EFFICIENT METHODS FOR SELECTING ADDITIONAL PARAMETERS OF BLOCK ADJUSTMENT

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Two methods are introduced for selecting additional parameters, one related with the principle component analysis and the other based on the stepwise regression analysis. The repetition of the adjustment is avoided by using the reduced normal equations of additional parameters in the analysis. With the methods the danger of overparametrization and possible numerical instability is circumvented also in cases of wide sets of candidates for additional parameters. Special measures for controlling the geometrical quality of the model extension are developed based on the use of Q_{xx} and Q_{yy} matrices.

1 INTRODUCTION

1.1 In recent years a great number of studies have been concerned with the problem of finding a proper extension of the functional model of the bundle block adjustment for compensation of systematic image errors. The special task of selecting additional parameters from a set of candidates has aroused a wide discussion of statistical testing methods as well as of the methods for controlling the stability of the model. Before introducing new methods certain paradoxes noticed by the author are discussed.

1.2 Paradox of instability of the model: It has been a fairly general conclusion that a safety compensation of systematic image errors can only be achieved in strong photogrammetric blocks with e.g. 60 % sidelap or with dense ground control. In weaker blocks the danger of instability due to overparametrization is obvious, if additional parameters are not treated properly.

On the other hand, from the practical point of view the weak blocks with 20 % sidelap and sparse ground control are important, especially in small-scale topographic mapping. In these cases the uncompensated systematic errors can deteriorate the results considerably. As a conclusion it can be said that we have to go very near the border of instability - but we must not cross it - to give the full benefit of the use of additional parameters for the practical field. For that reason we need very sophisticated methods for selecting additional parameters.

1.3 Paradox of orthogonality: The use of additional parameters that are mutually orthogonal and with respect to other parameters, is considered as means for improving the stability (Ebner 1976, Grün 1978, Heikkilä & Inkilä 1978). At the other extreme, when the additional parameters are exactly orthogonal with respect to the coordinates of ground points, they have no compensation power, being thus useless. On the other hand, a full correlation would result in a singular system of normal equations. Thus, we have to find a balance between these two extremes.

1.4 On the concept of stability: In the text above the concept of stability is not yet well-defined. We have to distinguish between 1) the concept of the stability of the adjustment model and 2) the numerical stability.

In photogrammetry the stability of the adjustment model is in close connection with the concepts of quality and reliability (Baarda 1973, Förstner 1981). In fact, it is of primary interest. Thus, we must have means to measure the stability of the adjustment model. The degradation of stability must be kept within certain limits when the model is extended by additional parameters.

The numerical stability is of secondary interest only from the point of view of photogrammetry. It concerns the numerical methods used for solving the adjustment model. It is crucial to use numerical methods that guarantee the reliability of the numerical results but the numerical methods used should not have effect on the applied adjustment model.

2 ON THE GEOMETRICAL QUALITY

2.1 The stability of the adjustment model depends on the geometry of the problem. For making a clear separation from the numerical stability it is reasonable to speak about the geometrical quality of the adjustment model.

The accuracy and homogeneity of accuracy of coordinates of ground points are related with the geometry of the block. So are the internal and external reliability of the block.

Each of them has its own rationale, and a measure indicating the geometrical quality can be based on any of them. Additionally, the measure can be absolute or relative. The absolute measure is valid for justifying whether or not a given block fulfils some preset requirements. Absolute measures have actually been used in the numerous theoretical studies concerning the accuracy and reliability of photogrammetric blocks. The relative measure of quality can be based on the ratio of corresponding absolute measures of two cases, i.e. the geometrical quality with respect to some reference case.

2.2 When expanding the basic model of block adjustment by additional parameters the relative measure of geometrical quality is of special importance. With them we decide whether or not the model extensions degrade the geometrical quality dangerously. The check of determinability of additional parameters proposed by Förstner (1980) is based directly on the concepts of reliability.

