THE OPTIMUM CONFIGURATION OF THE CONVERGENT CASE OF CLOSE RANGE PHOTOGRAMMETRY*

Zhu Chenglin
Yang Xinyu
Photogrammetric Laboratory
Railway Engineering Department
Northern Jiaotong University
Beijing, P.R. CHINA
Commission V

ABSTRACT

Both of the configuration angle α and convergent angle φ are regarded as variables affecting the accuracy of the convergent case of close range photogrammetry and the overlap of picture pair is considered as a factor should not be neglected. The formulas for calculating the values of percentage overlap P% and the condition satisfying full overlap of picture pair were derived according to the relation among P%, α, φ and camera field angle 2β. The optimum values of α_o and φ_o in the case of percentage overlap P%=1 were acquired.

It is pointed out that the optimum curve proposed by Marzan can not be used in some cases because the overlap of picture pair is too small. The results of the investigation of this subject in the paper show the theoretical demonstration and practical methods developed by authors should be better than those before.

INSTRUCTION

In the recent ten years, close range photogrammetry has developed rapidly and the configuration of data acquisition for which has become an interested problem to the photogrammetrists. Considering the special characteristics of close range, many scholars conducted researches to determine the optimum layout of cameras to the object in order to get the best accuracy of object space coordinates of points. Many famous scholars, such as Y.I. Abdel-Aziz, H.M. Karara, G.T. Marzan and others, have done a lot of work in this field.

In analyzing the convergent case, Abdel-Aziz and Karara(3)(4) developed formulas relating the plate coordinate errors of a convergent case and those of what they called a pseudo-normal case. They obtained the optimum value of convergent angle under the condition that the positional error of the central point of the object to be photographed is minimum. The premise of their demonstration is to maintain base B and object distance D constant. or the distance S between the central point of the object and each of the two cameras constant. They only considered the convergent angle φ as a variable. Based on their results, Marzan proposed the concept of equivalent normal case and the equivalent overlap and gave the curve of the optimum

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configuration of data acquisition for close range photogrammetry.

Having studied their statement, we think the configuration angle α ($\alpha = tg^{-1} B/2D$) is another factor affecting the accuracy of object space coordinates. Both of the configuration angle α and convergent angle φ should be regarded as variables affecting the configuration of photogrammetry and the overlap of picture pair should be considered as a factor that cannot be neglected. One of the advantages of convergent case is that the full overlap can be obtained, but up tillnow, the optimum configuration satisfying full overlap has not been drawn in the existing photogrmmetric literature.

In this paper, we will discuss the facters affecting the configuration of colse range photogrammetry, which are the configuration angle α, the convergent angle φ, the camera field angle 2β and the percentage overlap P%, and their relations. The formulas for calculating the percentage overlap P% and the condition satisfying the full overlap, in terms of α, φ and 2β have been obtained. The optimum values of α_o and φ_o satisfying the full overlap have been acquired by using the strict mathematic method. The results of the investigation in this paper show that the theoretical demonstration and practical method developed by authors should be better than those before.

THE FACTORS AFFECTING THE ACCURACY OF OBJECT SPACE COORDINATES OF POINTS IN CONVERGENT CASE

The layout of the tow cameras to the object in convergent case is shown in Fig. 1.

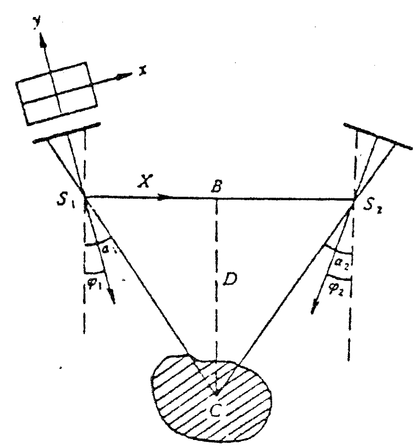

Fig. 1

B --- base

D --- perpendicular distance from the central point to the base

α_1, α_2 --- angles of which the lines joining central point of object space and perspective centers make with perpendicular directions to the base

φ_1, φ_2 --- angles of convergence of the two photos; angles which camera axes make with the directions perpendicular to the base.

