PROGRESS IN RADARCLINOMETRY:
Topography from Single Radar Images

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Abstract. A new method of photoclinometry has been devised for radar images. The most important aspect of the new method is the treatment of the fundamental problem of photoclinometric indeterminacy, whereby the specification of surface brightness produces a one-to-one correspondence between the possible strike and dip at a point of terrain under investigation, rather than a unique pair of values. The indeterminacy has been shown to reduce to the requirement for a one-dimensional boundary condition in the solution of a first-order partial differential equation, which is, however, unfortunately non-linear and inhomogeneous, with a driving function containing noise. Therefore, that approach has been ignored while a point-by-point evolution of strike-line orientation has been sustained through assumptions constraining the relation between the surface curvature of the terrain and the gradient in the image of the radar signal. The method requires radiometrically calibrated radar on a relative scale, in order to produce results of metric integrity. It is otherwise limited to the promotion of the geologic interpretation of radar images through partial correction for "layover" and production of subjective stereo-paired shaded-relief images.

Text. A method of inferring topography from individual images, rather than stereometric pairs, has existed in principle for nearly 20 years [1]. Called photoclinometry [2], it makes use of the integral relation between height and slope, coupled with the known dependence of a measured surface brightness in a single image on the local orientation of that surface, whereas photogrammetry uses the parallax between inter-identified surface elements in two images of different camera position. Photoclinometry has never given genuinely satisfactory results, largely for two reasons: (a) Surfaces that are homogeneous in both normal albedo and the geometric law of diffuse reflectivity are quite rare. (b) Photoclinometry, as a general theory, is not completely deterministic of the topography of a region of terrain presenting its image; for there is always a continuous set of interrelated pairs of values of the possible slopes, in two mutually perpendicular horizontal directions, that suffice to cause a particular value of the surface brightness of a given terrain element to be exhibited. Progress has recently been made toward solution of the former problem [3], but because the latter has remained unsolved, producing topography of metric integrity still eludes us.

This report describes the results of an investigation into alleviating the fundamental mathematical indeterminacy of photoclinometry and presents an encouraging example of its application. The description here is verbal and geometrically conceptual, and is presented for information rather than as an *a priori* theoretical justification [4]. Only the *a posteriori* justification based on the results presented is here forthcoming. Such justification is complete insofar as the goal of providing a tool facilitating the geologic interpretation of radar images is concerned. Nevertheless the actual technique has been formulated with maximization of

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*Also Astrophysicist, Branch of Astrogeology, U.S. Geological Survey metric precision as the goal, and will be described in those terms. Efforts to apply this new methodology thus far have been confined to radar images, largely for two reasons: (a) Surfaces homogeneous in back-scatter parameters are much more common at radar wavelengths than at and near visual wavelengths [5], and (b) a successful application to radar images is of especially high priority, inasmuch as the photoclinometric approach to fine-scale topography may be the only practical possibility for the Venus Radar Mapper mission expected to fly later in this decade. Inasmuch as the term "radargrammetry" has been widely applied to efforts to adapt photogrammetric methods to radar images, and the present effort is the first adaptation of the principles of photoclinometry to radar images, I call the resulting technique "radarclinometry." In addition, I call the auxiliary mathematics which alleviates photoclinometric indeterminacy "the hypothesis of local cylindricity," whose self-explanatory nature will emerge.

