AN EXTENDED MATHEMATICAL MODEL FOR AERIAL TRIANGULATION

Ralf Schroth
Photogrammetric Institute, Stuttgart University
Germany, Federal Republic

Commission III

Summary: In high precision point determination the variation of systematic errors can be described by an extended mathematical model. There the additional parameters are introduced as varying quantities with blockinvariant mean values and the correlation between the images is modelled by an autoregressive process. The model is discussed with respect to proximity to reality, numerical effort and the possible increase of accuracy based on simulations. The impact of the model onto future experiments for the determination of the stochastic properties of image coordinates is shown.

Zusammenfassung: Bei der hochgenauen photogrammetrischen Punktbestimmung läßt sich die Variation der systematischen Bildfehler mit Hilfe eines erweiterten mathematischen Modells erfassen. Hierbei werden zusätzliche Parameter als um einen blockinvarianten Mittelwert variierende Größen betrachtet und Korrelationen zwischen den Bildern durch einen autoregressiven Prozeß modelliert. Das Modell wird in Hinblick auf Realitätsnähe, numerischen Aufwand und zu erwartende Genauigkeitssteigerung an Hand von Simulationsergebnissen diskutiert. Die Auswirkungen des Modells auf künftige Versuche zur Erfassung der stochastischen Eigenschaften von Bildkoordinaten werden angesprochen.

Résumé: Dans la détermination photogrammétrique de points avec très grande précision, on peut saisir la variation des erreurs systématiques à l'aide d'un modèle mathématique élargi. Dans ce procédé, on considère des paramètres supplémentaires comme des données variant autour d'une valeur moyenne de variance de blocs nulle et on modèle des corrélations entre les images par un processus autorégressif. A l'aide de résultats de simulations on discute le modèle en vue de proximité de la réalité, du temps et des moyens numériquement investis et de l'augmentation de la précision à attendre. Les effets du modèle pour de futurs essais de saisir les propriétés stochastiques de coordonnées-image sont abordés.

1. INTRODUCTION

The mathematical model for aerial triangulation has reached a level of high quality. For most practical applications like point determination for cartographic plotting or densification of geodetic networks, the existing methods for block adjustment, especially with bundles, on which this paper will be restricted, are sufficient. The status of the refinement of the mathematical model has nearly been unchanged since the proposal by Ebner (1976) to describe the additional parameters as stochastic quantities. The refined functional model by using locally, e.g. stripwise, different sets of additional parameters for compensating changing systematic effects usually gives good results, provided that numerical instabilities by overparameterisation are prevented. But the stochastic model still is very simple. In practical application the image coordinates are assumed to be uncorrelated and of equal precision. The standard deviation of the additional parameters mostly are restricted to the extreme values zero or infinite (caused by operational reasons) or are set to the values of the expected size for the reasons of stability and increase of accuracy (see Kilpelä, Heikilä and Inkilä (1981)).

The nearly stagnating evolution of the mathematical model in the past few years is essentially caused by the unknown stochastic properties of the image coordinates just like the correlations within and between the images. But there are still discrepancies between the theoretical expectation of the possible accuracy by photogrammetric point determination and the empirical results, which is the motivation to work on further development of refined models for aerial triangulation.

Some empirical investigations concerning the stochastic model have been conducted by Klein (1980), Schilcher (1980) and Schroth (1982). The results of the testblock Appenweier have shown, that the values of stripwise additional parameters significantly vary from strip to strip (Klein (1980)). The same characteristics are proved by Schilcher (1980) with his analysis of the stochastic model of image points using testfield Rheidt material. His investigations have shown a significant variation of the systematic effects from image to image. Based on these results Schroth (1982) has analysed the systematic errors of time series of up to 76 images, which were derived from flights with reseau cameras. The results can be summarized as follows:

- The systematic effects significantly differ from their mean value.
- The correlation between the images caused by the systematic errors goes up to 70 %. In the average it ranges between 10 and 30 %.
- The achieved autocorrelation functions can be approximated by an exponential function exp(-c·Lag), being characteristic for a discrete first order Markov-process.

All these investigations show that the additional parameters may be modelled as stochastic variables with a non-zero expectation. The separation of the stochastic model into correlations within the images on one side, and correlations between images on the other seems to be meaningful.