In the symmetrical case, we have

$$\varphi_1 = \varphi_2 = \varphi \; ; \; \alpha_1 = \alpha_2 = \alpha \, .$$

Many famous scholars, such as Karara(3), Abdel-Aziz(4) and Marzan(5), studied the accuracy of convergent case using symmetrical model. They adopted the accuracy of the central point of object space as the comparing standard for different configurations. This model and standard are still adopted in this paper.

The formulas relating the standard errors in convergent photo coordinates to those in object space coordinates, for central point, are as follows(2):

$$m_X = \frac{D}{2f}(\cos\varphi + tg\alpha \sin\varphi)^2 m$$

$$m_Y = \frac{D}{2f}(\cos\varphi + tg\alpha \sin\varphi)\, m \qquad (1)$$

$$m_Z = \frac{D}{2f\, tg\alpha}(\cos\varphi + tg\alpha \sin\varphi)^2 m$$

Where m_X, m_Y and m_Z are the standard errors of object space coordinates, m is the standard error of photo coordinates, We assume $m_X = m_Y = m$.

Analysing Equations (1), we can see that the factors affecting the accuracy of points in object space are the base B, object distance D, camera principal distance f, convergent angle φ and the standard error m. The compositive effect of base B and object distance D can be expressed as $tg\,\alpha$.

The factors affecting the configuration of photogrammetry are configuration angle α and convergent angle φ .

The configuration angle α is a function of base B and object distance D, $tg\,\alpha = B/2D$, therefore $tg\,\alpha$ is a relative value which reflects the relation between B and D. Since the calculation of the coordinates of points is based on the principle of space intersection, the configuration of triangle $S_1 C S_2$ is one of the factors affecting the positional accuracy of points. The angle α is a factor of the configuration of $\triangle S_1 C S_2$. We call it configuration angle. The variation of configuration angle α and convergent angle φ influences the accuracy of points and changes the overlap of photos. The variations of the values of overlap P% caused by the variations of α and φ is a factor should not be neglected when we study the configuration of photogrammetry.

THE DETERMINATION OF THE OPTIMUM VALUES OF CONFIGURATION ANGLE α_o AND CONVERGENT ANGLE φ_o

Both of configuration angle α and convergent angle φ should be considered as two variables affecting the configuration of photogrammetry of convergent case.

One of the advantages of the convergent case is the full overlap can be obtained. The optimum configuration of convergent case satisfying full overlap is important and useful in practice.

The optimum configuration of convergent case should satisfy two conditions:

(1) The value of the percentage overlap P% of photos is equal to 1;

(2) The optimum configuration, by definition, is attained when the positional error $m_r^2 = m_X^2 + m_Y^2 + m_Z^2$ is minimum, i.e., least positional error.

Yang gave the condition satisfying full overlap of convergent case, in terms of α, φ, and 2β, as follows: (2)

$$tg\,\alpha = \frac{\sin 2\varphi}{\cos 2\varphi + \cos 2\beta} \tag{2}$$

From Equations(1), we have the expression for the positional standard error of central point of the object as follows:

$$m_r^2 = \frac{1}{2}\left[(\cos\varphi + tg\,\alpha\sin\varphi)^2 + \left(1+\frac{1}{tg^2\alpha}\right)(\cos\varphi + tg\,\alpha\sin\varphi)^4\right]\left(\frac{Dm}{f}\right)^2$$

For simplifying the calculation, let u = $tg\,\alpha$, we have

$$F(u,\varphi) = m_r^2 \Big/ \frac{1}{2}\left(\frac{Dm}{f}\right)^2$$
$$= (\cos\varphi + u\sin\varphi)^2 + \left(1+\frac{1}{u^2}\right)(\cos\varphi + u\sin\varphi)^4 \tag{3}$$

The Condition(2) can be written as

$$\Psi(u,\varphi) = u(\cos 2\varphi + \cos 2\beta) - \sin 2\varphi = 0 \tag{4}$$

Solving the problem of the optimum configuration of convergent case can be reduced to seeking the conditional extreme of a function, i.e., solving the minimum value of the function F under the condition $\Psi(u,\varphi)=0$.

If the function F has an minimum value on the level curve $\Psi(u,\varphi)=0$ at the point (u_0, φ_0), then under certain conditions there exists a number λ such that

$$\nabla F(u_0,\varphi_0) = \lambda \nabla \Psi(u_0,\varphi_0)$$

The number λ in above equation is called Langrange multiplier for F and Ψ.

