MULTIPLE EXPOSURES IN NON-METRIC CAMERA APPLICATIONS

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ABSTRACT

Systematic errors introduced through film deformation constitute a major factor limiting the photogrammetric accuracy of non-metric images. This paper examines the effectiveness of multiple exposure photography in reducing that component of the object point positioning bias which is due to film unflatness. Aspects of network precision and accuracy are discussed, as are details concerning the additional parameter model selection and the use of orientation constraints. A non-metric camera experiment, in which multiple exposures were employed, is outlined and the results of this practical application are presented.

INTRODUCTION

The general usage of multi-frames in analytical non-topographic photogrammetry encompasses a number of geometric arrangements for the repeated photography at a camera station. For example, *Torlegård* (1981) discusses the following approaches: repeated photography with rotation about the camera axis, repeated photography with altered camera axis directions, and multiple exposures with the same orientation. It is well known that the first of these geometric configurations affords both an averaging of tangential and non-symmetric systematic errors, be they due to lens imperfections or film deformation, and the recovery of the principal point location in self-calibration adjustments. By incorporating the second arrangement *Wester-Ebbinghaus* (1982) has formulated a single station self-calibration technique. It is, however, the last of the three geometries, that of multiple exposures with nominally the same exterior orientation, which forms the topic of this paper.

For the "normal" case, *Hottier* (1976) has demonstrated that multiple exposures can provide a practical means of improving photogrammetric accuracy. In the context of precision, *Fraser* (1983) has discussed the role of multiple exposure photography in network design optimization, in terms of the first- and second-order design problems. The main purpose of this paper is to demonstrate that for analytical non-metric camera applications, the multiple exposure concept can provide both a significant enhancement of photogrammetric accuracy, and a better indication of the precision (object point standard errors) of an "amateur" camera network than would be obtained in the corresponding single-frame case. In addition, through the results of the investigation, a further illustration of the relatively high accuracy potential of non-metric images is provided.

ACCURACY ASPECTS

In a network in which successive exposures are taken at each camera station, but not necessarily at precisely the same orientation and position, six additional exterior orientation parameters are introduced into the functional model of the bundle adjustment, thus leading to a different first-order design of the network. If the same number of photos, k, is taken at each exposure station, then the influence on the precision of the object target point coordinates is described by the expression $\underline{C}^{(2)}_{\underline{x}_k} = k^{-1}\underline{C}^{(2)}_{\underline{x}}$, where $\underline{C}^{(2)}_{\underline{x}}$ is the covariance matrix of the adjusted XYZ coordinates for the single-exposure network (e.g. *Fraser*, 1983).

As an alternative to multiple exposures, the same level of enhancement of precision — but not necessarily positioning accuracy — is obtained by making k times more image coordinate measurements than in the single-exposure case. However, for non-metric imagery where the mathematical model is invariably incomplete, the multiple exposure approach has the potential of partially "randomizing" systematic film deformation effects which are not common to each exposure. In addition, it is possible to carry a different set of additional parameters (APs) for each photograph when multiple exposures are used. Such an approach then leads to the notion of self-calibration with both block- or sub-block invariant APs, and APs which relate only to a single image.

Results of recently conducted experiments, carried out in order to investigate the photogrammetric accuracy potential of non-metric cameras, indicate that the self-calibration approach employing block- and sub-block invariant APs is perhaps the optimum method of analytical restitution. Reasonably high accuracies in object point positioning (of the order of one part in 10,000 of the object field diameter, or better) have been routinely attained using this analytical data reduction technique (e.g. *Fraser*, 1982a). However, the method is not without its shortcomings, especially in the area of model fidelity. The modelling of film deformation effects on a photowise basis can, in certain circumstances, lead to an adjustment result which displays high internal consistency at the expense of a degradation in the accuracy of object point positioning.

As a first step in reducing the influence of film unflatness effects in the analytical restitution, one can either increase the degrees of freedom by using more photos and/or a greater target point density, or incorporate object space control. In addition, in cases where the network is minimally constrained, the degree of the APs employed to model film deformation is of considerable importance (*Fraser*, 1982b). The use of multiple exposures falls into the category of increasing the network redundancy, in a favourable way as far as non-metric cameras are concerned.