After an introduction in the time series analysis, the paper will present an extended mathematical model for the bundle block adjustment. A comparison of different mathematical models by simulation and the discussion of the new model with an outlook on further tests is enclosed.

2. IMAGES AS TIME SERIES

The analysis of the stochastic properties of image coordinates and their definition in the mathematical model requires a structured stochastic model. We will separate the stochastic model into correlations within and between images. This section will be restricted to the variation of systematic image deformations. As an example figure 1 shows the variation of parameter p_8 (Ebner's set of additional parameters) within an analysed flight mission of the testfield Rheidt (see Schroth (1982)).

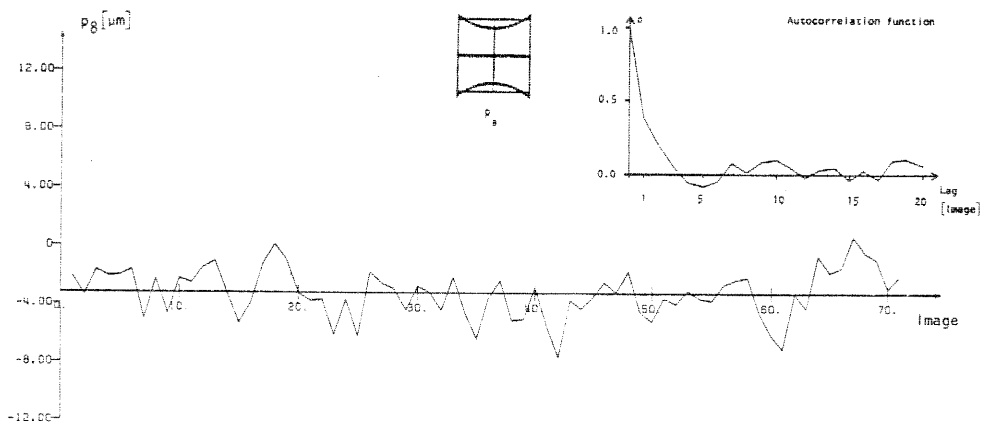

Fig. 1: Standardized values of Ebner's parameter p_8; flight mission Rheidt (13.05.74), camera WILD RC 8-R10

The structure of the photogrammetric process and the empirical results suggest to treat the images as time series, whose deformations may be modelled by a Markov-process. So the sequence of a single parameter z_t (type of systematic deformation) can be described by the following autoregressive scheme:

$$\boxed{z_t = \sum_{i=1}^{k} a_i z_{t-i} + \varepsilon_t} \tag{2.1}$$

Thus, it is assumed that z_t linearily depends on the k previous values z_{t-i} of the parameter. In eq. (2.1) the expectations $E(\underline{z})=E(\underline{\varepsilon})=\underline{0}$ and the dispersion $D(\underline{\varepsilon})=\sigma^2 \underline{I}_{p-k}$ are assumed to be given; k is the order of the autoregressive process, p the number of epoches (images), a_i the process coefficients, z_t the stochastic variable of the systematic deformation at the time t and ε_t the corresponding error of the process which is described by white noise. A necessary condition of the autoregressive process is the stationarity, i.e. the trend of the systematic deformations must be eliminated ($E(\underline{z})=\underline{0}$).

Eq. (2.1) may be rewritten as a vector equation

$$\underline{Dz} = \underline{n} \tag{2.2}$$

with $\underline{n}^T = [\nu_1, \ldots, \nu_k, \varepsilon_{k+1}, \ldots, \varepsilon_p]$, where ν_k are process errors till the starting point.

For an autoregressive process of order 1 (AR(1)-process) the coefficient matrix $\underline{D}$ ($o(\underline{D})=p\times p$) and the covariance matrix $D(\underline{z})=\underline{C}_{zz}$ ($o(\underline{C}_{zz})=p\times p$) of the systematic deformations can be written as

$$\underline{D} = \begin{bmatrix} 1 & 0 & . & . & 0 & 0 \\ -a & 1 & . & . & 0 & 0 \\ . & . & . & & . & . \\ . & . & & . & \vdots & . \\ 0 & 0 & . & . & -a & 1 \end{bmatrix}, \quad \underline{C}_{zz} = \sigma^2 \begin{bmatrix} \frac{1}{1-a^2} & & & \\ & 1 & 0 & \\ & & . & \\ & 0 & & . \\ & & & & 1 \end{bmatrix} = \sigma^2 \underline{Q}_{nn} \tag{2.3}$$

where a is the process coefficient.