Forming a function

$$\Phi(u,\varphi) = F(u,\varphi) + \lambda\Psi(u,\varphi)$$
$$= (\cos\varphi + u\sin\varphi)^2 + \left(1+\frac{1}{u^2}\right)(\cos\varphi + u\sin\varphi)^4 + \lambda u(\cos 2\varphi +$$
$$+ \cos 2\beta) - \lambda\sin 2\varphi \tag{5}$$

Considering the both effects of u and φ, we should study the variation of Φ introduced by that of u and φ simultaneously. Deriving the partical derivatives of Φ with respect to u and φ separately, putting each of them equal to zero, meanwhile considering Equation(4), we have

$$\begin{cases} \frac{\partial\Phi}{\partial u}=2u^3(\cos\varphi+u\sin\varphi)\sin\varphi+(\cos\varphi+u\sin\varphi)^3\cdot(4u^3\sin\varphi+2u\sin\varphi- \\ \quad -2\cos\varphi)+\lambda u^3(\cos2\varphi+\cos2\beta)=0 \\ \frac{\partial\Phi}{\partial\varphi}=(\cos\varphi+u\sin\varphi)(u\cos\varphi-\sin\varphi)\left[1+2\left(1+\frac{1}{u^2}\right)(\cos\varphi+u\sin\varphi)^2\right]- \\ \quad -\lambda u\sin2\varphi-\lambda\cos2\varphi=0 \\ u(\cos2\varphi+\cos2\beta)-\sin2\varphi=0 \end{cases} \qquad (6)$$

Equations(6) is a set of high power transcendental equations which is impossible to find the exact solution. Yang adopted the iterative method to find the approximate solution of Equations (6) by a microcomputer PDP 11/23 according to the principle of gradient method. The results of α_0 and φ_0 for different camera field angle are shown in Table 1.

The values of α_0 and φ_0 are the results of theoretical derivation. The premises of the derivation are that the optimum values of α_0 and φ_0 should satisfy, (1) the positional error $m_r^2= m_X^2 + m_Y^2 + m_Z^2$ is minimum; (2) the percentage overlap P% is equal to 1. Whether α_0 and φ_0 satisfy the two conditions can be checked by numerical computation.

Table 1.

2β (°)	α_0 (°)	φ_0 (°)
20	33.9254226685	32.7711067200
40	34.1103782654	29.7680759430
60	34.7474174500	25.6480255127
90	37.0375175476	18.5253448486
120	41.7591438293	11.1483926773

We know the positional error is expressed by

$$m_r=\sqrt{\frac{1}{2}\left[(\cos\varphi+\mathrm{tg}\,\alpha\sin\varphi)^2+\left(1+\frac{1}{\mathrm{tg}^2\alpha}\right)(\cos\varphi+\mathrm{tg}\,\alpha\sin\varphi)^4\right]}\cdot\frac{Dm}{f}$$
$$=\mathrm{PMR}\cdot\frac{Dm}{f} \qquad (7)$$

Where PMR is positional error factor. The values of PMR can be used to compare the positional accuracy of different configurations. We set three criteria to compare the accuracy of one configuration with that of another. These criteria are:

(1) The object distance D is constant from one configuration to another.
(2) The percentage overlap P%=1 is the same from one configuration to another.
(3) Set the positional error $m_r = \sqrt{m_X^2 + m_Y^2 + m_Z^2}$ as the standard to compare one configuration to another.

According to Equation(7), the author(2) has solved the values of PMR for five different camera field angles 2β, and every degree of convergence φ . The corresponding configuration angle satisfying full overlap is calculated according to formula(6). All the results are shown in Table 2.

Table 2. The calculated Values of the Error Factors PMR for Different Configurations of Convergent Case

