A CONCEPTION OF ADJUSTMENT BLOCK TERROTRIANGULATION WITH OUTLYING OBSERVATIONS

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Abstract

In large observational systems such as terrotriangulation blocks, in generality photogrammetric blocks, may occur outlying observations. The problem consists in the fact that before an adjustment we do not know which observations are outlying or not. It is not easy to solve the problem after the adjustment of the system using of the method of least squares. As a result of this method, observations having small errors may sometimes get corrections greather than the outlying observations; this occurs particulary in numerically weakly conditioned systems. An exclusion of some observations on the basis of the values of corrections may cause a rejection of good observations. Thus, we suggest to use another principle of the adjustment, the co-called "Principle of choice of an alternative" (Kadaj 1978,1980). This method enables to decrease essentially or eliminate the influence of the outlying observations. All observations take part in the adjustment.

1. Introduction

Terrotriangulation is a basis for elaboration of photogrammetric terrestial photographs. As the geometrical picture of the system there is a spatial network in which, besides photogrammetric observations (coordinates and parallaxes) there may occur arbitrary geodetic observations such as angles, distances and differences of altitudes. Thus, a significant feature of such an observational system is a great inhomogenity of the set of observations as regards the kind, a source and accuracy, and also an irregularity of the geometrical system of observations, i.e. the shape of the network. Another feature of the method in comparison with e.g. the geodetic network is a considerable number of processed sets of data (observations) for a fixed number of points to be determined. The increased number of data results from the properties of a terrotriangulation network - as a geodetic-photogrammetric network - based on miscellaneous observations.

A large number of the obteined numerical data is not free from various distortions (deviations) being significant for the results of the adjustment of the network. Besides systematic deviations and obvious errors which one can identify and eliminate, in a set of observations there may occur deviations whose reasons remain unknown (random), and their magnitudes coming out significantly beyond limits of a'priori assumed accuracy of measurement e. g. a mean error. We call them outlying observations. In the adjustment of a network by means of the least - squares method, the outlying observations may lead to significantly distorted or even completely worthless results. Thus, in this case, there is a need to eliminate or at least to neutralize the influence of those elements. In multidimensional, nonhomogeneous, and as in the case of terrotriangulation, the irregular systems identification of the outlying observations is not an easy task. However, the existence of outlying elements is indicated by mean error of a typical observation σ computed after the adjustment in relation to its value σ_0 determined a'priori before the measurement, $\sigma \gg \sigma_0$, but a large value of the correction does not necessarily indicates that the corresponding observation has actually a large real error. Identification of the outlying observations using the least - squares method for an initial adjustment may be impossible or difficult for the realization.

The another approach to the problem rely on the change or some modification of the general adjustment criterion. The characteristic feature of the least - squares method is that it fits the estimator "uniformly" to all observations. Therefore, this also concerns the outlying observations which are not a'priori overt. Some class of adjustment algorithms is known as the "Robust Estimation" (Andrews 1974, Huber 1964, Krarup, Kubik and Juhl 1980, Kubik 1982, Rey 1978, a.a.). The method is based on the application of an iterative process modifying the least - squares principle in such a way that if in a given k-th cycle the correction v exceeds the assumed value (the assumed limit of outlying) then in k+1 cycle the respective observation will get some artificially lowered weight. In this way, none of the observations is completely excludet and the estimator, in effect, gets the desired properties i.e. is a little sensible to possible deviations.

The authors have applied for the terrotriangulation network so-called "Principle of choise of an alternative" (Kadaj 1978,1980). The method does not require a definition of the limit of outlying observations and is based on a criterion defined (contrary to the maximum likelyhood method) as follows:

$$\sum_i \varphi(v_i) \longrightarrow \max. \qquad (1)$$

where: φ - denisity distribution of standardized observational errors, v_i - corrections, i - indicator of observations.
This criterion leads to the estimators being a little tender for outlying. In practice, this means that if all observations besides the outlying ones form a well conditioned overdetermined system then the outlying observations included for adjustment will not essentially deform the results. The name of the method comes from its probabilistic interpretation (Kadaj 1980), where, contrary to the maximum likelyhood principle (for the independent observations)

$$\mathcal{P}\Big(\bigcap_i Z_i\Big) = \prod_i \mathcal{P}(Z_i) \longrightarrow \max. \qquad (2)$$

(Z_i - some occurences with $\mathcal{P}(Z_i) \cong h \cdot \varphi(v_i)$, $h > 0$), the maximum probability for a sum of occurrences Z_i (an alternative of those occurrences) with some approximation is required:

$$\mathcal{P}\Big(\bigcup_i Z_i\Big) + \delta = \sum_i \mathcal{P}(Z_i) \longrightarrow \max. \qquad (3)$$

($\delta \geq 0$-an error of the approximation criterion; $\delta = 0$ if occurences Z_i are mutually exclusive).