The extension of eq. (2.1) to several time series of systematic deformations with the same stochastic properties between the epoches leads to the following covariance matrix:

$$\underline{C}_{zz} = \underline{Q}_{nn} \otimes \underline{C}_{ss} \tag{2.4}$$

with the Kronecker product $\otimes$ and the covariance matrix $\underline{C}_{ss}$ ($o(\underline{C}_{ss})=q\times q$, q=number of deformation types) of the systematic image deformations reduced by their trends.

The estimation of the process coefficients and more detailed information about time series analysis is described in Box/Jenkins (1976). For the following extended mathematical model eq. (2.1) is essential, because it enables a functional formulation of the stochastic properties of the systematic image deformations shown by the covariance function for an AR(1)-process:

$$C(z_i, z_j) = \frac{\sigma^2}{1-a^2} a^{|i-j|} \ . \tag{2.5}$$

Thus the parameter a essentially is the correlation between two parameters of adjacent images. It is assumed to be identical for all parameters. Based on the results of the analysis of 5 flight missions we will restrict the AR-process to order one.

3. AN EXTENDED MATHEMATICAL MODEL FOR PHOTOGRAMMETRIC POINT DETERMINATION

(1) In the following mathematical model the expectation of the observations will be a function of fixed and stochastic unknown parameters just like the collocation model. The fixed parameter group $\underline{x}$ contains the 6 transformation parameters of the perspective projection for each image and the unknown object coordinates. The stochastic parameter group contains the random systematic deformations $\underline{z}$ which vary about their mean values (expectations) $\underline{z}_o$. The expectations $\underline{z}_o$ simultaneously stand for the trends of the AR(1)-processes and belong to the functional part of the model. The vector $\underline{z}_o$ is equivalent to the blockinvariant additional parameters for selfcalibration. The variation of the systematic effects is described by the correlations between the imagewise parameter groups $\underline{z}^{(i)}$ (i = epoch, image) and by the stochastic properties of the individual parameters $z_j^{(i)}$ represented by their covariance matrix $\underline{C}_{ss}$. The correlations between the epoches are modelled by an AR(1)-process (see Förstner (1982)).

So the extended model may be defined in the form of the following Gauß-Markov model:

$$\boxed{E\begin{pmatrix}\underline{l}\\ \underline{0}\end{pmatrix} = \begin{pmatrix}\underline{A}\underline{x} + \underline{B}\underline{z}_o + \operatorname{diag}(\underline{B}^{(i)})\underline{z}\\ (\underline{D}\otimes\underline{I}_q)\underline{z}\end{pmatrix} \quad \text{and} \quad D\begin{pmatrix}\underline{l}\\ \underline{0}\end{pmatrix} = \begin{pmatrix}\underline{C}_{\varepsilon\varepsilon} & \underline{0}\\ \underline{0} & \underline{C}_{zz}\end{pmatrix}} \tag{3.1}$$

where is in addition to section 2:

$\underline{l}$, $\underline{0}$	=	vectors of observations including additional observations for control points ($o(\underline{l})$=n×1, $o(\underline{0})$=pq×1)
$\underline{x}$, $\underline{A}$	=	vector of fixed unknowns (transformation parameters, object coordinates) and corresponding coefficient matrix ($o(\underline{x})$=u×1, $o(\underline{A})$=n×u)
$\underline{z}_o$, $\underline{B}$	=	vector of unknown additional parameters (blockinvariant) and corresponding coefficient matrix ($o(\underline{z}_o)$=q×1), $o(\underline{B})$=m×q)
$\underline{z}$	=	vector of the unknown imagewise additional parameters, ordered with increasing image numbers ($o(\underline{z})$=pq×1)
$\operatorname{diag}(\underline{B}^{(i)})$	=	corresponding coefficient matrix of type block diagonal ($o(\underline{B}^{(i)})=m_i\times q$, $o(\operatorname{diag}(\underline{B}^{(i)}))$=m×pq)
$\underline{C}_{\varepsilon\varepsilon}$	=	covariance matrix of image and control point coordinates ($o(\underline{C}_{\varepsilon\varepsilon})$=n×n)
$\underline{C}_{zz}$	=	covariance matrix of the additional parameters, see eq. (2.4), ($o(\underline{C}_{zz})$=pq×pq)
m	=	number of observations of image coordinates
n	=	number of observations of image and control point coordinates