P%=1.00

$2\beta=20°$			$2\beta=40°$			$2\beta=60°$			$2\beta=90°$			$2\beta=120°$		
φ (°)	PMR	α (°)	φ (°)	PMR	α (°)	φ (°)	PMR	α (°)	φ (°)	PMR	α (°)	α (°)	PMR	α (°)
1.0	39.31367	1.0	1.0	35.79766	1.1	1.0	30.41140	1.3	1.0	20.29209	2.0	1.0	10.18923	4.0
2.0	19.68846	2.1	2.0	17.93288	2.3	2.0	15.24430	2.7	2.0	10.19858	4.0	2.0	5.18561	8.0
3.0	13.16083	3.1	3.0	11.99314	3.4	3.0	10.20584	4.0	3.0	6.85758	6.0	3.0	3.55818	11.9
4.0	9.90762	4.1	4.0	9.03472	4.5	4.0	7.68962	5.3	4.0	5.20489	8.0	4.0	2.77494	15.8
5.0	7.96424	5.2	5.0	7.26888	5.7	5.0	6.20639	6.7	5.0	4.22773	10.0	5.0	2.32970	19.7
6.0	6.67581	6.2	6.0	6.09940	6.8	6.0	5.21976	8.0	6.0	3.58857	12.0	6.0	2.05401	23.5
7.0	5.76171	7.2	7.0	5.27078	7.9	7.0	4.52274	9.3	7.0	3.14283	14.0	7.0	1.87608	27.2
8.0	5.08160	8.3	8.0	4.65526	9.1	8.0	4.00684	10.7	8.0	2.81828	16.0	8.0	1.76041	30.9
9.0	4.55756	9.3	9.0	4.18191	10.2	9.0	3.61182	12.0	9.0	2.57485	18.0	9.0	1.68777	34.4
10.0	4.14283	10.3	10.0	3.80815	11.3	10.0	3.30156	13.4	10.0	2.38859	20.0	10.0	1.64719	37.9
11.0	3.80766	11.3	11.0	3.50691	12.5	11.0	3.05309	14.7	11.0	2.24432	22.0	11.0	1.63231	41.2
12.0	3.53223	12.4	12.0	3.26016	13 6	12.0	2.85112	16.1	12.0	2.13202	24.0	11.1	1.63188	41.8
13.0	3.30282	13.4	13.0	3.05543	14.8	13.0	2.68510	17.4	13.0	2.04483	26.0	12.0	1.63966	44.5
14.0	3.10966	14.4	14.0	2.88383	15.9	14.0	2.54750	18.8	14.0	1.97795	28.0	13.0	1.66779	47.7
15.0	2.94557	15.5	15.0	2.73883	17.0	15.0	2.43284	20.1	15.0	1.92800	30.0			
16.0	2.80517	16.5	16.0	2.61558	18.2	16.0	2.33702	21.5	16.0	1.89253	32.0			
17.0	2.68439	17.5	17.0	2.51035	19.3	17.0	2.25896	22.8	17.0	1.86983	34.0			
18.0	2.58004	18.6	18.0	2.42030	20.5	18.0	2.19026	24.2	18.0	1.85874	36.0			
19.0	2.48962	19.6	19.0	2.34315	21.6	19.0	2.13510	25.5	18.5	1.85743	37.0			
20.0	2.41115	20.6	20.0	2.27713	22.8	20.0	2.09002	26.9	19.0	1.85852	38.0			
21.0	2.34302	21.7	21.0	2.22081	23.9	21.0	2.05390	28.3	20.0	1.86881	40.0			
22.0	2.28394	22.7	22.0	2.17305	25.1	22.0	2.02586	29.7	21.0	1.88957	42.0			
23.0	2.23285	23.8	23.0	2.13291	26.2	23.0	2.00523	31.1	22.0	1.92109	44.0			
24.0	2.18887	24.8	24.0	2.09964	27 4	24.0	1.99149	32.4	23.0	1.96401	46.0			
25.0	2.15129	25.8	25.0	2.07263	28.5	25.0	1.98427	33.8	24.0	2.01930	48.0			
26.0	2.11950	26.9	26.0	2.05139	29.7	25.6	1.98291	34.7	25.0	2.08840	50.0			
27.0	2.09302	27.9	27.0	2 03553	30.9	26.0	1.98332	35.2						
28.0	2.07144	28.9	28.0	2.02474	32.0	27.0	1.98851	36.6						
29.0	2.05442	30.0	29.0	2.01880	33.2	28.0	1.99980	38.1						
30.0	2.04169	31.0	29.8	2.01737	34.1	29.0	2.01727	39.5						
31.0	2.03302	32.1	30.0	2.01752	34.4	30.0	2.04108	40.9						
32.0	2.02825	33.1	31.0	2.02082	35.6	31.0	2.07153	42.3						
32.8	2.02714	33.9	32.0	2.02865	36.7	32.0	2.10902	43.8						
33.0	2.02725	34.2	33.0	2.04101	37.9	33.0	2.15408	45.2						
34.0	2.02992	35.2	34.0	2.05796	39.1	34.0	2.20742	46.7						
35.0	2.03621	36.2	35.0	2.07963	40.3	35.0	2.26990	48.1						