When photogrammetry is practiced on very narrow field photography, so that the images are practically orthographic projections of the terrain, the 3-dimensional positions of the cameras relative to the mean plane of the topography do not emerge as part of the solution, but must be given as independent knowledge. This, and more, is required to be given independently in order to do radarclinometry. Not only must we have the radar elevation angle above the horizon in order to generate a mean incidence angle for the image, we must also have a radiometric calibration for the radar system so that a back-scattering cross-section density can be inferred from each element of image brightness. And the back-scattered intensity versus incidence-angle curve (reflectance function) estimated to be applicable to the type of terrain under scrutiny must also be independently given. It is not necessary to know these curves in absolute radiometric units. The radiometric calibration is first used to produce an image whose pixel signal values are proportional to back-scattering cross-section density (equivalent to reflected specific intensity). The determination of an operating point on the curve representing the microwave reflectance function that corresponds to the average incidence angle of this image, together with a scaling factor for radar image brightness, can be inferred to first order by making the angle correspond to the average of the brightness over the entire image. The set of pixel brightnesses in the decalibrated image can be used to generate a mean incidence angle, whose disagreement with the pre-known value can be used iteratively to correct for non-linearities in the calibration curve and reflectance function. By assuming a normal and isotropic slope distribution for the surface we can even use the mean-square incidence angle to correct for the fact that the average incidence angle of the imaged terrain is slightly different for the real topography in comparison to the corresponding mean plane, where it is assumed to be the complement of the horizon-elevation of the radar (effective radar depression angle). The calibration, as a software problem, can be solved separately from the photoclinometry. Though not trivial, it is much simpler and I will not focus on it again here (see reference 4). As for the hardware problem, radiometrically calibrated imaging radars do not yet exist, at least in accessible form, but efforts are presently under way by others to produce them [6]. In the meantime, I substituted a heuristically-based calibration curve (straightline), together with the x-band microwave reflectance function for broken desert [7], for the purpose of testing the present method. The test is therefore more qualitative than quantitative, and the mode of presentation of the results will reflect that fact.

The basic theory for the determination of topography from a single radar image, as an array of reflected microwave energies, will now be described. First, note that while the coordinate of a terrain element in a radar image delineated parallel to the radar motion (azimuth) is the same as that for the element's vertical projection onto the mean plane of the topography, the image coordinate perpendicular to azimuth (radar ground-range) does *not* correspond to that vertical projection (Fig. 1). To obtain this coordinate, the range must be multiplied by the cosine of the horizon-elevation of the radar, to which must be added the product of the terrain height above the mean plane and the tangent of the horizon-elevation. The horizon in question is an astronomical rather than a real topographic horizon, so the elevation angle is known *a priori* as the effective depression-angle of the radar. We must nevertheless solve *simultaneously* for the height of each imaged terrain element and its mapping onto the mean plane as a vertical projection.

We begin by picking a starting point in the radar image that is at the near-range edge. This starting point is assigned a height of zero relative to the mean plane. A best estimate of the local orientation of the strike-line (line of zero height change) is made for this terrain element. This strike-line orientation resolves the ambiguity in how the down-range slope component and the azimuthal slope component shall combine to produce the necessary incidence angle that is consistent with the exhibited magnitude of the image pixel. For a heuristically idealized imaging radar as a microwave transceiver of antenna pattern identifiable with resolution in both dimensions, and also one that scans rapidly in depression angle while moving in azimuth (i.e. a facsimile camera) this process can be envisioned as solving for the intersection of the plane perpendicular to the strike-line that contains the terrain point with the cone whose apex is the terrain point, whose axis passes through the radar, and whose half-angle is the incidence angle[8]. Both represent geometric loci of possible directions for the line perpendicular to the local terrain surface. There are two such intersection lines, but only one represents a terrain slope shallower than the incoming radar wave-front. The other geometric possibility actually represents part of a situation in which the image pixel signal represents multiple reflections (foldover). Images containing such a phenomenon are *not* processable by the present method, and their unsuitability is usually obvious to the eye.

With both azimuthal and down-range slope components known, a range-integration step can be taken which establishes both the height and mean-plane coordinates for the next down-range pixel of the image-line taken at constant azimuth. Although only the down-range slope-component is used in this integration step, the azimuthal component is set aside for later use after the entire range line has been integrated.