(2) The presented mathematical model has a strongly refined functional part, because the variation of the systematic deformations described by the additional parameters is integrated by their imagewise determination. Equally the correlations between the images are comprised by the functional model with the AR(1)-process. The restriction on the AR(1)-process is caused by foregoing empirical investigations and the easy managing of this type of stochastic process. The type of parameter set for the additional parameters is not restricted, but an orthogonal set will be of clear practical advantage, because in this case the covariance matrix $\underline{C}_{ss}$ can realistically assumed to be a diagonal matrix.

The stochastic model is separated into two parts. First the stochastic properties of the systematic errors are described by the covariance matrix $\underline{C}_{zz}$ which in case of an orthogonal parameter set and of independent time series for each parameter is a diagonal matrix. The remaining part of the stochastic model contains the stochastic properties within the images and those of the geodetic control. The structure of the covariance matrix of the image coordinates has to be analysed by further investigations. It should contain the errors of the measuring process and the local short periodical image deformations caused, for example, by unflatness of the pressure plate, the moving part of the pressure plate or climatic influences during the exposure, i.e. the physical influences. But the refined functional model and the consideration of the correlations between the images probably do not put great influence on the parameter estimation caused by a simplified covariance matrix just like a diagonal matrix for the image coordinates.

(3) The parameter estimation in the Gauß-Markov model of eq. (3.1) is a well known procedure. The reduction of the normal equations onto the orientation and the additional parameters is possible, if the observations can be treated uncorrelatedly, at least if no correlations between points can be assumed. The stochastic treatment of the imagewise additional parameters including the AR(1)-process causes only an addition onto the reduced normal equation system. The estimation of the process coefficient a is possible, but it must be an iterative solution because it occurs in the functional <u>and</u> the stochastic model (see eq. (2.3)). Also the estimation of the covariance matrix $\underline{C}_{ss}$ is feasible by variance-covariance-component estimation. Especially if the covariance matrix $\underline{C}_{ss}$ is a diagonal matrix the estimation procedure is not so expensive (see Förstner (1979)) presumed one has got enough images to reach convergence.

The problem of overparameterisation will be not critical, because the stochastic treatment of the imagewise additional parameters will <u>not</u> reduce the redundancy under the level of the mathematical model with blockinvariant parameters.

The increase of the numerical effort with the extended model against the model with blockinvariant parameters essentially is a function of the number of parameters per image. The setting up of the normal equations increases the number of operations (multiplications) by the quadratic factor of the ratio of transformation parameters to the sum of all parameters per image. For the solution of the equation system the increase goes up to power cube of this ratio.

(4) The application of the extended model will be restricted to the field of high precision point determination. So the main task of the model is to reach a high level of accuracy in photogrammetric network densification or deformation analysis. In the near future however the scientific aspect of the model will dominate. The possibility of the variance-covariance estimation of the additional parameters and the estimation of the correlation between the images allows further investigations on the stochastic model with empirical tests.

So the extended mathematical model is a further step into the refinement of the mathematical model for aerial triangulation. It contains the main part of the earlier neglected stochastic model. The efficiency of the extended model will be proved by simulations in the following section.

4. COMPARISON OF DIFFERENT MATHEMATICAL MODELS BY SIMULATION

The extended mathematical model (eq. (3.1)) is now compared with different models for aerial triangulation, which are described in table 3. The aim of this comparison is to show the increase of accuracy of point determination with the increase of refinement of the mathematical model.
A photogrammetric block of 4 stripes and 6 images each is simulated with 60% foreward and side overlap (see fig. 2). Each image contains 25 points regularly distributed by the 5×5 scheme. For planimetric and vertical control 12 points are used ($i \approx 1.5b$). Only the inner area (fig. 2, hatched area) of the block is applied for situating the check points as the geometric stability there is guaranteed. The coordinates of the control and check points are simulated free of error.

