ESTIMATION OF INITIAL VALUES BEFORE BUNDLE ADJUSTMENT OF CLOSE RANGE DATA

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Abstract

A space resection and multistation intersection analysis is presented which works well for highly convergent close range photography. A program based on this analysis has been written at the National Physical Laboratory and, in addition to providing initial values for a bundle adjustment program, it is also used to detect gross errors such as incorrect labelling of measured points.

1. Introduction

At NPL considerable effort has been put into the production of an accurate bundle solution package for use in close-range photogrammetry applied to metrological problems. An essential requirement for this accurate program to work was the provision of good initial esimates of the photogrammetric variables.

In the present analysis it is assumed that the minimum amount of information necessary to fix the positions and orientations of the cameras is presented in the form of initial estimates of the positions of at least four object points. It will be noted where appropriate how additional information can be used to reduce the amount of computation.

2. Outline of the Notation

In photogrammetry using n_p photographs there are n_p+1 co-ordinate systems in which object and image points may be expressed. One is the 'exterior' co-ordinate system of the object and the other n_p are the 'interior' co-ordinate systems of the cameras at the instants of taking the photographs. If the co-ordinate systems are assumed to be right handed and orthogonal then any vector in one system must correspond to a vector in any other system differing by a translation, a rotation and, possibly, a change of scale. If the scale is unchanged, vector differences can only differ by a rotation.

Let the vectors in the 'exterior' co-ordinate system be denoted by upper case letters and those in the 'interior' systems by lower case letters using superscripts to indicate the photograph with which they are associated.

i.e. $(\underline{X}_i - \underline{X}_o^{(p)}) = \underline{\underline{R}}^{(p)} (\underline{x}_i^{(p)} - \underline{x}_o^{(p)})$ ---(1)

to simplify the notation let

$$\underline{g}_i^{(p)} = (\underline{x}_i^{(p)} - \underline{x}_o^{(p)}) \text{and } \underline{G}_i^{(p)} = (\underline{X}_i - \underline{X}_o^{(p)})$$

then (1) becomes

$$\underline{G}_i^{(p)} = \underline{\underline{R}}^{(p)} \underline{g}_i^{(p)} \qquad \text{---(2)}$$

If $\underline{X}_i$ is the position of a point on the object and $\underline{x}_i^{(p)}$ is the corresponding point in the 'internal' co-ordinate system of the pth photograph then $\underline{X}_o^{(p)}$ and $\underline{x}_o^{(p)}$ are the 'external perspective centre' and the 'inner perspective centre' of the pth camera.

In order to obtain the image points $\underline{h}_i^{(p)}$ on the pth photograph it is necessary to project the object points in the co-ordinate system of that photograph onto the corresponding image plane. The simplest such projection is of the form:

$$\underline{p}_i^{(p)} \equiv (\underline{h}_i^{(p)} - \underline{x}_o^{(p)}) = C_i^{(p)} \underline{g}_i^{(p)} \qquad \text{---(3)}$$

Which corresponds to an aberration free 'pin-hole' camera for which any point $\underline{h}_i^{(p)}$ in the image plane satisfies:

$$\underline{p}_i^{(p)} . \hat{f}^{(p)} = (\underline{h}_i^{(p)} - \underline{x}_o^{(p)}) . \hat{f}^{(p)} = - f^{(p)} \qquad \text{---(4)}$$

where $\hat{f}^{(p)}$ is the normal to the image plane and $f^{(p)}$ is the principal distance. It is usual to take an 'internal' system of co-ordinates for the camera such that $\hat{f}^{(p)}$ has the simple form $\hat{f}^{(p)} = (0,0,1)$ therefore from (3) and (4)

$$C_i^{(p)} = - f^{(p)} / (\underline{g}_i^{(p)})_z$$

hence one obtains the usual photogrammetric equation as:

$$\underline{p}_i^{(p)} = - f^{(p)} \underline{g}_i^{(p)} / (\underline{g}_i^{(p)})_z \qquad \text{---(5)}$$

As measurements of the image are done in two dimensions it is convenient to define a two dimensional vector $\underline{u}_i^{(p)}$ having the same components in the image plane as $\underline{p}_i^{(p)}$. Then $\underline{u}_i^{(p)}$ is the two dimensional vector from the principal point on the pth photograph to the image of the ith point on that photograph. The relation between these vectors is illustrated in figure(1).