In the situation where multiple exposures are taken from a stable camera station, the invariance of the exterior orientation parameters relating to all images from that station can be applied as a constraint in the network adjustment. Such a constraint function takes the simple form of

$$\underline{x}_{1_{i-1}} - \underline{x}_{1_i} = 0 \quad ; \quad i = 2, \ldots, k \qquad (1)$$

where $\underline{x}_{1_i}$ is the vector of the exterior orientation parameters for photo i, and k is the number of exposures at the station. This constraint can be treated as either weighted or absolute, depending on the stability of the camera platform during the multiple exposures. It is important to note that the extra degrees of freedom afforded by this constraint are not reflected in the covariance matrix $\underline{C}^{(2)}_{\underline{x}_k}$ of the XYZ object point coordinates. Notwithstanding the negligible impact on network precision, it was initially envisaged that such a constraint would contribute to a further minimization of the effects of those components of film deformation which were not common to all images taken from a single camera station, thus leading to a further enhancement of accuracy.

MULTIPLE EXPOSURE EXPERIMENT

With the above mentioned considerations in mind it was decided to conduct a non-metric camera experiment, first to examine whether the use of multiple exposures could decrease the discrepancy between the design precision and the positioning accuracy, i.e. effectively enhance the model fidelity; and second to ascertain whether the use of multiple exposures and constraint functions would facilitate a more complete modelling of film deformation effects. In the experiment it was decided to specifically investigate the following aspects:

(1) The accuracy enhancement that accompanied the use of multiple exposures coupled with block-invariant APs (i.e. "stable" camera calibration parameters) and the use of constraint functions.

(2) Accuracy aspects of using multiple exposures, camera station constraint functions, and a different set of film deformation APs for each image to supplement the block-invariant APs.

Both (1) and (2) were to be examined for the cases of minimal and redundant object space control.

THE PHOTOGRAMMETRIC NETWORK

A pre-calibrated object point array of approximately 100 targets was employed as a testfield for the non-metric camera investigation. Of the targets, 80 were positioned in a 0.25 m grid pattern on a wall, whereas 20 were affixed to plumblines which hung 0.9 m out from the wall. The plane target area had dimensions of approximately 2 m square. The positioning accuracy of the target points was in the order of 50 µm (mean standard error) in the X, Y and Z coordinate directions. This level of precision corresponds to about 1 part in 50,000 of the effective object field diameter.

The target array was photographed from three locations, at each of which four exposures were taken. Figure 1 illustrates the imaging geometry. At each exposure station the camera body was clamped to a tripod to ensure the stability of its position and orientation during the multiple exposures. The camera used was a Hasselblad 500 ELM of 80 mm focal length, which – except for the provision of two fiducial marks – can be viewed as being an "off-the-shelf" non-metric camera. A photographic scale of 1:50 was adopted, and the film used was standard professional quality Kodak polyester-based plus-X with an ASA rating of 125.

Image coordinate measurements on the 12 photographs were carried out monoscopically on a Wild AC/1 analytical plotter, nominally to an accuracy level of about two micrometres (mean standard error).