Analysing Table 2, we can get two conclusions, namely

(1) Under the full overlap condition, for an increase in convergence φ , there is a corresponding increase in configuration angle α , resulting in a decrease in the values of PMR, i,e., an increase in accuracy. When α and φ are equal to the values of α_o and φ_o shown in Table 1, the PMR is minimum. Beyond the optimum values of α_o and φ_o, increasing α and φ further will not yield better accuracy than no increase at all.

(2) The optimum values of α_o and φ_o are different for different camera field angle 2β .

ANALYSIS OF THE APPLICABILITY OF MARZAN'S CURVE FOR OPTIMUM CONFIGURATION

Studying the optimum configuration of convergent case, the overlap of photos is a factor should not be neglect. Marzan did not consider this factor in his derivation of the optimum curve. Yang(2) studied the applicability of Marzan's optimum curve by analysing the variation of P% on his curve and gave the formula of P% reduced to the equivalent normal case:

$$P\% = \begin{cases} \frac{1}{2} + \frac{\mathrm{tg}\{\mathrm{arc\,tg}[\mathrm{tg}(\beta+\varphi) - A] + \varphi\}}{2\,\mathrm{tg}\beta} & \text{(inside overlap case)} \\ \frac{1}{2} + \frac{\mathrm{tg}\{\mathrm{arc\,tg}[\mathrm{tg}(\beta-\varphi) + A] - \varphi\}}{2\,\mathrm{tg}\beta} & \text{(outside overlap case)} \end{cases} \qquad (8)$$

where

$$A = \frac{\mathrm{tg}\theta\cos^2\varphi + 2\sin\varphi}{\left(1 - \frac{1}{2}\mathrm{tg}\theta\sin\varphi\right)\cos\varphi}$$

According to the values of P% caculated from Equation(8), Yang drew figures of percentage overlap P% shown in Fig.(2),(3),(4),(5),(6), and drew Marzan's optimum curve in each figure.

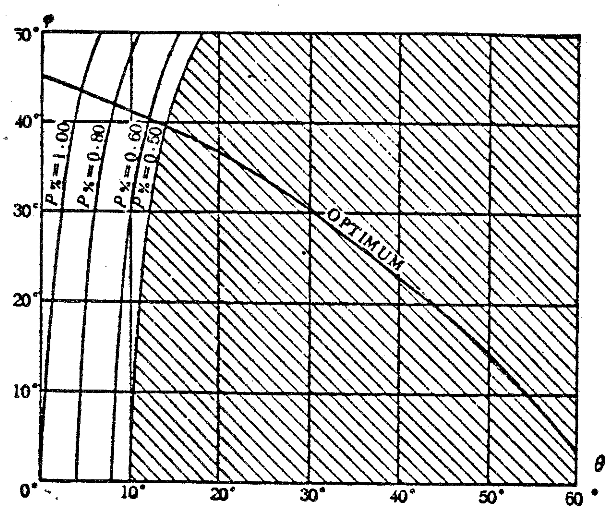

Fig. 2.

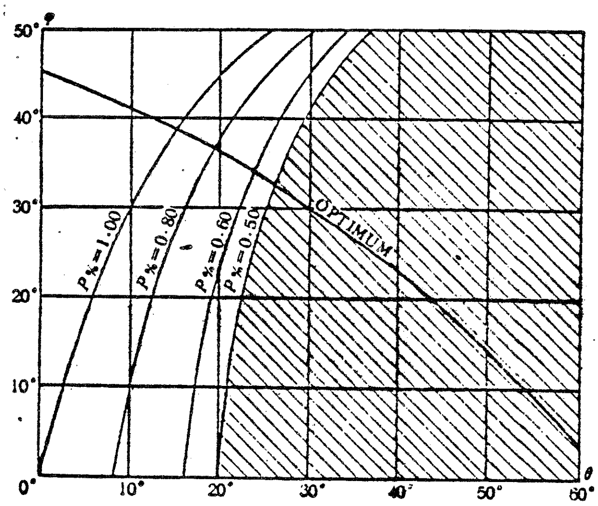

Fig. 3.