3. A Method of Finding the Perspective Centres

In the photogrammetric literature several methods for finding the position of the camera in terms of its perspective centre $\underline{X}_o^{(p)}$ and the rotation matrix $\underline{\underline{R}}^{(p)}$ are described but they usually rely on some prior knowledge of approximate camera positions. This may be reasonable in air survey work where the overall geometry is fairly standard but at close range it is useful to have a method which works with the minimum number of initial assumptions. These assumptions or initial conditions may be presented in a variety of ways. In the present section it is assumed that every photograph images at least four points for which initial values of the co-ordinates are known.

The method presented here to estimate the perspective centre $\underline{X}_o^{(p)}$ contains four distinct steps: (i) choosing three suitable points $\underline{X}_1$, $\underline{X}_2$ and $\underline{X}_3$ from the object points for which initial estimates of the co-ordinates are known as initial points to determine $\underline{X}_o^{(p)}$, (ii) finding the distances S_1, S_2 and S_3 from $\underline{X}_o^{(p)}$ to the three chosen points $\underline{X}_1$, $\underline{X}_2$ and $\underline{X}_3$, (iii) finding the intersection of three spheres radius S_1, S_2 and S_3 with centres $\underline{X}_1$, $\underline{X}_2$ and $\underline{X}_3$ and (iv) choosing the best solution from (ii) and (iii) using all the points for which initial co-ordinates are known.

Step(i): choice of initial points.

If sets of initial co-ordinates have been given for n_g points on the object, three of them must be chosen to start the space resection. The chief requirements of these points are that they are imaged on the photograph under consideration and that their images are not collinear. Provided there are not excessive numbers of initial points it is possible to calculate the area of the triangle on the image plane with the images of each set of three initial points $\underline{X}_i$, $\underline{X}_j$ and $\underline{X}_k$ at its vertices for all possible sets of three initial points. The points $\underline{X}_1$, $\underline{X}_2$ and $\underline{X}_3$ giving the maximum area are in some sense furthest from collinearity and provide a suitable starting point for the resection.

Step(ii): determination of distances S_1, S_2 and S_3.

The model adopted for the photographic process assumes that the angles between rays passing through $\underline{X}_o^{(p)}$ are unchanged as in figure(2a). In the object space:

$$\cos\theta_{12}^{(p)} = (\underline{X}_1 - \underline{X}_o^{(p)}).(\underline{X}_2 - \underline{X}_o^{(p)})/S_1^{(p)}S_2^{(p)} \quad \text{where} \quad S_i^{(p)} = |\underline{X}_i - \underline{X}_o^{(p)}| \qquad ---(6)$$

and in terms of the image co-ordinates and the principal distance:

$$\cos\theta_{12}^{(p)} = (\underline{u}_1^{(p)}.\underline{u}_2^{(p)} + f^{(p)2})/(\sqrt{(\underline{u}_1^{(p)2}+f^{(p)2})}\sqrt{(\underline{u}_2^{(p)2}+f^{(p)2})})$$

For a given photograph the superscript p may be dropped and the tetrahedron of figure(2a) exploded and flattened onto the plane containing $\underline{X}_1$, $\underline{X}_2$ and $\underline{X}_3$ to give figure(2b). The radius r_{12} of the circle containing points $\underline{X}_o$, $\underline{X}_1$ and $\underline{X}_2$ is given by:

$$2r_{12} = |\underline{X}_1 - \underline{X}_2|/\sqrt{(1-\cos^2\theta_{12})} \qquad ---(7)$$

Also by expressing $\underline{X}_o$, $\underline{X}_1$ and $\underline{X}_2$ in the Cartesian co-ordinates shown in figure(2b) or otherwise it may be shown that:

$$S_1 = r_{12}\sqrt{(2+2\cos(\theta_{12}-\alpha_{12}))} \qquad ---(8)$$

and $$S_2 = r_{12}\sqrt{(2+2\cos(\theta_{12}+\alpha_{12}))} \qquad ---(9)$$

where in (7), (8) and (9) the positive square root is always taken and $|\alpha_{12}| \leqslant |\pi - \theta_{12}|$. Similarly taking a cyclic labelling convention:

$$S_2 = \tau_{23} \sqrt{(2 + 2\cos(\theta_{23} - \alpha_{23}))} \quad \text{---(10)}$$

$$S_3 = \tau_{23} \sqrt{(2 + 2\cos(\theta_{23} + \alpha_{23}))} \quad \text{---(11)}$$

$$S_3 = \tau_{31} \sqrt{(2 + 2\cos(\theta_{31} - \alpha_{31}))} \quad \text{---(12)}$$

$$S_1 = \tau_{31} \sqrt{(2 + 2\cos(\theta_{31} + \alpha_{31}))} \quad \text{---(13)}$$