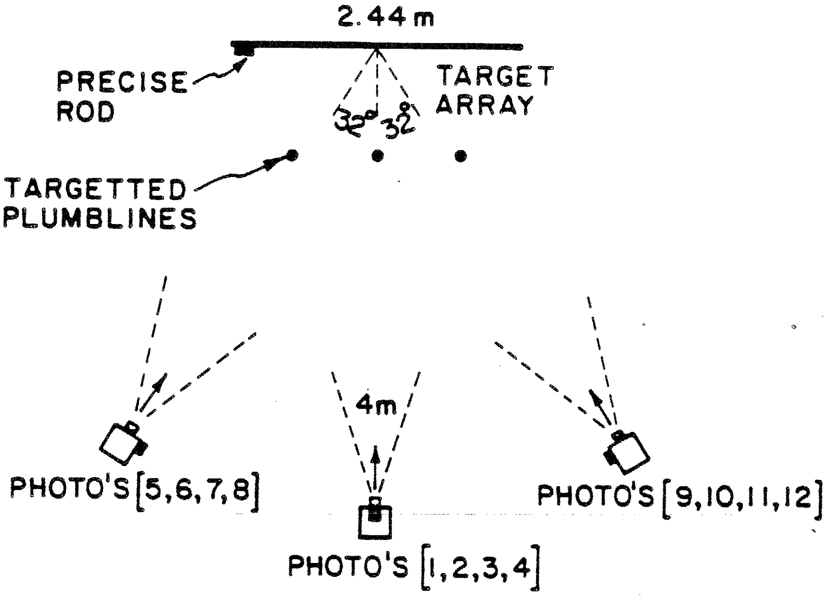


Figure 1 : Imaging Geometry.

DATA REDUCTION

Initially, a number of self-calibration adjustments were carried out in which only block-invariant APs were carried. The AP set comprised four terms: principal distance correction, dc; the coefficient K_1 of the well-known odd-order lens distortion polynomial; and the principal point coordinates x_0, y_0. For each of two control configurations, one minimal and one comprising six "fixed" object points (four at the corners of the plane field and two on the plumblines), four adjustments were carried out. In the first network only one photo per camera station was used. This 3-photo block adjustment was then followed by adjustments of 6-, 9- and 12-photo blocks, i.e. 2, 3 and 4 photos per camera station, respectively.

Following the self-calibration adjustments with only block-invariant APs, the same networks as described above were readjusted, this time with both block-invariant APs and the following set of image correction terms, which were carried to model film deformation (mainly unflatness) in each image:

$$\begin{Bmatrix}\Delta x\\ \\ \Delta y\end{Bmatrix} = \sum_{i=0}^{3}\sum_{j=0}^{i}\begin{Bmatrix}a_{ij}\\ \\ b_{ij}\end{Bmatrix} x^{i-j}\, y^{j} \qquad (2)$$

Of the terms in Eq. 2, those which were not statistically significant and/or exhibited high correlation with either exterior orientation parameters or other APs were suppressed. It is interesting to note that in all cases terms of degree 3 were suppressed, and in most adjustments a few of the second degree terms were also not significant at the chosen confidence level of 95%. Thus, in the network adjustments with mixed AP sets, the final number of APs ranged from 19 in the 3-photo network to 47 in the 12-photo block adjustment.

All network adjustments were computed both with and without the exterior orientation parameter constraints, Eq. 1. The constraint functions were applied in an absolute sense, i.e. the difference equation was assigned zero variance.

RESULTS

Block-Invariant APs

Considering first the adjustments with only block-invariant APs, the aim was to achieve an effective "randomization" of film deformation effects and therefore to improve functional model fidelity; i.e. decrease the bias in the estimated object point coordinates. With the removal of bias in the parameter estimates, the RMS error of photogrammetrically determined target point coordinates, which provides a measure of accuracy, should be in basic agreement with the positioning precision, as expressed by the mean standard error. The RMS error $\bar{s}_c$ is obtained from the vector $(\underline{\Delta X}, \underline{\Delta Y}, \underline{\Delta Z})^T$ of coordinate differences between the measured and "true" coordinates of the n non-control points:

$$\bar{s}_c = \left(\frac{\underline{\Delta X}^T\underline{\Delta X} + \underline{\Delta Y}^T\underline{\Delta Y} + \underline{\Delta Z}^T\underline{\Delta Z}}{3n} \right)^{\frac{1}{2}} \tag{3}$$

The mean standard error $\bar{\sigma}_c$, on the other hand, is determined from the trace of the covariance matrix of the checkpoint coordinates:

$$\bar{\sigma}_c = \left(\frac{1}{3n} \mathrm{tr}\ \underline{C}^{(2)}_{x_k} \right)^{\frac{1}{2}} \tag{4}$$