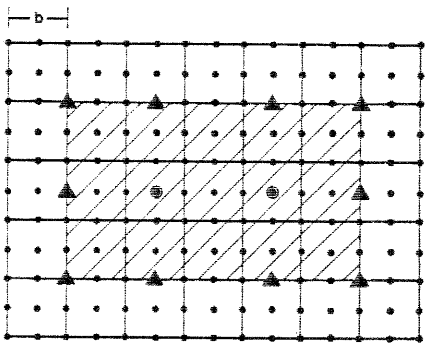

Fig. 2: Simulated block with 60% foreward and side overlap
b = base length
• = image, resp. check point
▲ = planimetric and vertical control
◉ = vertical control
/// = area of evaluation

The image coordinates are mounted with random errors of 1µm standard deviation. Table 1 shows the simulation models with different types of systematic effects onto the image coordinates which are used for the investigations. With each simulation model independently has been created 10 blocks

type	random errors $\sigma_\varepsilon^2 = 1\mu m$	systematic effects blockinvariant	systematic effects imagewise $E(\underline{z})=\underline{0},\ D(\underline{z})=\underline{C}_{zz}$	correlations (AR(1)-process) $a=0.7$
SA	x	—	—	—
SB	x	x	—	—
SC	x	—	x	—
SD	x	x	x	—
SE	x	x	x	x

Table 1: Simulation models Si (i = A, ..., E)

to get a sufficient sample. The systematic errors are generated by additional parameters of Grün (1978) for 25 image points, which are equivalent to the parameters no. 5, 6, 7, 8, 11 and 12 of Ebner's set. Their mean values, the corresponding standard deviations and the correlations between the images are results of the autoregressive analysis of several flight missions. For the simulation of the correlation a process coefficient of $a = 0.7$ is used. Table 2 shows the chosen values for the simulated systematic effects.

parameter type (Ebner's set)	5	6	7	8	11	12
Mean value [µm]	-1.7	1.2	-5.8	-1.3	-1.1	-0.6
variance [µm²]	0.52	0.79	0.80	0.67	0.22	0.25

Table 2: Applied values for generating the systematic errors (standardized onto the maximum distortion effect in the image)

The different mathematical models which are applied by the analysis start with the simple perspective model and the model with blockinvariant additional parameters and are refined using the extended model analog to the simulation models (see table 3). In addition statistical informations for each type of determination are given.

All the determinations of the different simulated blocks are done by an extended version of the programme PAT-B at our mini-computer Harris H 100. The results of the block adjustments are shown in table 4.

type	mathematical model		number of		redundancy
	functional	stochastic	observations	unknowns	
RA	PP	$\underline{C}_{11}=\sigma_\varepsilon^2\underline{I}$	1200	591	609
RB	PP, BAP	$\underline{C}_{11}=\sigma_\varepsilon^2\underline{I}$	1200	597	603
RC	PP, IAP	$\underline{C}_{11}=\sigma_\varepsilon^2\underline{I}$, $\underline{C}_{zz}$=diag	1344	735	609
RD	PP, BAP, IAP	$\underline{C}_{11}=\sigma_\varepsilon^2\underline{I}$, $\underline{C}_{zz}$=diag	1344	741	603
RE	PP, BAP, IAP, CORR	$\underline{C}_{11}=\sigma_\varepsilon^2\underline{I}$, $\underline{C}_{zz}$=diag, a=0.7	1344	741	603

Table 3: Mathematical models Ri (i = A, ..., E)
PP = perspective projection
BAP = blockinvariant additional parameters
IAP = imagewise additional parameters
CORR = correlation between images formulated by AR(1)-process