The choice of relative measure depends on which of the four mentioned items we want to stress. The decision thus includes some experience-based heuristic reasoning. Two such measures are introduced below.

$$Z_X = \max_{x \in X} \left(\frac{q_{xx}^{(i)}}{q_{xx}^{(o)}} \right), \quad Z_V = \max_{v \in V} \left(\frac{q_{vv}^{(o)}}{q_{vv}^{(i)}} \right) \qquad \text{(1a), (1b)}$$

Z_X relative measure of accuracy,
Z_V relative measure of internal reliability,
X set of coordinates of ground points,
V set of residuals of photogrammetric image observations
(o) indicates the basic model (see next chapter for details of adjustment model),
(i) indicates some extended model indexed i,
q_{xx}, q_{vv} diagonal element of weight coefficient matrix.

For testing whether the geometrical quality has degraded with the extension of the model we set the null hypothesis

HO: geometrical quality is not degraded, which will be rejected if

$$Z_X > \alpha_X \text{ or } Z_V > \alpha_V, \qquad \text{(2a), (2b)}$$

where α_X and α_V are heuristically defined constants

$1 < \frac{\alpha_X}{\alpha_V} < a$, with $1 < a < 2$ assumingly.

It is obvious that the block includes some points having very poor $q_{xx}^{(o)}$ and $q_{vv}^{(o)}$ values, e.g. the points with observations from two photographs only. These points can be considered as "hopeless" ones for which some coordinates are, however, needed. Evidently these q_{xx} and q_{vv} values would be very sensitive to model extensions. For avoiding problems arising we can cluster them from sets X and V before applying the test. The clustering would presumably involve 1 % - 10 % fraction of sets X and V.

3 THE MATHEMATICAL MODEL

3.1 Assume we have the extended model of bundle adjustment in linear form:

$$Ax + Bs = l - v, \; P_{ll} \,, \qquad (3)$$

x vector of unknown parameters of the basic model including the coordinates of ground points and orientation elements of images, size $[u_x, 1]$,

A respective design matrix, size $[n, u_x]$,

s vector of unknown parameters of the model extension (additional parameters) size $[u_s, 1]$,

B respective design matrix, size $[n, u_s]$,

l vector of observations, size $[n, 1]$,

v vector of residuals, size $[n, 1]$,

P_{ll} weight matrix of observations, size $[n, n]$.

The model extension is taken to be dynamic in the sense that some parameters may be removed during the adjustment process. This removal can be performed smoothly by introducing fictitious observations

$$E_s = 0 - v_s;\ P_{ss},$$

E identity matrix, size $[u_s, u_s]$,

P_{ss} diagonal weight matrix, size $[u_s, u_s]$.

In the following, however, only a strict removal corresponding to an infinite element in the weight matrix is assumed. Initially $P_{ss} = 0$.

3.2 Partial reduction: The normal equation system arising from (3) has the form

$$N \left[\begin{array}{c} x \\ \hline s \end{array}\right] = b \tag{4a}$$

where

$$N = \left[\begin{array}{c|c} N_{xx} & N_{xs} \\ \hline N_{sx} & N_{ss} \end{array}\right] = \left[\begin{array}{c|c} A^T P_{ll} A & A^T P_{ll} B \\ \hline B^T P_{ll} A & B^T P_{ll} B (+P_{ss}) \end{array}\right] \tag{4b}$$

$$b = \left[\begin{array}{c} b_x \\ \hline b_s \end{array}\right] = \left[\begin{array}{c} A^T P_{ll} l \\ \hline B^T P_{ll} l \end{array}\right] . \tag{4c}$$

Let us solve the basic model by the symmetric triangular factorization

$$N_{xx} = L_{xx} L_{xx}^T; \qquad \text{factorize } N_{xx}, \tag{5}$$

$$L_{xx} y_x = b_x; \qquad \text{solve } y_x, \tag{6}$$

$$L_{xx}^T \hat{x}^{(o)} = y_x; \qquad \text{solve } \hat{x}^{(o)}, \tag{7}$$

$$Q_{xx}^{(o)} = N_{xx}^{-1} = L_{xx}^{-1^T} L_{xx}^{-1}; \qquad \text{compute } Q_{xx}^{(o)}, \tag{8}$$

$$\hat{\sigma}_o^{(o)2} = \frac{l^T P_{ll} l - y_x^T y_x}{n-u_x}; \qquad \text{compute } \hat{\sigma}_o^{(o)^2}, \tag{9}$$

where $\hat{x}^{(o)}$ solution of the basic model,

$Q_{xx}^{(o)}$ weight coefficient matrix of the unknown parameters of the basic model,

$\hat{\sigma}_o^{(o)2}$ estimated variance of the unit weight.

