

## CONTROL FEATURES - AN ALTERNATIVE SOURCE FOR URBAN AREA CONTROL

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### 1. ABSTRACT

The establishment of the field control is economically demanding and its preservation is difficult, specially in the developing countries. A large part of the control points is lost between two successive mappings. Consequently, an intensive work for providing control points is repeated indefinitely. This repetition has higher frequency in great cities regions, where new maps are more frequently required and the changes imposed by the development make the preservation of control even more critical.

This work shows the potential of the digitized features, an almost perenial control, for substituting control points. Digitized features were used as control to determine, analytically, the parameters of the spatial similarity transformation. The mathematical model was established and implemented in a computer program and tested.

Satisfactory results were obtained and are presented.

### 2. INTRODUCTION

The establishment of field control for mapping purposes is time consuming, expensive and great part of it is lost from one mapping to the next. The signs and monuments are destroyed in a short period of time, specially in developing countries, and the net of control has to be restored almost integrally for every new cartographic task.

Transferring points between adjacent photographs and identifying object points on the image are also time consuming, source of errors and consequently they raise the cost of the mapping task.

This work presents some results of applying digitized features as substitution for control points. The features have the advantage of being quite perenial, particularly in urban regions, as for example: buildings, avenues, roads, railroads. They also eliminate the problems of point identification and transference. The identification of features is safer than the one of points.

The feature is represented by the coordinates of the discrete set of points which represent it. These coordinates are used to define a mathematical function that represents the actual feature. This procedure makes possible a new mathematical relation, between image and function.

Recent studies (Masry, 1981 and Lugnani, 1980) have demonstra-

ted the potential of the concept. In this investigation the concept is applied specifically to the spatial similarity transformation and the use of straight features (field entities which can be represented by a straight line), general curves can also be used. Their application at this model showed positive results (Souza, 1982), the model requires further tests.

### 3. DIGITIZED FEATURES

The modern computer science and technology makes possible storage, retrieval and processing of large amount of data, efficiently. It became practically attractive the establishment of control features files. They can be created, edited and retrieved at production environment technology. The knowledge of the quality of the stored data is requisit in the present case.

The term feature is used to name any object (or part of it) which can be represented by a unique spatial curve segment, as for example: a road or its central line, a railroad, the edge of a building and so on.

It is called digitized feature a discret set of point representing a feature. The digitized features are stored in files, through the coordinates of their points. These coordinates of points of a set of features, chosen from the area of interest, can be obtained by surveying or geodetic procedures; by digitization from stereoscopic models established in stereoplotting instruments; or from digitization from available maps. In every case it is required the editing of the digitized entities, and the knowledge of their quality.

The application of digital mapping technology has started in some Brazilian companies or agencies as PROCEMPA ("Processamento de Dados do Município de Porto Alegre"), and "Diretoria de Proteção ao Vôo". In the present work, however, it was chosen to use fictitious features (Mathematically Generated Features), due to the benefit they present with respect to the facility of obtaining and possibility of sharp ovality control.

### 4. MATHEMATICAL MODEL

The mathematical model, for the application of the straight line control feature concept, was developed and tested, for the spatial similarity transformation. This transformation was selected mainly due to its common use in photogrammetry and geodesy.

The similarity transformation from  $oxyz$  to  $OXYZ$  spaces can be represented by:

$$\bar{X} = \lambda M\bar{x} + \bar{X}_0$$

Where  $\bar{X}$  and  $\bar{x}$  are vectors of coordinates of a point  $p$  referred to the spaces  $OXYZ$  and  $oxyz$ ;  $\bar{X}_0$  is the vector of coordinates of origin with respect to  $OXYZ$ ;  $M$  is a orthogonal matrix of rotations and  $\lambda$  is a scale factor.

To illustrate the present development, it is assumed that  $oxyz$  space is the model space and  $OXYZ$  is a ground local system. It is also assumed that the area of the model has a set of fea-

tures  $F_i$  and the corresponding file, where the features  $F_i$  are stored as coordinates of discrete sets of points. The feature  $F_i$ , in the file, is represented by two points (or by many points if the curve is not straight) of the OXYZ space or ground space. These points are not identifiable. Only one point (or few points in case of general curve) have to be observed on the model feature  $f_i$  corresponding to  $F_i$ , so that the feature can be used as control.