	SA				SB				SC				SD				SE			
	RA/SA				RA/SB				RA/SC				RA/SD				RA/SE			
RA	0.56	0.61	1.23	0.59	1.28	1.76	8.36	1.54	0.73	0.84	1.63	0.79	1.31	1.84	8.51	1.60	1.46	1.83	8.53	1.66
	1.48	1.50	3.48	1.01	3.71	4.76	15.33	2.76	2.09	2.09	4.16	1.34	3.56	5.19	15.64	2.86	3.59	4.95	15.30	3.04
					RB/SB				RB/SC				RB/SD				RB/SE			
RB					0.57	0.58	1.17	0.58	0.72	0.85	1.63	0.79	0.75	0.80	1.64	0.78	0.96	0.87	1.65	0.92
					1.56	1.55	3.17	1.00	2.09	2.12	4.17	1.33	1.98	2.14	4.21	1.33	2.37	2.29	4.21	1.53
									RC/SC				RC/SD				RC/SE			
RC									0.61	0.66	1.27	0.64	0.73	0.94	2.62	0.84	0.82	0.89	2.52	0.85
									1.74	1.74	3.57	0.99	2.11	2.48	5.39	1.57	2.28	2.17	5.02	1.64
													RD/SD				RD/SE			
RD													0.61	0.67	1.31	0.64	0.70	0.68	1.30	0.69
													1.55	1.71	3.51	1.00	1.79	1.75	3.30	1.06
																	RE/SE			
RE																	0.67	0.65	1.30	0.66
																	1.73	1.71	3.37	1.01

Example:

Determ./Simul.			
μ_x	μ_y	μ_z	μ_{xy}
ε_x	ε_y	ε_z	$\hat{\sigma}_o$

μ: mean square values of residuals of check points; $\mu_{xy}=\sqrt{(\mu_x^2+\mu_y^2)/2}$
ε: maximum coordinate differences at check points
$\hat{\sigma}_o$: estimated standard deviation of the unit weight

Table 4: Results of the block adjustments (in µm); mean values of 10 cases each

The analysis of the 15 combinations of simulation and determination models leads to the following results:

1. Using identical models for simulation and determination (diagonal part of table 4), the retults are equivalent to the theoretical expectation of the accuracy of bundle adjustment shown by Ebner et al. (1977). The mean square values of residuals of check points range in planimetry (μ_{xy}) between 0.58 and 0.66 µm and in height (μ_z) between 1.17 and 1.31 µm (theoretical values: $\mu_{xy} \approx 0.6$ µm, $\mu_z \approx 1.2$ µm). The estimated variance of the unit weight is the same as the simulated noise ($\bar{\sigma}_o^2 \approx \sigma_o^2$); $\bar{\sigma}_o$ ranges between 0.99 and 1.01 µm.

2. The increase of accuracy between the simple model RA and the model with common systematic RB goes up to factor 2 - 7; for example RASB → RBSB (μ_{xy}: 1.54 → 0.58, μ_z : 8.36 → 1.17) or RASE → RBSE (μ_{xy}: 1.66 → 0.92, μ_z : 8.53 → 1.65). But with increasing proximity to reality of the simulation model (SB → SE), this factor is decreasing.

3. The expected behaviour of the model RB, which theoretically is only considering the common systematic effects, can be confirmed at the simulation model SC. The results of the cases RASC and RBSC are identical, so the model RB will not be influenced by the local systematic effects. Equally the case RBSD gives the same result, so that also by superimposition of blockinvariant and local systematic effects the model RB takes only the common part into account.

4. The case RCSD gives significantly better results than the case RASB (μ_{xy}: 0.84 ← 1.54, μ_z : 2.62 ← 8.36), although the model RC is expected to compensate only the local systematic effects. So the model RC might have compensated for some essential parts of the common systematic.

5. There is a significant increase of the accuracy from model RB to RD by factor 1.2 - 1.4, which can be shown at the cases RBSD → RDSD (μ_{xy}: 0.78 → 0.64, μ_z : 1.64 → 1.31) and RBSE → RDSE (μ_{xy}: 0.92 → 0.69, μ_z : 1.65 → 1.30). This factor rises with increasing level of simulation (SD → SE).

6. The consideration of the correlation between the images with the model RE causes only an increase of the accuracy of 5% compared with the model RD (RDSE → RESE: μ_{xy}: 0.69 → 0.66, μ_z : 1.30 → 1.30); this increase is not significant. But in the case RDSE the estimated variance of the unit weight ($\bar{\sigma}_o^2$=1.13 µm²) differs significantly from the simulated noise (σ_o^2 = 1 µm²), so an improvement considering the correlation between the images is small but seems to be realistic.