2. The adjustment of a block terrotriangulation network according to criterion 1 with the assumption that errors of the observations have the normal distribution.

The non - linear system of equations for a block terrotriangulation network is created on the basis of three different groups of observations: photogrammetric observations (coordinates and parallaxes) which make the most numerous group, geodetic observations (horizontal and vertical angles, spatial distances, differences of altitudes) and some directly measured orientation elements of photographs. The system has the following form

$$\left.\begin{aligned} V_1^* &= F_1^*(X) - L_1^* \\ V_2^* &= F_2^*(X) - L_2^* \\ V_3^* &= F_3^*(X) - L_3^* \end{aligned}\right\} \quad V^* = F^*(X) - L^* \tag{4}$$

where: X- vector of unknowns (orientation elements of photographs, geodetical coordinates), k=1,2,3 - indicator of a group of observations, V_k^* - subvector of corrections, L_k^* - subvektor of observations, F^* - model function for the terrotriangulation observations.

Because of the non - homogenity of observations we multiply them by a matrix

$$P^{1/2} = \operatorname{diag}\left[P_1^{1/2},\ P_2^{1/2},\ P_3^{1/2}\right]. \tag{5}$$

where:

$$P_k = \operatorname{diag}_i \left[\sigma_{ki}^{-2}\right] \cdot \sigma_o^2 \quad (\text{for } k=1,2,3) \tag{5a}$$

is the matrix of weights for the k-th group, $\sigma_o^2 = \text{const.} > 0$. We get the new system, written in the form:

$$V = F(X) - L \tag{6}$$

where:

$$V = [v_i]_{(m,1)},\quad F(X) = [f_i(X)]_{(m,1)},\quad L = [l_i]_{(m,1)},\quad X = [x_j]_{(n,1)}$$

m - number of observations, n - number of unknowns. The new transformed observations in (6) are the random variables with identical normal distributions (variance σ_o^2 =const.).

Thus, the criterion (1) has the form (Kadaj 1978):

$$\Phi(X) := \sum_{i=1}^{m} \exp\left(-\frac{v_i^2}{2\cdot\sigma_o^2}\right) \longrightarrow \max. \tag{7}$$

The correction v_i are functions of the unknown vector X which may be written as

$$v_i = v_i(X) \qquad (\; V = V(X) = [v_i(X)] = [f_i(X) - l_i] = F(X) - L \;)$$

The required condition for the existence of an extremum (max of function $\phi(X)$) leads to n equations of the type

$$\frac{\partial \phi(X)}{\partial x_k} = \sum_{i=1}^{m} \exp\left(-\frac{v_i^2}{2\cdot \sigma_o^2}\right)\cdot\left(-\frac{v_i}{\sigma_o^2}\right)\cdot\frac{\partial v_i}{\partial x_k} = 0 \tag{8}$$

$(k=1,2,\ldots,n)$.

Substituting in (4)

$$a_{ik}(X) := \frac{\partial v_i}{\partial x_k} = \frac{\partial f_i(X)}{\partial x_k} \tag{9}$$

(derivatives as functions of the unknown vektor X),

$$g_i(X) := \exp\left(-\frac{v_i^2}{2\cdot \sigma_o^2}\right), \qquad v_i = v_i(X) \tag{10}$$

(functions of the vector X)

we get the equations

$$N_k(X) = \sum_{i=1}^{m} a_{ik}(X)\cdot g_i(X)\cdot v_i(X) = 0 \tag{8a}$$

$(k=1,2,\ldots,n)$. These equations can be written in the matrix form

$$N(X) = [N_k(X)]_{(n,1)} = A^T(X)\cdot G(X)\cdot V(X) = \underline{0}_{(n,1)} \tag{8b}$$

where:

$A(X) = [a_{ik}(X)]_{(m,n)}$ - matrix of partial derivatives,

$G(X) = \operatorname{diag}_i [g_i(X)]$ - diagonal matrix,

$V(X) = [v_i(X)]$ - correction vector of observations.