With the completion of the first integration step, above, we are not yet ready to repeat the process at the second pixel in order to jump to the third. We must first find out how the strike-line orientation has changed from the first pixel position to the second. To do this, we must investigate the surface curvature of the terrain in the vicinity of the first pixel. Since the surface curvature represents the manner by which the local surface normal is changing direction, it directly determines the brightness gradient in the image through the photometric function. We can measure the brightness gradient in the image. But when we attempt to invert the problem and determine from it the curvature, we face the fact that while the brightness gradient has two independent components, the terrain surface curvature has three, in the form of the three second

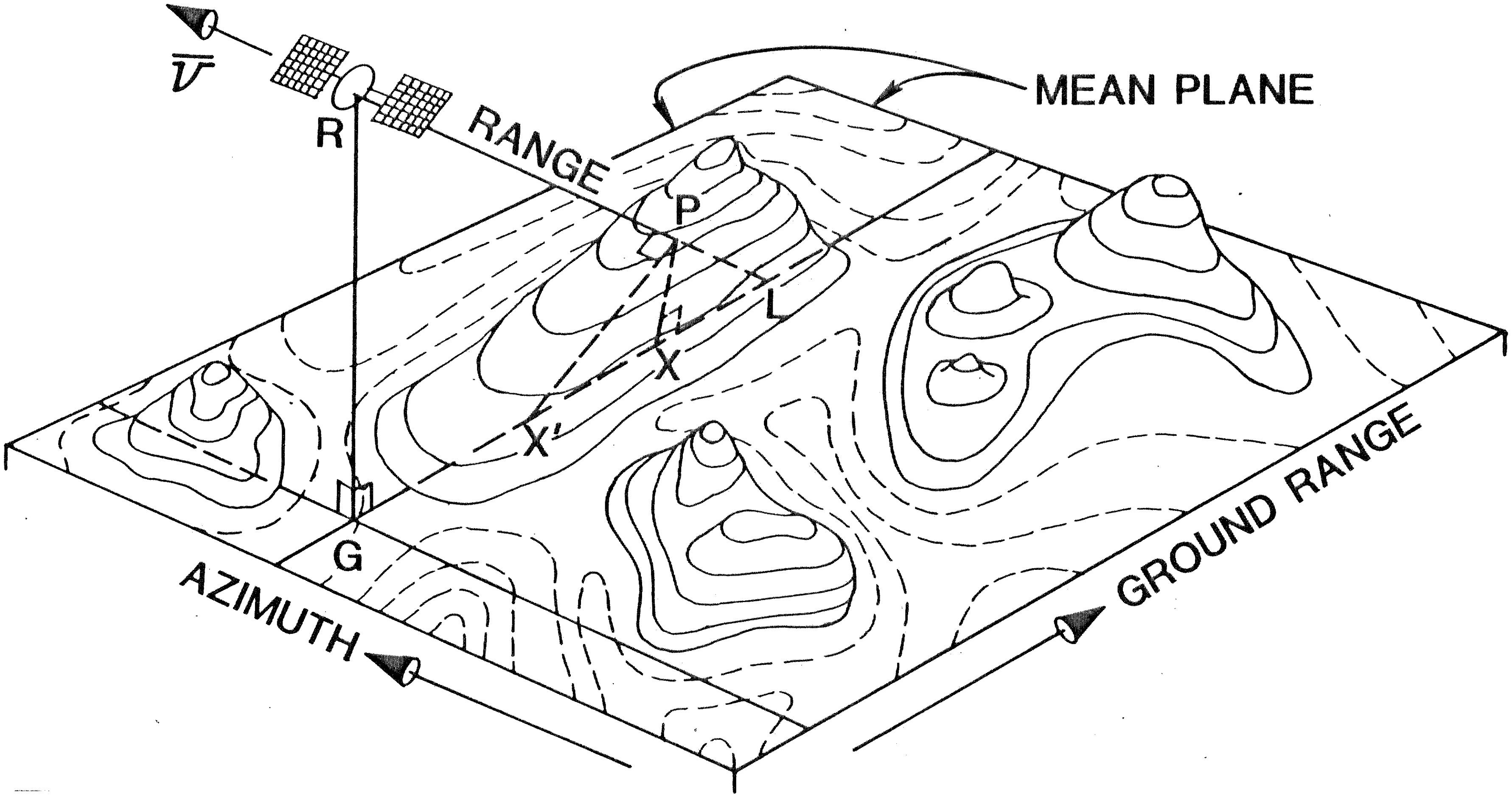

Figure 1. Illustration of the necessity to solve simultaneously for topographic height and topographic mapping coordinates. The solid elevation contours lie above the mean plane of the topography. The dashed contours are below the mean plane. The dashed portions of straight lines lie below the terrain surface. Point P is an arbitrary point of terrain. Its range from the radar is $\overline{RP}$. ΔRGL is a right triangle. $\overline{GX'}$ represents the ground-range coordinate of P in a radar image (relief-to-range ratio assumed small). The corresponding coordinate on a topographic map is $\overline{GX}$. Azimuth, as defined by radar engineers, is a rectilinear coordinate and not to be confused with the angle familiar to astronomers and surveyors.