The factorisation (5) can be achieved by the standard Cholesky methods or e.g. by some sequential algorithm (Inkilä 1984). The sparsity structure of N_{xx} can be fully exploited in the factorisation as well as the sparsity of L_{xx} in the computation of $Q_{xx}^{(o)}$ (Sarjakoski 1984).

The reduced normal equations of the additional parameters,

$$N_{ss}^{'} s = b_s^{'}, \tag{10}$$

will now be computed with formulas

$$L_{xx} L_{sx}^T = N_{sx}^T; \qquad \text{solve } L_{sx}, \tag{11}$$

$$N_{ss}^{'} = N_{ss} - L_{sx} L_{sx}^T; \qquad \text{compute } N_{ss}^{'}, \tag{12}$$

$$b_s^{'} = b_s - L_{sx} y_x; \qquad \text{compute } b_s^{'}. \tag{13}$$

The possible numerical instability caused by the poor geometrical quality of the extended model has no influence when forming (10) by (11) - (13). The only consequence is that N_{ss} might be singular.

The actual methods for analyzing (10) and computing an estimate for s are discussed in detail in the next chapter. Once we have computed some estimate $\hat{s}^{(i)}$, the corresponding $\hat{x}^{(i)}$ is given by formula

$$N_{xx} \hat{x}^{(i)} = b_x^{(i)} = b_x - N_{xs} \hat{s}^{(i)} \tag{14a}$$

or more directly

$$L_{xx}^T \hat{x}^{(i)} = y_x - L_{sx}^T \hat{s}^{(i)}; \qquad \text{solve } \hat{x}^{(i)}. \tag{14b}$$

Assuming that $Q_{ss}^{(i)}$ (see next chapter for details) is available after the estimation of $\hat{s}^{(i)}$, the $Q_{xx}^{(i)}$ matrix can be computed by updating Q_{xx} matrix:

$$L_{xx}^{T}M_{sx}^{T} = L_{sx}^{T}; \qquad \text{solve } M_{sx}, \tag{15}$$

$$Q_{xs}^{(i)} = - M_{sx}^{T} \; Q_{ss}^{(i)}, \tag{16}$$

$$Q_{xx}^{(i)} = Q_{xx}^{(o)} - Q_{xs}^{(i)}M_{sx}. \tag{17}$$

Because of the additive nature of (17), $Q_{xx}^{(i)}$ can be updated selectively for desired elements, thus saving computational work considerably (Sarjakoski 1984).

4 METHODS FOR ESTIMATING s

4.1 Orthogonalisation method: For the reduced normal equations (10) we can make orthogonal transformation

$$C^{T}N_{ss}^{'}CC^{T}s = C^{T}b_{s}^{'} \tag{18}$$

where C is the orthogonal matrix of eigenvectors of $N_{ss}^{'}$. Making substitutions

$$D = C^{T}N_{ss}^{'} \; C \tag{19}$$

$$t = C^{T}s \tag{20}$$

$$w = C^{T}b_{s}^{'} \tag{21}$$

we get

$$Dt = w. \tag{22}$$

D being the diagonal matrix of eigenvalues of $N_{ss}^{'}$ the elements of t are mutually orthogonal. It is important that the possible singularity of $N_{ss}^{'}$ does not prevent the computation of eigenvalues and eigenvectors by using e.g. QL - algorithm (Bowdler et al. 1971). Numerical experiments carried out by the author have revealed that it is extremely important to scale the original N_{ss} matrix to have diagonal elements of unity for getting a clear separation of the elements of t. Noting by S the diagonal elements of $N_{ss}^{'}$ we get the scaled matrix $N_{ss}^{''}$ also after the partial reduction:

$$N_{ss}^{''} = S^{-1/2}N_{ss}^{'}S^{-1/2}. \tag{23}$$

The computation of eigenvalues and eigenvectors as well as the application of (19) - (22) is now performed with respect to $N_{ss}^{''}$. However, without losing generality, in the following text the scaling is assumed to be done beforehand.