The model below deals with ground features which can be considered straight. This kind of features are frequent in urban areas. These features are stored in files by two of their points. Any point  $P (X, Y, Z)$  of  $F$  satisfies the equation:

$$t = \frac{X - X_1}{X_2 - X_1} = \frac{Y - Y_1}{Y_2 - Y_1} = \frac{Z - Z_1}{Z_2 - Z_1}$$

Where  $P_1(X_1, Y_1, Z_1)$  and  $P_2(X_2, Y_2, Z_2)$  are ground points defining the stored feature.

The coordinates of  $P$  can be obtained explicitly from the equation above, in a parametric form:

$$X = X_1 + t(X_2 - X_1)$$

$$Y = Y_1 + t(Y_2 - Y_1)$$

$$Z = Z_1 + t(Z_2 - Z_1)$$

Substituting  $X, Y, Z$  of the similarity transformation by their above expressions, we have:

$$\lambda(m_{11}x + m_{12}y + m_{13}z) + X_0 - X_1 - t(X_2 - X_1) = 0 = F_x$$

$$\lambda(m_{21}x + m_{22}y + m_{23}z) + Y_0 - Y_1 - t(Y_2 - Y_1) = 0 = F_y$$

$$\lambda(m_{31}x + m_{32}y + m_{33}z) + Z_0 - Z_1 - t(Z_2 - Z_1) = 0 = F_z$$

Where  $t$  is the parameter for a specific straight line representing a given feature, and  $x, y, z$  are coordinates of a generic point observed on the model.

This model includes the seven similarity transformation parameters ( $\lambda, \omega, \phi, \kappa, X, Y, Z$ ) plus one  $t$  for each feature used as control. It is not required the correspondence of points from both spaces. It is not necessary to identify a point to observe it as illustrated in Figure 1. Three equations are written for each control feature. Consequently, a minimum of four features is required to apply least squares adjustment.

The least squares solution to the above implicit model is obtained iteratively. The correction to the approximate parameters for the 1<sup>TH</sup> iteration is given by:

$$X_i = - \left[ A_i^T (B_i P^{-1} B_i^T)^{-1} A_i \right]^{-1} A_i^T (B_i P^{-1} B_i^T)^{-1} W_i$$

Where  $P$  is the weight matrix;  $W_i$  is the misclosure vector:

$$W_i = F(L_{a_{i-1}}, X_{a_{i-1}}) + B_i(L_b - L_{a_{i-1}})$$

Where  $L_a$  is the vector of adjusted observation from previous iteration,  $X_a$  the best approximate value of the parameters, and  $L_b$  is the vector of the original observations.

The design matrix A and B are given by:

$$A = \begin{bmatrix} \frac{\partial F_x}{\partial \lambda} & \frac{\partial F_x}{\partial \kappa} & \frac{\partial F_x}{\partial \phi} & \frac{\partial F_x}{\partial \omega} & \frac{\partial F_x}{\partial X_0} & \frac{\partial F_x}{\partial Y_0} & \frac{\partial F_x}{\partial Z_0} & \frac{\partial F_x}{\partial t_1} & \frac{\partial F_x}{\partial t_2} & \dots & \frac{\partial F_x}{\partial t_{nf}} \\ \frac{\partial F_y}{\partial \lambda} & \frac{\partial F_y}{\partial \kappa} & \frac{\partial F_y}{\partial \phi} & \frac{\partial F_y}{\partial \omega} & \frac{\partial F_y}{\partial X_0} & \frac{\partial F_y}{\partial Y_0} & \frac{\partial F_y}{\partial Z_0} & \frac{\partial F_y}{\partial t_1} & \frac{\partial F_y}{\partial t_2} & \dots & \frac{\partial F_y}{\partial t_{nf}} \\ \frac{\partial F_z}{\partial \lambda} & \frac{\partial F_z}{\partial \kappa} & \frac{\partial F_z}{\partial \phi} & \frac{\partial F_z}{\partial \omega} & \frac{\partial F_z}{\partial X_0} & \frac{\partial F_z}{\partial Y_0} & \frac{\partial F_z}{\partial Z_0} & \frac{\partial F_z}{\partial t_1} & \frac{\partial F_z}{\partial t_2} & \dots & \frac{\partial F_z}{\partial t_{nf}} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