partial derivatives of height with respect to horizontal coordinates. It is at this point that the assumption of a locally cylindrical nature to the curvature must be invoked. This assumption removes the independence of the three components of curvature. The axial alignment of local cylindricity need not be specified *a priori*. The image pixel brightness and brightness-gradient are deterministic of this alignment. Its projection into the image plane can be measured because it must be the local direction of the image isophote. Its true three-dimensional alignment can be then inferred because the local strike and dip of the terrain are already known. This leaves only the magnitude of the curvature, which is now reduced to a one-to-one correspondence with the magnitude of the image brightness gradient.

With the curvature fully specified, a range integration step can be taken which establishes both the azimuthal and the down-range component of slope at the next pixel in the image-line, but this information is used only to rotate slightly, in a horizontal plane, the strike-line which will apply at the next integration step of the topography.

The above process is recycled repetitively until the end of the range-line (actually the range-line pair needed to acquire the brightness-gradient) is reached. As we now shift by one line in azimuth in order to repeat the entire procedure and thus develop a topography two-dimensionally, we note that, in principle, the slope and curvature information developed at the beginning of the preceding range-line integration is sufficient to give us starting values for height, coordinates on the mean plane, and strike-line orientation. However, all images are noisy, especially synthetic-aperture radar images; and radar image brightness is only weakly sensitive to the azimuthal component of slope. (If the local surface normal is shifted in the vertical plane containing the radar, a change in the incidence angle is directly effected, but a shift perpendicular to this plane by the same angle produces a much smaller change in incidence angle.) Therefore, while the starting value of strike-line orientation is produced in this way, since it tends to be stochastic, the starting value of height is not. It is set to zero. In fact the other end of this line and all lines are also forced to zero as an auxiliary constraint. Only after the topographic profile has been formed is its average height adjusted using the average azimuthal component of slope from the preceding line-integration. The average tilt is also adjusted at this time on the basis of the average pixel brightness. (This feature is not only not optimum, it is a perversion of the determinative powers of the theory, from which we may expect to retreat. But given that it took 20 hours of a continuous computer run to produce a topographic file from a 630 x 630 pixel radar image, refining the procedure will likely be slow and painful; nevertheless, I foresee possibilities for greatly reducing the computation time. All these considerations will be explored in future research).

Because of the weak photometric power to determine the azimuthal component of slope, the topographic noise spectrum is asymmetrically enhanced in azimuthal spatial frequency. Therefore along each line of azimuth a re-apodization is applied which brings the Fourier transform of the line smoothly to zero at one-half the sampling frequency. In addition, the run in *azimuth* of the parameters of the linear trends of topography down each *range line* is adjusted by normalizing its power spectrum to agree with the power spectrum of the corresponding run in *range* of linearization parameters of the *azimuthal* trends. Fourier phases are not altered, and the amplitudes are *only* renormalized above frequencies (choosable) of about 18 cycles per frame, adjusted to maintain continuity. This heavy-handedness will likely be abandoned as we learn better how to do radarclinometry[10].

The results of this work are shown in Figure 2. It appears to justify continuing research and development in this area. My goal remains the maximization of the metric precision in the implied topography. Even though the lack of radiometric calibration restricts the resulting products to a partial correction for the primary radar image distortion of forshortening of terrain tilted toward the radar, and artificial subjective stereometry, my colleagues in geology (e.g. L.A. Soderblom) have indicated that this is a more important contribution as far as the geologic interpretation of radar images is concerned.