The sufficient condition for the existence of maximum of function $\phi(X)$ at point X: $N(X) = \underline{0}$ is that the Hessian of the function $\phi(X)$ at point X,

$$H(X) := \left[\frac{\partial \phi(X)}{\partial x_i \cdot \partial x_j}\right]_{(n,n)} = \left(-\frac{1}{\sigma_o^2}\right)\cdot\frac{\partial N(X)}{\partial X} \tag{11}$$

is negatively determined, i.e. $H(X) \leqslant \underline{0}_{(n,n)}$ in sense of a matrix inequality: if a vector $Y \neq \underline{0}_{(n,1)}$ then the quadratic form $Y^T H(X)\, Y \leqslant 0$. The matrix $\partial N(X)/\partial X$ is then positively determined.

The equation (8b) can be solved by the iterative formula

$$X^{(s+1)} = X^{(s)} - B^{+}(X^{(s)}) \cdot N(X^{(s)}), \quad \text{for } s=0,1,2,.. \tag{12}$$

where for $B(X) := -\sigma^2 \cdot H(X)$ we get the Newton's algorithm ("+"- the symbol of the pseudoinversion - Moore - Penrose inversion), or with the matrix

$$B(X) := A^T(X) \cdot G(X) \cdot A(X) \tag{13}$$

a quasi - Newton's algorithm.

In considered system one has, generally, the well approximation $X^{(0)}$ of the optimal vector X. Nevertheless, in the iterative process may occur some dificulties, for instance disturbances by the local extremums of $\Phi(X)$. To overcome these obstacles one can artificially increase the value of the variance σ^2 at the beginning of the iterative process. It leads to the artificial flattering of $\Phi(X)$. At the end of the iterative process one returns to the a' priori assumed value of the variance.

3. Schema of the computational algorithm

a) Determination of the integer parameters and a topological structure of the terrotriangulation network

b) Formulation of the observational equations $v_i^* = f_i^*(X) - l_i^*$ for $i=1,2,...,m$

c) Determination of the weight matrix P, with diagonal elements σ_0^2/σ_i^2 , $\sigma_0^2 =$ const., and the normalization of observational equations:

$$v_i = \frac{\sigma_0}{\sigma_i}\left(f_i^*(X) - l_i^*\right) = f_i(X) - l_i \quad (i=1,2,...,m)$$

d) Determination of the vector $X^{(0)}$

e) For $s=0,1,2,...$ while $s=0$ or $\|X^{(s+1)} - X^{(s)}\| > \delta$ (δ - a numerical accuracy of computations):

e1. Computation of corrections $v_i^{(s)} = v_i(X^{(s)})$, for $i=1,2,...,m$

e2. Computation of numerical functions

$$g_i(X^{(s)}) = \exp\left(-\frac{(v_i^{(s)})^2}{2\cdot\sigma_0^2}\right) \quad \text{for } i=1,2,...,m$$

e3. Computation of the matrices: $A(X^{(s)})$, $H(X^{(s)})$ and

$$B(X^{(s)}) = -\sigma_0^2 \cdot H(X^{(s)})$$

e4. Checking up whether the point $X^{(s)}$ is situated in the concavity region of the function $\phi(X)$, i.e.:

$$B(X^{(s)}) \geqslant \underline{0}$$

(If this condition is fulfilled then the Cholesky's factorization of the matrix B for real numbers exists). Otherwise, one increases the mean error σ_0 by a certain step

$$\sigma_0 := \sigma_0 + \Delta$$

e5. Computation of the increment $\underline{\delta X}^{(s,s+1)}$ of the vector $X^{(s)}$ and the subsequent approximation this vector:

$$X^{(s+1)} = X^{(s)} + \underline{\delta X}^{(s,s+1)}$$

e6. Return with $\sigma_0 := \sigma_0 - \Delta$

f) Computation of all geometrical parameters for the terrotriangulation (Rychlewski 1982,1983)

g) Computation of mean errors for three groups of observations and for the geodetical coordinates. The variance - covariance matrix for the unknown vector in this method is presented in (Kadaj 1980).

The presented algorithm has been introduced to the existing program for the adjustment of the terrotriangulation networks: Rychlewski 1982,1983. At present the testing of the program, using simulated sets of data, is performed.

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