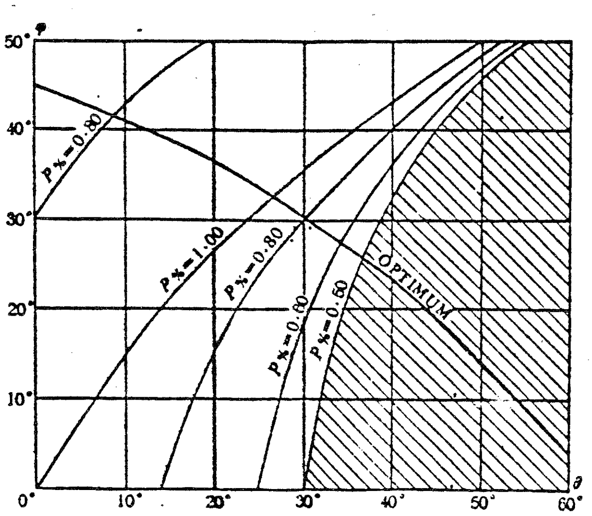

Fig. 4

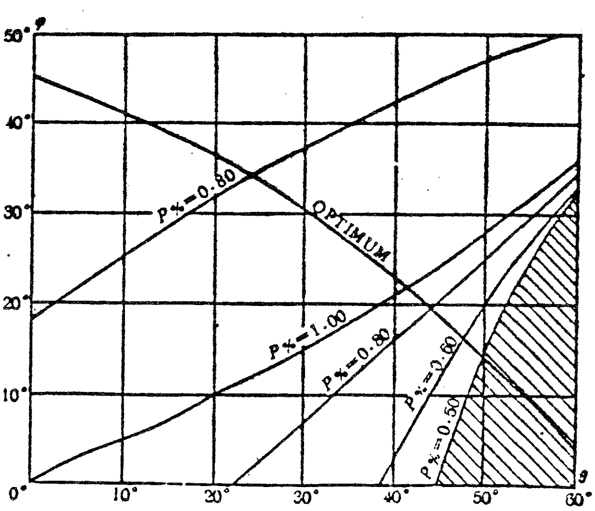

Fig. 5.

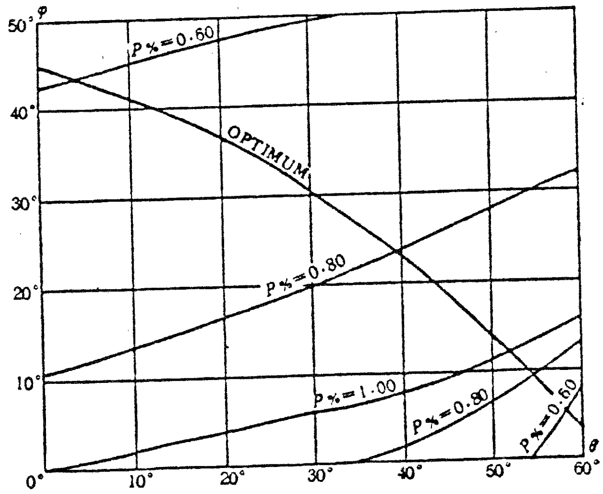

Fig. 6.

From those figures, we can see

(1) The value of percintage overlap P% is a variable on Marzan's optimum curve. The value of P% is larger than o.5 only on part of the curve and smaller than 0.5 on the other part of it. There is a corridor of α and φ, the shaded portion shown in each figure, within which P% is smaller than 0.5. Since the overlap P% is too small, the shaded portion in those figures cannot be used in practice.

(2) One of the advantages of the convergent case is the full overlap can be obtained. The optimum configuration satisfying full overlap is important in practice. For practical purpose, the Marzan's curve is not feasible since it doesn't show the point of θ_o and φ_o at which the overlap P% is equal to 1. It might be one of the reasons that Marzan's curve hasn't been applied widely.

THE COMPARISION BETWEEN THE PROPOSED CONFIGURATION AND MARZAN'S

According to formula(8), we can find the point of Marzan's optimum curve at which the optimum values of θ_o and φ_o satisfy full overlap. The values of θ_o and φ_o for different camera field angles are shown in Table 3.

Table. 3.