As can be seen from Figure 2, this aim was achieved to a large extent for the minimally controlled networks, but to a lesser extent for blocks with 6 control points. In the former case, an increase in the number of exposures per camera station was accompanied by a marked increase in the degree of agreement between $\bar{s}_c$ and $\bar{\sigma}_c$, at least up until three exposures per station. For the networks with six fixed control points there is a 20 - 30 µm discrepancy between the RMS error and mean standard error values. However, this may not necessarily be due to an incomplete functional or stochastic modelling, given that the "true" coordinates display standard errors of around 50 µm.

One striking feature of the results shown in Figure 2 for the multiple exposure networks is the accuracy attained in object point positioning. The minimum RMS error obtained (based on 76 checkpoints) was 126 µm for the 12-photo network with six control points. This corresponds to approximately 1 part in 20 000 of the object field diameter. For the minimally controlled networks, i.e. those without explicitly surveyed control, accuracy levels of 1 part in 15 000 are approached in the 9- and 12-photo blocks. If only points within the plane defined by the three control points are considered, $\bar{s}_c$ values of 146 and 143 µm are obtained for the 9- and 12-photo networks, respectively. This level of accuracy corresponds to roughly 1 part in 17 000. In all but the minimally controlled 3-photo network, accuracy levels of better than 1 part in 10 000 were obtained, thus indicating the effectiveness of multiple exposures as an accuracy enhancement tool in non-metric camera networks. In the adjustments with block-invariant APs, the RMS value $s_{x,y}$ of the image coordinate residuals ranged from 2.8 µm for the 3-photo network to 3.6 µm for the block of 12 photographs.

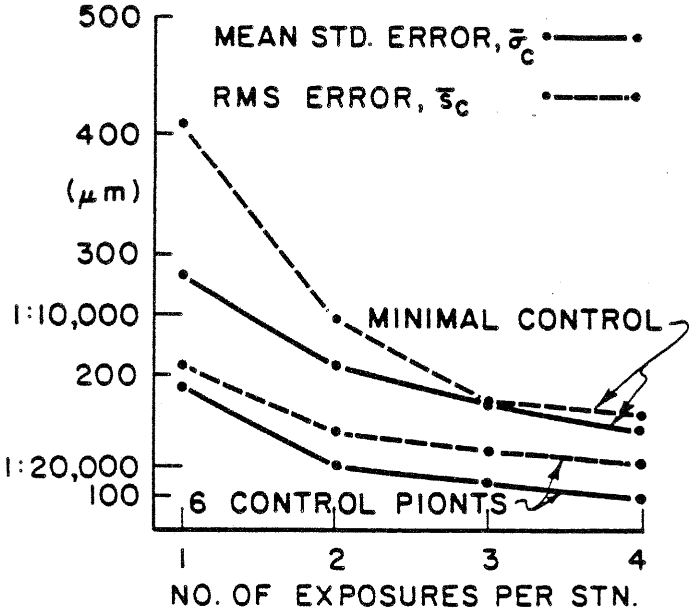


Figure 2 : Accuracy ($\bar{s}_c$) and precision ($\bar{\sigma}_c$) obtained in the adjustments with only block-invariant APs.