It is noticeable that the orthogonalisation procedure fully corresponds to the following modification of adjustment model (3):

$$B' = BC \tag{24a}$$

$$Ax + B't = l - v,\ P_{ll}. \tag{24b}$$

Once the parameter vector $\hat{t}$ has been estimated from (22), some elements of t being set to zero (rejected), for a model extension noted (i), we can compute $\hat{s}^{(i)}$ and $Q_{ss}^{(i)}$ by reversal transformations:

$$\hat{s}^{(i)} = C\hat{t}^{(i)} \tag{25}$$

$$Q_{ss}^{(i)} = CQ_{tt}^{(i)}C^T, \tag{26}$$

where $Q_{tt}^{(i)}$ is the diagonal weight coefficient matrix with zero-elements corresponding to the rejected elements of t. The complete model is now solved by using (15) and (16).

4.2 Because of the orthogonality the procedure for selecting elements of t to be included in the extended model on the basis of their statistical significance is straightforward i.e. a strict application of Student test. A simple pretest is, however, necessary for rejecting elements t_k with very low value of $d_{t_k t_k}$ (c.f. formula (22)).

The variance of the unit weight is given by

$$SS_{ll}^{(o)} = l^T P_{ll} l - y_x^T y_x \tag{27}$$

$$SS^{(i)} = SS^{(i-1)} - q_{t_k t_k} w_{t_k t_k}^2 \tag{28}$$

$$\hat{\sigma}_o^{(i)2} = SS^{(i)} / (n - u_x - u_t^{(i)}) \tag{29}$$

where $SS^{(i)}$ the weighted sum of residuals, (i) noting the current version of model extension with (o) for the basic model,

t_k the element of t being under consideration,

$u_t^{(i)}$ the number of orthogonal parameters in the extended model (including the one under consideration).

The use of the measures of geometrical quality according to (2a) - (2b) complicates the selection procedure considerably. This is due to the fact that the orthogonality does not help in the application of tests (2a) - (2b). There are $2^{u_s} - 1$ combinations of elements of t, which all should be tested in principle.

For avoiding a complete search a heuristic algorithm based on the technique of principal component analysis (see e.g. Afifi & Azen 1979) is proposed:

1. Add the element t_k with the highest value of $d_{t_k t_k}$ into the model if
   - it is statistically significant and
   - the extended model including t_k and the previously accepted elements of t passes the test (2a) - (2b).

2. Repeat step 1 until all elements of t have been processed.

4.3 The procedure of stepwise regression analysis is also well-known in the statistical analyses. Its purpose is to select the "best" set of predictors from a given set of candidates. It is operational also for selecting additional parameters, now applied for the reduced normal equations. The procedure augmented by the test of geometrical quality is as follows:

Let us note by

S the set of candidates, i.e. the elements of s vector,
R the subset of S currently included in the extended model (R for regression),
Q the subset of S currently waiting to be included into the extended model (Q for queeing), $Q = C_S R$ in every step of the process.

Algorithm:

0. Initially $R = 0; Q = S.$

1. Inclusion phase:
   1a. Terminate process if $S = Q = 0.$
   1b. Compute Student test values t_q for all $q \in Q$ (see below for details).
   1c. Select $cand_{in}$ such that

   $$t_{cand_{in}} = \max_{q \in Q} (t_q).$$

   1d. Terminate process if $t_{cand_{in}} < t_{in-limit}$.
   1e. Add $cand_{in}$ into the model for trial, i.e. add $cand_{in}$ into R.