And B is a block diagonal matrix, with the first block given by:

$$B = \begin{bmatrix} \frac{\partial F_x}{\partial x} & \frac{\partial F_x}{\partial y} & \frac{\partial F_x}{\partial z} & \frac{\partial F_x}{\partial X_1} & \frac{\partial F_x}{\partial Y_1} & \frac{\partial F_x}{\partial Z_1} & \frac{\partial F_x}{\partial X_2} & \frac{\partial F_x}{\partial Y_2} & \frac{\partial F_x}{\partial Z_2} \\ \frac{\partial F_y}{\partial x} & \frac{\partial F_y}{\partial y} & \frac{\partial F_y}{\partial z} & \frac{\partial F_y}{\partial X_1} & \frac{\partial F_y}{\partial Y_1} & \frac{\partial F_y}{\partial Z_1} & \frac{\partial F_y}{\partial X_2} & \frac{\partial F_y}{\partial Y_2} & \frac{\partial F_y}{\partial Z_2} \\ \frac{\partial F_z}{\partial x} & \frac{\partial F_z}{\partial y} & \frac{\partial F_z}{\partial z} & \frac{\partial F_z}{\partial X_1} & \frac{\partial F_z}{\partial Y_1} & \frac{\partial F_z}{\partial Z_1} & \frac{\partial F_z}{\partial X_2} & \frac{\partial F_z}{\partial Y_2} & \frac{\partial F_z}{\partial Z_2} \end{bmatrix}$$

A and B are numerically estimated for the most updated set of values of the point of expansion of the Taylor Series, the approximate parameters  $X_{i-1}$ , and the observed values  $L_{a_{i-1}}$ .

$X_i$  and  $V_i$  are computed in the adjustment and the point of expansion updated:

$$X_{a_i} = X_{a_{i-1}} + X_i$$

$$L_{a_i} = L_{a_{i-1}} + V_i$$

The model requires three or four iteration to converge.

## 5. RESULTS OF TESTS

Three tests are presented here, for the developed model. In the first one the geometric distribution of the control features as well as the number of them are deteriorated from a, practically, ideal to a critical distribution. The results are computed and given in Table 1. (in every table the computed values can be compared to the actual values once the data are fictitious. Random fluctuations were introduced in the generated data to simulate observations).

Figure 2 illustrates four different arrangements of entities and the corresponding results are in columns three to six of Table 1. In the second test a comparison is made between conventional and feature control solutions. The geometrical configuration of control points and entities are shown in Figure 3 and the results shown in Table 2, together with the actual parameters. The last test shows that the quality of approximate parameters is not very critical to the convergence of the model. Table 3 is self explanatory.

## 6. CONCLUSIONS AND RECOMMENDATIONS

The tested model is suitable to establish field control for computing the parameters of a spatial similarity transformation.

The reduction in the number of control features as well as degradation of their geometric configuration degrades the results as it happens to the control points solution.

The accuracy of the parameters obtained by conventional and by the new approach means are about the same for the same degree of freedom.

The quality of the approximate parameters is not critical for the convergence of the solution.

It seems reasonable to recommend the establishment of special files with feature control in urban areas. They can constitute a more perennial net of control.

Research on the structure and organization of such files are recommended.