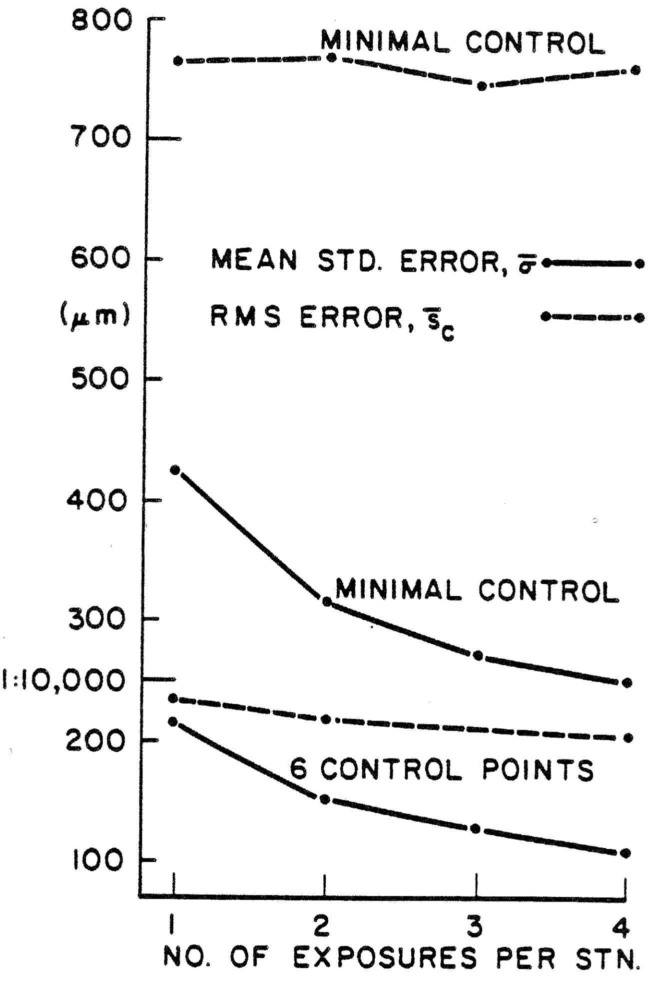


Figure 3 : Accuracy ($\bar{s}_c$) and precision ($\bar{\sigma}_c$) obtained in the adjustments with both block-invariant APs and film deformation APs for each image.

FILM DEFORMATION APs

Figure 3 illustrates the accuracy levels attained in the network adjustments in which both block-invariant APs and photo-variant film deformation APs (see Eq. 2) were carried. The $\bar{s}_c$ values are markedly higher than the corresponding mean standard errors $\bar{\sigma}_c$, and there is a degradation of both accuracy and precision compared to the network adjustments in which only "stable" camera calibration constants were included in the AP model. Furthermore, in Figure 3 there is a mild divergence between the plots of positioning accuracy and precision as the number of exposures per station is increased. For the minimally controlled networks, accuracy levels of only about 1 part in 3 000 of the object field diameter are obtained, with there being no significant reduction in the RMS error of the object point coordinates as the number of exposures per station is increased. As in the adjustments with block-invariant APs, the X, Y and Z errors with the largest magnitude lie out of the plane of the minimal control. With six control points, $\bar{s}_c$ is reduced to just over 200 µm (1 part in 12 500), but the discrepancy between the estimates of accuracy and precision is as high as a factor of two times the mean standard error value in the 12-photo network.