REFERENCES AND FOOTNOTES

1. Rindfleisch, T., Photogram. Eng. 32, 262-277 (1966). Watson, K., U.S. Geol. Surv. Prof. Paper 599B, 1-10 (1968). Wildey, R., Icarus 25, 613-626 (1975).
2. This word was invented by Jack McCauley in my presence in about 1965 from the Greek roots ϕοτοσ and κλινοσ .
3. Eliason, P. T., Soderblom, L. A., and Chavez, P. S., Jr. Photogram. Eng. and Remote Sensing 48, 1571-1579 (1981). These authors assumed a Lambert scattering law for the photometric function. Their auxiliary assumption resolving fundamental photoclinometric indeterminacy was that the strike-line was perpendicular to the sun-line. Neither assumption is realistic, but metric integrity was not a goal, immediately or ultimately, of that research. It succeeded in separating topographic and albedo information from the viewpoint of geologic interpretation of separate maps of albedo and shaded relief.
4. The corresponding mathematical analysis is projected to be published later in Photogrammetric Engineering and Remote Sensing.
5. Schaber, G. G., Berlin, G. L., and Brown, W. E., Jr. Geol. Soc. Amer. Bull. 87, 29-41 (1976). It can be subjectively appreciated in unsophisticated examination of many SEASAT/LANDSAT correspondences.
6. Brown, W. E. Jr. in private conversation. SEASAT is very close, in many ways, though it has the distinct disadvantage of steep depression angle; causing, among other things, frequent multiple reflections. Shuttle Imaging Radar B will probably qualify. Inasmuch as the term "photometry" is not specific regarding radiative wavelength range, it has been used entirely interchangeably with the term "radiometry" in this report.
7. Ruck, G. T., Barrick, D. E., Stuart, W. D., and Krichbaum, C. K., Radar Cross-Section Handbook (Plenum Press, New York-London, 1970).
8. For the radar thus idealized, linear transduction produces a pixel signal (DN number) proportional to back-scattered specific-intensity. For real imaging radars this can be approximately compensated with a factor of cot i. More exact compensation prolongs computation, but is warranted by fully calibrated radars. Its incorporation is presently under development. While the cotangent of the incidence angle is a factor whose practical significance is to change the shape of the effective reflectance function, the additional factor required for proper correction is the secant of the projected angle, as seen from the radar, between the normal vector and the vertical vector at the terrain target, and this factor is a function of a second and higher order in the individual slope components. The down-ground-range slope component becomes an implicit function of the incidence angle, requiring solution by numerical iteration. The algorithm incorporating this improvement is working satisfactorily, but the new program of which it is a part is still in the debugging phase.