## 7. REFERENCES

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TABLE 1 - EFFECTS OF GEOMETRIC DEGRADATION OF CONTROL CONFIGURATION AND REDUCTION OF NUMBER (NF) OF CONTROL ON THE COMPUTED PARAMETERS

ACTUAL VALUES OF PARAMETERS		COMPUTED PARAMETERS			
		Fig 2(a)	Fig 2(b)	Fig 2(c)	Fig 2(d)
K (rad)	0,87266	0,87250	0,87286	0,88680	0,87272
$\emptyset$ (rad)	0,00000	0,00025	0,00080	0,02593	0,01185
W (rad)	0,00000	0,00039	0,00173	0,00424	0,01317
XO (m)	3500,00	3499,637	3498,801	3428,998	3509,444
YO (m)	2000,00	2000,428	1999,508	1976,147	2000,903
ZO (m)	700,00	699,727	699,592	574,967	639,132
$\lambda$	10,0000	9,9979	10,0005	9,8841	9,9973

TABLE 2 - COMPARISON OF RESULTS OBTAINED WITH THE USE OF CONVENTIONAL CONTROL AND FEATURE CONTROL TEN FEATURES OR TEN CONTROL WERE USED (NFR = NP = 10)

ACTUAL VALUES OF PARAMETERS		COMPUTED PARAMETERS	
		Fig 3(a)	Fig 3(b)
K (rad)	0,78539	0,78523	0,78541
$\emptyset$ (rad)	0,00000	0,00020	0,00001
W (rad)	0,00000	0,00023	0,00000
XO (m)	50,000	49,634	50,148
YO (m)	300,000	300,430	300,731
ZO (m)	100,000	99,728	100,062
$\lambda$	10,0000	9,9989	9,9997

TABLE 3 - TEST NUMBER THREE-MODEL CONVERGENCE WITH POOR QUALITY APPROXIMATE PARAMETERS

ACTUAL VALUES OF PARAMETERS		PARAMETERS	
		STRAIGHT FEATURES	
		APPROXIMATE	COMPUTED
K (rad)	0,87266	1,39626	0,87250
$\emptyset$ (rad)	0,00000	0,03580	0,00025
W (rad)	0,00000	0,05266	0,00039
XO (m)	3500,000	1750,000	3499,637
YO (m)	2000,000	1000,000	2000,428
ZO (m)	700,000	1050,000	699,727
$\lambda$	10,0000	15,0000	9,9979