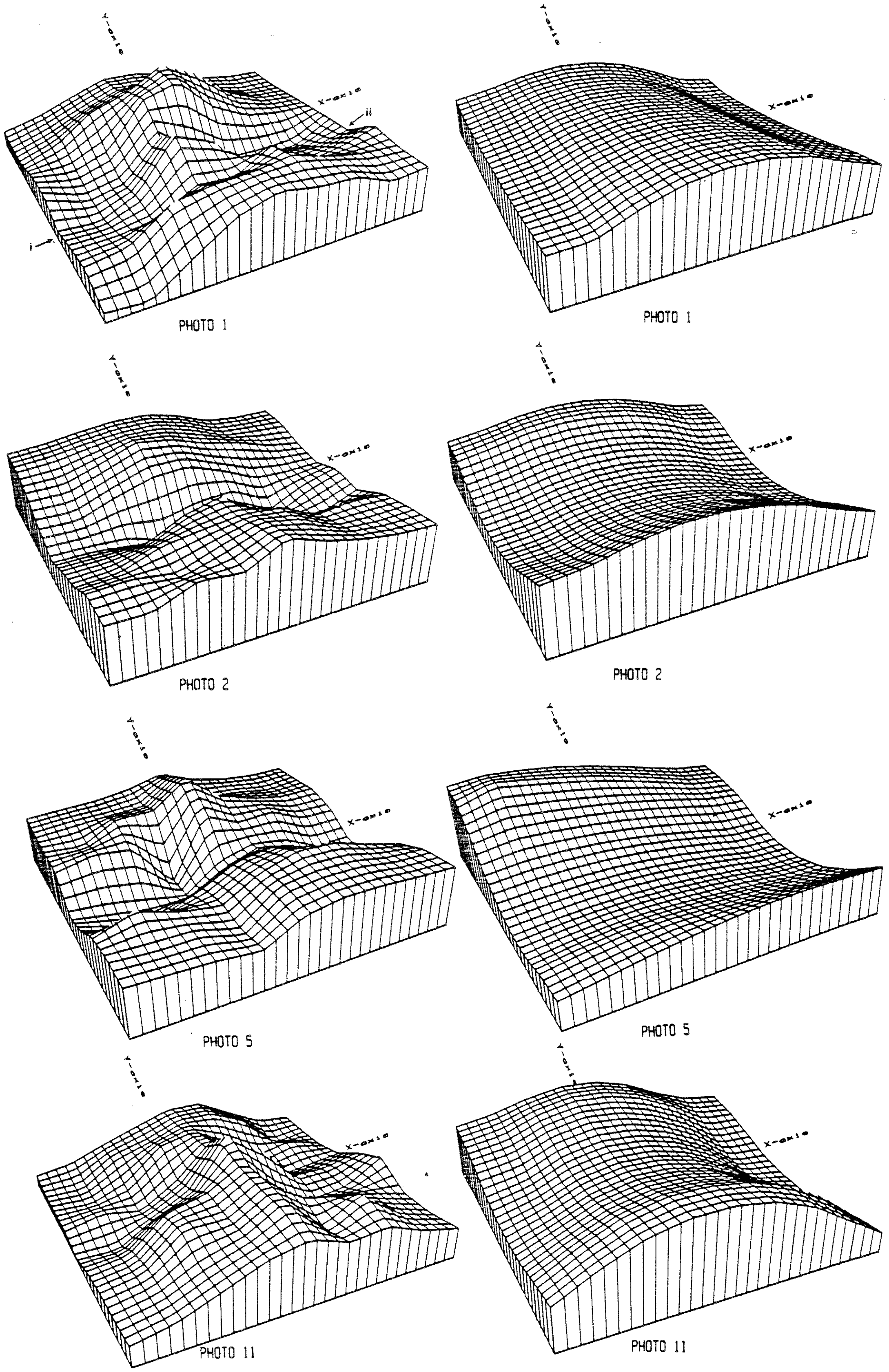
The results illustrated in Figure 3 corroborate the findings of an earlier non-metric camera investigation (*Fraser*, 1982b) in which it was shown that attempts to model film topography by empirical APs on a photo-wise basis tend to yield unsatisfactory results due to a mechanism of projective compensation. In essence, a mathematical film surface is obtained such that the internal consistency of the relatively oriented bundle is maximized at the expense of positioning accuracy. The $s_{x,y}$ values for the adjustments with mixed AP sets were between 0.2 and 0.5 µm lower than the corresponding adjustments with block-invariant APs, and they ranged from 2.2 to 3.4 µm.

CONSTRAINTS ON THE EXTERIOR ORIENTATION PARAMETERS

Turning now to the application of exterior orientation constraint functions. As an aid in the "randomization" of film unflatness effects these constraints proved somewhat unsuccessful. In fact, they degraded the model fidelity to a limited degree. As anticipated, the inclusion of Eq. 1 had no measurable impact on the covariance matrix $\underline{C}_{x_k}^{(2)}$ of the object point coordinates, and only in the minimally controlled networks did the constraints favourably influence accuracy. Here, the RMS errors of the X, Y and Z coordinates of points out of the plane containing the three control points (7 arbitrarily fixed parameters) were reduced by between 10 and 40 µm. The application of the exterior orientation constraints also resulted in an increase in the RMS error of the x and y image coordinates. This perhaps indicates that although the camera body was stable the orientation of the "mean film plane", and therefore the mathematical perspective centre, were changing from exposure to exposure as the film topography changed.