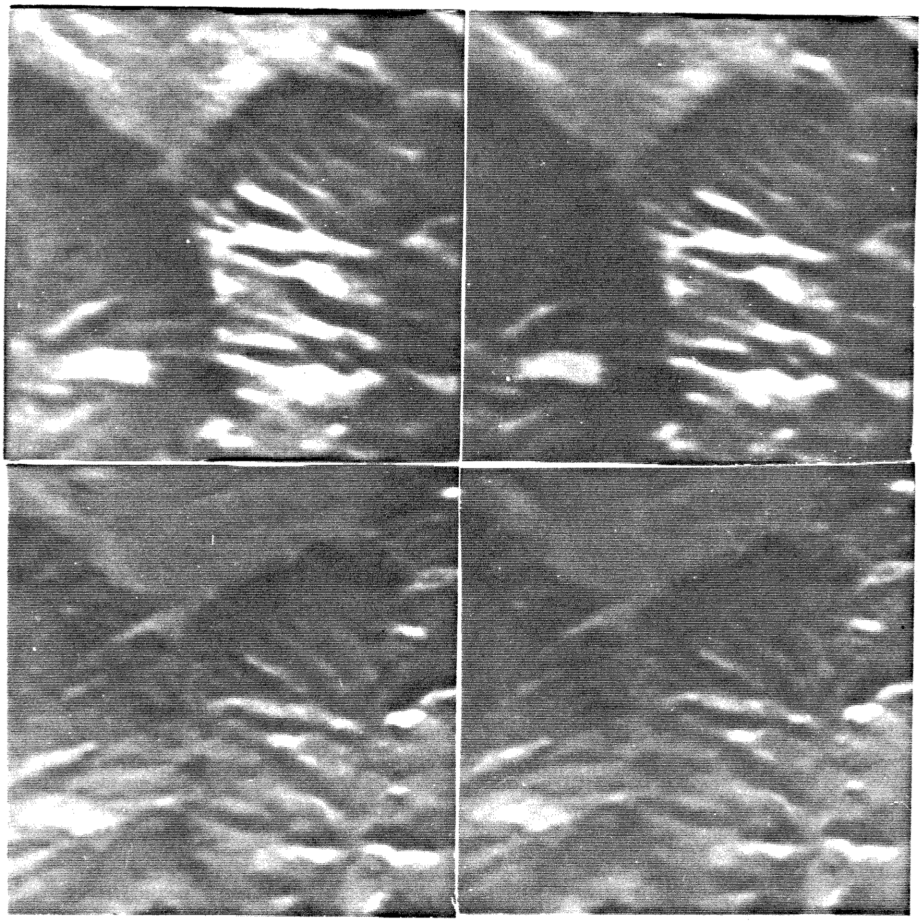

Figure 2. Shaded relief stereopairs produced from digitized topography [9]. The topography providing the upper pair was derived radarclinometrically, employing the hypothesis of local cylindricity, from a single radar image (Motorola SLAR). For comparison, the lower pair was derived by digitizing the standard topographic map of roughly the same region (Crazy Jug Point in the Grand Canyon, Arizona) at approximately the same resolution. The effective elevation angles of the artificial sun used to produce the shaded relief differ because the mean plane of the topography for the upper pair tilts relative to the plane of mean sea-level characterizing the lower pair.

9. The algorithm producing a shaded relief image is due to Batson, R. M., Edwards, K., and Eliason E. M., Jour. Res. U.S. Geol. Surv. 3, 401-408 (1975); while the one producing synthetic stereo-mates, by the same authors, is published in Photogram. Eng. and Remote Sensing 42, 1279-1284 (1975).
10. In the advanced theory presently under development, whose algorithm is not yet operational, the initial height and strike line orientation of an adjacent profile along ground-range is determined by a sequence of iterations of the entire profile. The path-independence of any line-integral of the gradient of the height function over the mean-datum plane, originating at the near-point of one profile and terminating at the far point of the adjacent profile, is used to bring the difference in mean heights into consistency with the mean azimuthal component of slope. Similarly, the difference in mean gradient of the height function along the two profiles is rendered consistent with the mean curvature tensor. -- An additional improvement in the new theory lies in the selection of options, at each integration step, with respect to curvature assumptions used at that step. Selection is on the basis of user-input tolerances, advised through noise considerations. If the local height and brightness gradients are large enough and the curvature of the local isophote in the image is small enough, the Hypothesis of Local Cylindricity is selected. If the isophotic curvature is large or the height gradient component in azimuth is very small, a "Hypothesis of Local Biaxial Ellipsoidal Hyperbolicity" is selected. Herein local curvature is assumed to be representable by a surface of second degree, with two axes of common semi-major axis parallel to the mean-datum plane and the third perpendicular thereto. The center of figure is located by the solution; not independently specified; and the two values of semimajor axes are permitted to have the same or opposite sign. There is finally an option selectable when all gradients are so small that a solution stable in the presence of noise is of paramount consideration. I call this the "Hypothesis of Least-Squared Local Sphericity." As the name implies, the local curvature is assumed spherical. The center of curvature is located by solution; it is not required to be independently specified. Even so, this solution is overconstrained and is used as a model that is least-square fitted to the two values of brightness gradient components. Thusly, Local Planarity need never be explicitly assumed. Continued developments in radarclinometry are being supported by contracts from the U.S. Department of Defense, Defense Mapping Agency, and the National Aeronautics and Space Administration, Planetology Programs Office, to the U.S. Geological Survey.

Figure Captions

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