2. Exclusion phase:
   2a. Compute Student test values t_r for all $r \in R$ (see below for details).
   2b. Select $cand_{out}$ such that $t_{cand_{out}} = \min_{r \in R} (t_r).$
   2c. If $t_{cand_{out}} < t_{out-limit}$ then:

   remove $cand_{out}$ from set R, i.e. exclude $cand_{out}$
   go to step 2a.

3. Intermediate check:
   3a. If $cand_{in}$ is not in set R then:

   remove $cand_{in}$ from set S (permanent removal)
   go to phase 1.

4. Control of the geometrical quality:
   4a. Compute the test values for the current model according to (1a) - (1b).

4b. If the geometrical quality passes test (2a) - (2b) then:
set $R_{last\text{-}accepted} := R$
else
remove $cand_{in}$ from S (permanent removal)
reset $R := R_{last\text{-}accepted}$.

4c. Go to phase 1.

5. Termination
$R_{last\text{-}accepted}$ is the set of additional parameters to be finally included in the extended model.

In the inclusion phase of the algorithm used $t_{in\text{-}limit}$ has to be determined heuristically. A reasonable choice is $1 < t_{in\text{-}limit} < 2$. A low value results in many trials of combinations for a proper set of predictions. The value of $t_{out\text{-}limit}$ determines the predictors that will be kept in the model. It is rational to select $t_{out\text{-}limit}$ according to the conventional Student test.

This approach makes it sure that each parameter in the final model has a value differing statistically significantly from zero.

For computing the Student test values in the inclusion phase, the reduced normal equations of the included parameters must be augmented by one candidate at a time. Let

$$N^{(r)}\hat{s}^{(r)} = b^{(r)} \tag{30}$$

be the reduced normal equations according to formula (10) for the set R of parameters, which are included in the current model, noted (r). Let the reduced normal equations for trial be

$$N^{(t)}\hat{s}^{(t)} = b^{(t)} \text{ with partitioning} \tag{31a}$$

$$\left[\begin{array}{c|c} N_{RR}(=N^{(r)}) & N_{Rq} \\ \hline N_{qR} & N_{qq} \end{array}\right] \left[\begin{array}{c} \hat{s}_R \\ \hline \hat{s}_q \end{array}\right] = \left[\begin{array}{c} b_R \;(=b^{(r)}) \\ \hline b_q \end{array}\right], \tag{31b}$$

where R notes the set of parameters included in the model (r)
q notes a candidate $(q \in Q)$ which is under examination to be added, resulting in the model (t).

Assuming we use Cholesky factorisation we have for the model (r)

$$L_{RR}L_{RR}^T = N_{RR}, \tag{32}$$

$$L_{RR}y_R = b_R, \tag{33}$$

$$diag\;(Q_{RR}) = diag\;(N_{RR}^{-1}) \tag{34}$$

where L_{RR}, y_R and $diag\;(Q_{RR})$ as well as $SS^{(r)}$ have been computed.

The solution of (31a) is an easy updating procedure following analogically (5) - (17):

$$L_{RR}L^T_{qR} = N^T_{qR}; \qquad \text{solve } L_{qR} \tag{35}$$

$$N'_{qq} = N_{qq} - L_{qR}L^T_{qR} \tag{36}$$

$$Q_{qq} = N'^{-1}_{qq} \tag{37}$$

$$L_{qq} = N'^{1/2}_{qq} \tag{38}$$

$$y_q = L^{-1}_{qq} (b_q - L_{qR}y_R) \tag{39}$$

$$\hat{s}_q = L^{-1}_{qq} \; y_q \tag{40}$$

$$SS^{(t)} = SS^{(r)} - y^2_q \tag{41}$$

$$\hat{\sigma}^{2(t)}_o = SS^{(t)} \; (n-u_x-u^{(t)}_s)^{-1} \tag{42}$$

where $u^{(t)}_s$ is the number of additional parameters in the model (t).