2β (°)	θ_o (°)	φ_o (°)
20	5	45
40	15	39
60	27	32
90	41	22
120	52	12

Before we derive the necessary formulas, we set the criteria used for comparing the optimum configuration proposed by Yang with that proposed by Marzan. These criteria are:

(1) The overlap P% of photos is equal to 1.
(2) The object distance is the same from one configuration to another.

According to Marzan's formula(5)

$$m_r=\sqrt{\frac{1}{2\left(1-\frac{1}{2}\operatorname{tg}\theta\sin\varphi\right)^2\cos^2\varphi}+\frac{1}{2}+\frac{2}{(\operatorname{tg}\theta\cos^2\varphi+2\sin\varphi)^2}}\cdot\frac{D'm}{f}$$

$$=\text{error factor}\cdot\frac{D'm}{f} \qquad (9)$$

where D' is object distance of the equivalent normal case. For the purpose of comparision, D' needs to be reduced to the object distance D of corresponding convergent case. We know(5)

$$D' = \frac{D}{\left(1-\frac{1}{2}\mathrm{tg}\theta \sin\varphi\right)\cos\varphi} \qquad (10)$$

Substituting Equation(10) in Equation(9), we have

$$m_r = \frac{\text{error factor}}{\left(1-\frac{1}{2}\mathrm{tg}\theta \sin\varphi\right)\cos\varphi} \cdot \frac{Dm}{f} = PE \cdot \frac{Dm}{f} \qquad (11)$$

where PE is the comparative factor.

Equation(7) can be written as

$$m_r = \sqrt{\frac{1}{2}\left[(\cos\varphi + \mathrm{tg}\,\alpha \sin\varphi)^2 + \left(1+\frac{1}{\mathrm{tg}^2\alpha}\right)(\cos\varphi + \mathrm{tg}\,\alpha \sin\varphi)^4\right]} \cdot \frac{Dm}{f}$$

$$= PE' \cdot \frac{Dm}{f} \qquad (12)$$

where PE' is also the comparative factor.

The values of PE and PE' are calculated according to formulas (9),(11) and (12). The results are shown in Table 4.

Table 4. The Comparison between the Configurations Proposed by the author and that of Marzan

the proposed configurations				the Marzan's			
2β (°)	α_0 (°)	φ_0 (°)	PE'	α_0 (°)	φ_0 (°)	θ_0 (°)	PE
20	33.9	32.8	2.02714	46.8	45	5	2.31010
40	34.1	29.8	2.01737	44.9	39	15	2.21232
60	34.7	25.6	1.98291	44.2	32	27	2.11731
90	37.0	18.5	1.85743	43.9	22	41	1.92077
120	41.8	11.1	1.63188	44.0	12	52	1.64188

Analysing Table 4. we can see

(1) The accuracy of the optimum configuration proposed by the author is better than that of Marzan.

(2) Under the condition that the object distance is the same, the base B of the configuration proposed by Marzan is larger than that of the author, so the α_0 and φ_0 reduced to the convergent case is larger than that of the author. For example, when 2β =20°, 40°, the α_0 and φ_0 proposed by Marzan is larger than that of the author for about 10° mor . In practice, the larger value of φ not only causes the larger variation in photo scale but also results in some part of the object in foreground covering the other part of it in background, which is difficult to photograph in some cases. Therefore, it is an advantage of the optimum configuration proposed by author that φ_0 is obvious smaller than that of Marzan.

(3) The elements α_0 and φ_0 of the optimum configuration proposed by author can be chosen according to the value of 2β. which is very convenient for application purpose, while Marzan chose of the equivalent normal case as one of the elements of the configuration of convergent case, which is not convenient in practice since θ must be reduced to α of the convergent case.

CONCLUSIONS

(1) It is pointed out that for determining the optimum configuration of convergent case we must consider the factor of overlap P%, and the optimum values of configuration angle α_0 and convergent angle φ_0 are different for different camera field angles 2β.

(2) Both of the configuration angle α and convergent angle φ are variables affecting the configuration of convergent case and the configuration factor tg α is one of the factors affecting the accuracy, which cannot be regarded as a constant.

(3) The author made thorough analysis of the Marzan's optimum curve and gave his own point of view. The optimum configuration for convergent case proposed by the author yields better accuracy and the convergent angle φ_0 adopted by author is smaller than that of Marzan, which are favourable to photogrammetry.

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