	-0.0007	
1369.2322	1	-0.0036	1369.2322	1	-0.004	0.044	-0.003	0.093	-0.002	
σ_0 = 0.0029				0.0038		0.0006		0.0006		
C = -0.0022				-0.0012		-0.00239		-0.000008		
S = -0.0000059				-0.0000058		-0.0000069		-0.0000098		

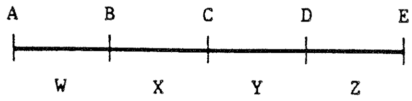

Fig. 1. Calibration baseline.

each iteration. Computations were done after introducing an error of 0.012 in the first measurement. It can be seen from Table 1 that the error of 0.012 was picked up from the second iteration onward and that the weight of each observation was ten times that of the first. These results confirm the theoretical findings.

Conclusion and Recommendation

It appears that the method of recursive weight assignment successfully computes the parameters, compensating for small errors. Thus, the method described is an effective data snooping technique justified by theory. The method is very well suited in least squares adjustment.

It is recommended that this method be further studied and applied in large aerial and terrestrial triangulation adjustments.

References

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