The accomplishment of (35) - (42) is necessary for the Student test value:

$$\underset{q \in Q}{Z_{tq}} = \hat{s}_q \; \hat{\sigma}^{(t)^{-1}}_o Q^{-1/2}_{qq}. \tag{43}$$

Once the candidate $q = cand_{in}$ has been selected for inclusion we also need $\hat{s}_R$ and Q_{RR} according to formula (31a) - (31b) for the Student tests of the exclusion phase:

$$L^T_{RR} \; \hat{s}_R = y_R - L^T_{qR} \; \hat{s}_q; \qquad \text{solve } \hat{s}_R, \tag{44}$$

$$L^T_{RR} \; M^T_{qR} = L^T_{qR} \; ; \qquad \text{solve } M_{qR}, \tag{45a}$$

$$diag \; (Q_{RR}) = diag \; (Q^{(r)})+diag \; (M^T_{qR}Q_{qq}M_{qR}). \tag{45b}$$

Worth noticing is that the procedure here is based on the formulas (14a) - (17), now, however, applied for a different partitioning.

When a parameter $(cand_{out})$ is removed from the set R (exclusion phase), the factorisation changes for the parameters taken into the model after the inclusion of $cand_{out}$.

The computation procedure according to (35) - (42) must be renewed for these parameters. An updating procedure (Inkilä 1984) can be used for decreasing the amount of computations. Nevertheless, because of the usually small size of the system, it has, in this case, very little of practical significance.

4.4 More on computational considerations: The presented algorithms include a strict test of the geometrical quality according to (2a) - (2b) requiring $diag \; (Q^{(i)}_{xx})$ and $diag \; (Q^{(i)}_{vv})$ each time. The computation of $diag \; (Q^{(i)}_{xx})$ is effective by using (15) - (16). Actually, only one column of L_{sx} is under consideration. This holds also for the principal component technique assuming L_{sx} being modified explicitly following the idea of (24a) - (24b):

$$L'_{sx} = C^T L_{sx}. \tag{46}$$

The computation of $diag\ (Q_{vv}^{(i)})$ according to

$$Q_{vv}^{(i)} = P_{ll} - \left[A \middle| B\right] . Q_{\substack{xx\\ss}}^{(i)} \left[A \middle| B\right]^T \tag{47}$$

requires more extensive computations even if the sparsity can be exploited (Sarjakoski 1984). It is not economical to apply (2b) directly. Instead, a pretest can be used:

Let $W = \bar{B}\ Q_{ss}^{(i)}\ \bar{B}^T,$ (48a)

$$z_{pre} = max\ (W_{ii}),\ i = 1,\ldots,2m \tag{48b}$$

where $\bar{B}$ is an artificial design matrix, size $[2m,\ u_s]$, for additional parameters, based on the use of coordinates of m image points located in a regular grid pattern,

W_{ii} can be interpreted as an effect of the current selection of additional parameters *(set R)* transformed on the image coordinate system.

The geometrical quality is considered degraded, if

$$z_{pre} > \alpha_{pre}\ , \tag{49}$$

where α_{pre} is a heuristically defined constant, $1 < \alpha_{pre} < 5$ assumingly.

5 FINAL REMARKS

The methods presented base essentially on the modification - explicit or implicit - of the reduced normal equations of the additional parameters. These equations offer the full information we need for the statistical tests.

The methods above are selection methods, i.e. some of the candidates may be removed. However, an effective realisation of other methods based on the analysis of the reduced normal equations is also possible. E.g. the method of iterative estimation of variance components of the fictitious observations of additional parameters (Ebner 1978, Förstner 1979, Heikkilä & Inkilä 1978) is also effective when using the approach above.

A simultaneous analysis of the additional parameters has been used in the methods above. More complex analyses can also be based on the use of the reduced normal equations of additional parameters: In a hierarchical approach discrete sets of additional parameters are analysed and possibly included into the extended model one after another. This approch is applicable e.g. for first handling some global parameters and after that the others with a local influence like stripwise sets of parameters. Stripwise sets can be joined with proper constraints if there is no significant difference between their values (Ebner 1976). Many other examples can be found.

All the presented methods result in a single solution of the complete normal equation system, being thus computationally effective. The problem of numerical instability never occurs until in the analysis phase of the reduced normal equations. Even then it is controlled by the test for the geometrical quality of the extended model.

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