7. As a consequence of the results of this simulations all the analysed mathematical models can be ordered with increasing level of accuracy: RA → RC → RB → RD → RE. The higher level of the model RB with the blockinvariant additional parameters than the model RC with the imagewise additional parameters is caused by the dominating effect of the common systematic deformations (mean value) against their variances in this special study. Decisive for the efficiency of the models RD and RE with the common and the imagewise additional parameters is the ratio of the variances of the additional parameters to the noise of the measurement.

Of course this investigation restricts only to one photogrammetric block with fixed control pattern, but it is a typical arrangement for precise point determination and the simulated data are of realistic material. So the results may give some detailed informations about the refinement of the mathematical model for aerial triangulation.

5. DISCUSSION

In this paper it is shown that a further refinement of the mathematical model is successful with respect to the accuracy. But it is also evident that the rate of increase cannot be as high as in earlier stages with the introduction of the additional parameters. Furtheron the increase of accuracy requires a high numerical effort.

The simulations did not prove the effectiveness of integrated correlations between the images onto the accuracy. But there is nearly no additional numerical effort to consider these correlations compared with the imagewise setup of additional parameters. The low additional numerical effort justifies the integration of correlations between images by 5% gain in efficiency. Further investigations especially with empirical data should be done to analyse this effect. Additionally the variation of the systematic deformations, the order of the AR-process and its process coefficients, the imagewise dispersions and their mathematical formulation, e.g. by covariance functions with short correlation distance, should be investigated.

Therefore some practical tests for the analysis of the stochastic properties of image coordinates are suggested:

1. A first step might consist of investigations of reseau images by time series analysis recently shown by Schroth (1982). If an additional reseau camera is mounted at a conventional photogrammetric flight, these images can be obtained in a cheap way under realistic conditions, because no ground control is necessary for the analysis. Of course in this first step one cannot get the effects of point definition, refraction or image motion.

2. After the correction of the correlations between the reseau images and the local systematic deformations one can estimate and analyse the covariance matrices within the images with methods shown by Förstner and Schroth (1982).

3. For getting the whole systematic effects with the above mentioned methods, investigations with test fields are necessary. The geodetic coordinates of these points must not really be known, because one photogrammetric block could be taken as control and the remaining differences to other blocks can be analysed. Because these are expensive investigations, a proper planning has to be performed by simulating different cases, which have to be analysed in view of getting the optimum parameters for such a project.

REFERENCES

Box/Jenkins: Time Series Analysis, Forecasting and Control. Holden-Day, San Francisco 1976

Ebner, H.: Self Calibrating Block Adjustment. Proceedings of the ISP-Congress (Comm. III), Helsinki 1976

Ebner, H., Krack, K. and Schubert, E.: Genauigkeitsmodelle für die Bündelblocktriangulation, BuL 1977, pp. 141-148

Förstner, W.: Ein Verfahren zur Schätzung von Varianz- und Kovarianzkomponenten. Allgem. verm.nachrichten 1979, pp. 446-453

Förstner, W.: Systematic Errors in Photogrammetric Point Determination. Proceedings Survey Control Network (Aalborg), Schriftenr. d. Hochschule d. Bundeswehr, Heft 7, Munich 1982, pp. 197-209

Förstner, W. and Schroth, R.: On the Estimation of Covariance Matrices for Photogrammetric Image Coordinates. Symposium on Geodetic Networks and Computations, DGK Reihe B, Heft 258/VI, Munich 1982, pp. 43-70

Grün, A.: Experiences with self-calibrating bundle adjustment. Proceedings of ACSM-ASP Convention, Washington 1978

Kilpelä, E., Heikilä, J. and Inkilä, K.: Compensation of Systematic Errors in Bundle Adjustment. Photogrammetria Vol. 37, 1981 pp. 1-13

Klein, H.: New Results of Bundle Block Adjustment with Additional Parameters. Proceedings of the 37th Photogrammetric Week, Stuttgart 1980, pp. 13-30

Schilcher, M.: Empirisch-statistische Untersuchung zur Genauigkeitsstruktur des photogrammetrischen Luftbildes. DGK Reihe C, Heft 262, Munich 1980

Schroth, R.: On the Stochastic Properties of Image Coordinates. Proceedings of the ISPRS-Symposium (Comm. III), Helsinki 1982