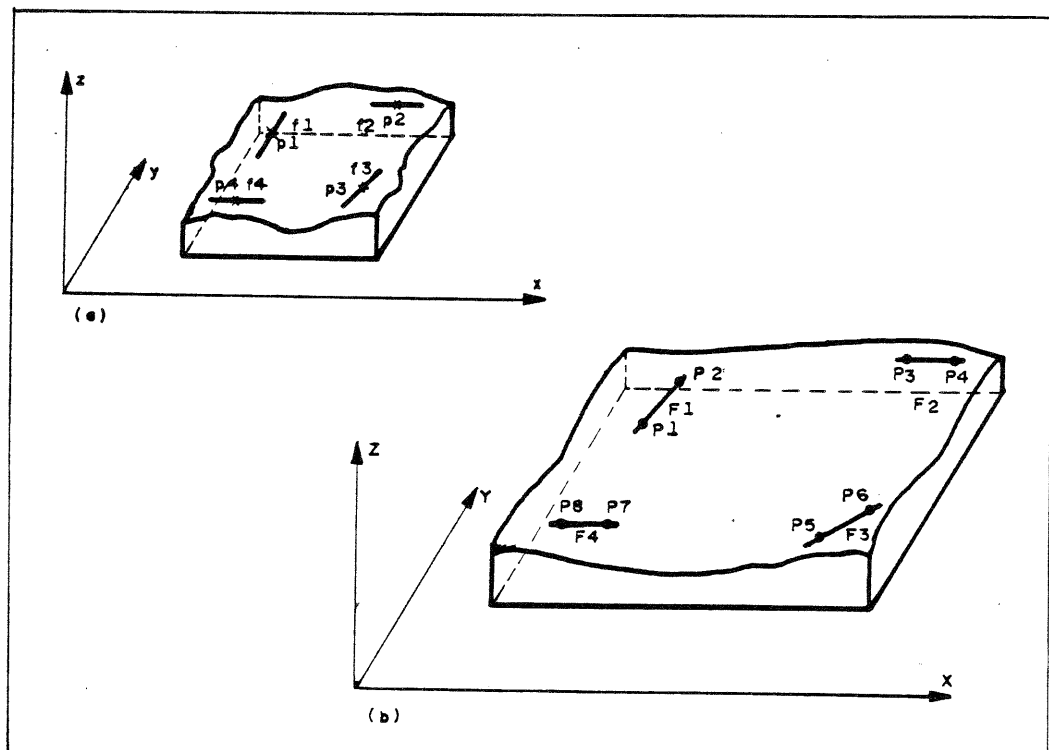


Figure 1. (a) Shows the Model and the Features  $f_i$  on it. (b) Shows the corresponding ground area. The point  $p_i$  can be any point on  $f_i$ .

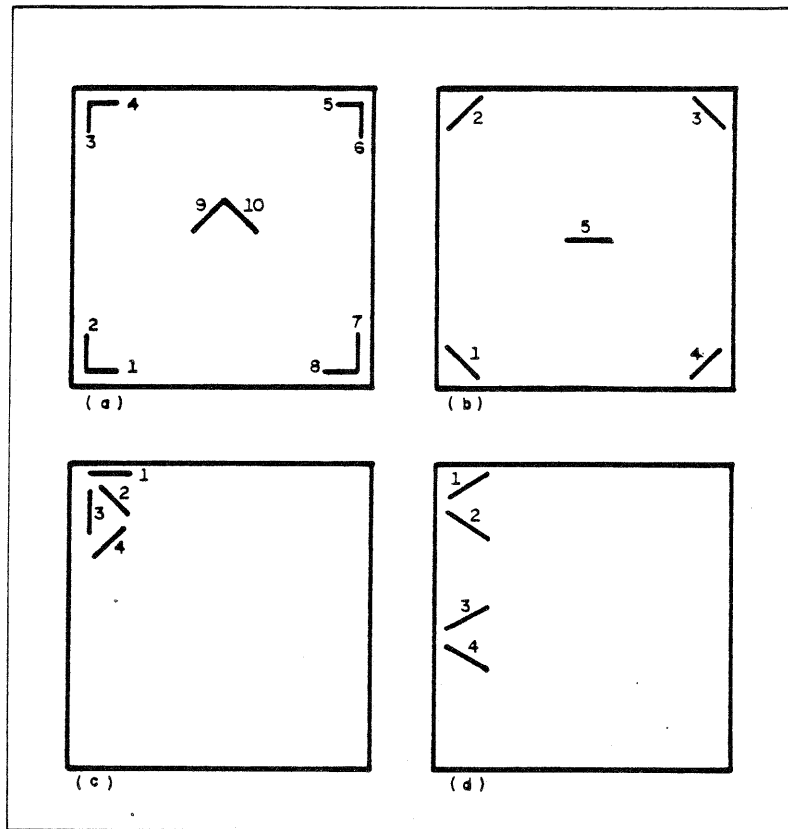


Figure 2. Number and distribution of straight features for the first test, corresponding to Table 1.

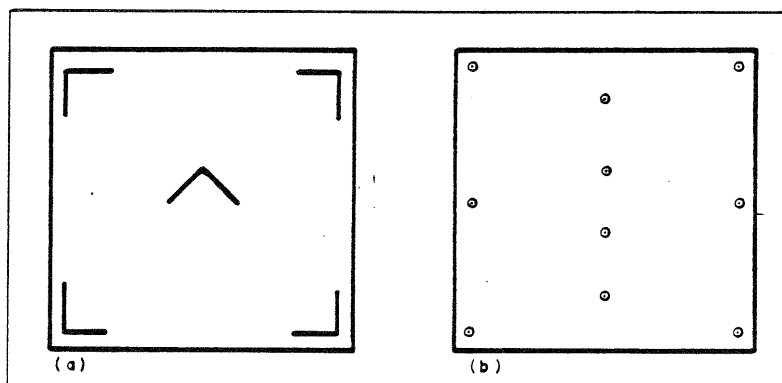


Figure 3. (a) Control features. (b) Control points. Distribution of control for test number two (refer to Table 2).