FILM SURFACE CHARACTERISTICS

In order to examine characteristics of the film surface shape, and the extent of variations in film topography from exposure to exposure, surface plots were generated. "True" image coordinate errors were obtained by computing a 12-photo adjustment with block-invariant APs and tight positional constraints on all object target points. Implicit in this procedure was the assumption that the residuals obtained ($s_{x,y}$ = 6.3 µm) were due solely to film unflatness. While this assumption is less than fully realistic it is, nevertheless, sufficiently valid if an examination of trends in the film topography is primarily what is sought. To transform x and y image coordinate residuals into their equivalent Δz or "height" components the following approximate formula was applied:



(a) Actual Unflatness

(b) Modelled Film Surface

Figure 4 : Plots of Film Topography.

$$\Delta z = \frac{c\ \Delta r}{(r + \Delta r)} \tag{5}$$

where c is the principal distance and Δr is the radial component corresponding to the image coordinate residuals. Plots of the film topography are illustrated for four photographs in Figure 4a. Photos 1 and 2 were imaged at the same camera station (see Figure 1).

The points labelled i and ii in the upper plot of Figure 4a show the positions of the two fiducial marks which were installed in the Hasselblad's film magazine. Visual inspection indicated that one of the fiducial mountings, namely that at position i physically depressed the emulsion surface, whereas the other just touched the underlying film. The resulting depressions can be seen on the surface plots for all photographs.

The results obtained from the adjustments which incorporated film deformation APs illustrated that low-order polynomials will likely prove less than adequate for the modelling of film unflatness effects. This view is reinforced when the surface plots shown in Figure 4b are examined. These plots show the modelled film surfaces which result from a 12-photo self calibration adjustment with film unflatness APs (only statistically significant terms included) for each image, and all object points assigned an a priori standard error of 50 µm. The Δz component plotted is obtained via Eq. 5 using the radial component of the image coordinate corrections determined by the following AP model:

$$\begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix} = \begin{Bmatrix} x/r \\ y/r \end{Bmatrix} \sum_{i=0}^{3} \sum_{j=0}^{i} a_{ij}\, x^{i-j} y^{j} \tag{6}$$

It can be seen that even in such an over-controlled network, only very low-order trends in the topography are adequately revealed. As compared to the 12-photo network with block-invariant APs (surface plots from which are shown in Figure 4a), the film deformation APs resulted in an adjustment of higher internal consistency. In the latter case an $s_{x,y}$ value of 5.4 µm was obtained versus 6.3 µm in the former.

CONCLUSIONS

On the basis of the results of this investigation, it can be concluded that the *adoption of multiple exposures is a practical means of significantly enhancing photogrammetric positioning accuracy in analytical non-metric camera applications*. The results shown in Figure 2 illustrate that, to a measurable degree, a "randomization" of film unflatness effects does take place and the measure of accuracy is brought into line with the level of precision in the network when multiple exposures are imaged. The systematic surface trends which can be seen in Figure 4a do, however, suggest that some bias in the functional model is likely to remain. Notwithstanding these limitations the accuracies obtained in the Hasselblad 500 ELM experiment are quite sufficient to satisfy the requirements of numerous high-accuracy close-range photogrammetric measuring tasks.

A second conclusion that can be drawn is that *film unflatness does remain as perhaps the major factor limiting photogrammetric accuracy which cannot be adequately compensated for analytically*. Naturally, in non-metric camera networks the problem is most pronounced. The reported results indicate that *higher object point positioning accuracies are likely to be obtained in a minimally or moderately controlled self-calibration adjustment if empirical film deformation APs are not carried.* As has been previously reported (*Fraser*, 1982b), low-degree AP functions are generally not descriptive enough to model the systematic film unflatness effects, and therefore do not enhance functional model fidelity. On the other hand the inclusion of higher degree terms in the AP model can lead to a significant deterioration in object point positioning accuracy.

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