If the variable α replaces α_{12} then from (8) and (9)

$$S_1(\alpha) = \tau_{12} \sqrt{(2 + 2\cos(\theta_{12} - \alpha))} \quad \text{---(14)}$$

$$S_2(\alpha) = \tau_{12} \sqrt{(2 + 2\cos(\theta_{12} + \alpha))} \quad \text{---(15)}$$

A cyclic permutation of labels can always be found such that:

$$\tau_{12} \leqslant \tau_{23} \text{ and } \tau_{31}$$

in which case

$$\xi_{31}(\alpha) = \cos^{-1}(S_1^2(\alpha)/2\tau_{31}^2 - 1) \quad \text{---(16)}$$

and $\xi_{23}(\alpha) = \cos^{-1}(S_2^2(\alpha)/2\tau_{23}^2 - 1)$ ---(17)

always exist and may be chosen to be in the range $0 \leqslant \xi < \pi$.

From (13) $\cos(\theta_{31} + \alpha_{31}) = S_1^2(\alpha)/2\tau_{31}^2 - 1$

therefore $\theta_{31} + \alpha_{31} = \pm\, \xi_{31}(\alpha)$

and from (12) $S_3(\alpha) = \tau_{31}\sqrt{(2 + 2\cos(2\theta_{31} + (-1)^i \xi_{31}(\alpha)))} \quad i = 1 \text{ or } 2$ ---(18)

Similarly from (11) $S_3(\alpha) = \tau_{23}\sqrt{(2 + 2\cos(2\theta_{23} + (-1)^j \xi_{23}(\alpha)))} \quad j = 1 \text{ or } 2$ ---(19)

Let $F_{ij}(\alpha) = \tau_{31}^2(1 + \cos(2\theta_{31} + (-1)^i \xi_{31}(\alpha))) - \tau_{23}^2(1 + \cos(2\theta_{23} + (-1)^j \xi_{23}(\alpha)))$

then if $F_{ij}(\alpha) = 0$, $\alpha_{12} = \alpha$ corresponds to a solution of equations (8) to (13) provided:

$$|\alpha_{23}| = |\theta_{23} + (-1)^j \xi_{23}(\alpha)| \leqslant |\pi - \theta_{23}| \quad \text{---(20)}$$

and $|\alpha_{31}| = |\theta_{31} + (-1)^i \xi_{31}(\alpha)| \leqslant |\pi - \theta_{31}|$

Thus the solutions for S_1, S_2 and S_3 may be found by searching for zeros along the four branches of $F_{ij}(\alpha)$ which is continuous in α and discarding those for which (20) is not satisfied. Fortunately the form of $F_{ij}(\alpha)$ as a function of α is quite straightforward. Thus in cases of practical interest where the geometry is such that the S_i are well defined by the photograph it is only necessary to step through about 50 values of α between $-(\pi - \theta_{12})$ and $(\pi - \theta_{12})$ in equal increments of $\Delta\alpha$ and look for cases where sign($F_{ij}(\alpha)$) = - sign($F_{ij}(\alpha - \Delta\alpha)$) or sign($F_{ij}(\alpha)$) = 0 indicating a root. A sufficiently good estimate of this root is then obtained by linear interpolation. Once a root α has been found the corresponding $S_1(\alpha)$ and

$S_2(\alpha)$ may be found from (14) and (15). Similarly $S_3(\alpha)$ may be found from (16) and (18).

The accuracies of the resulting estimates of S_1, S_2 and S_3 depend on the accuracy to which the root has been found and the accuracy of the initial image points $\underline{u}_1$, $\underline{u}_2$ and $\underline{u}_3$ which are used to compute $\cos\theta_{12}$ etc. If the $\underline{u}_i$ are to some extent subject to experimental uncertainty then there is a limit to the accuracy of the S_i however well the zeros of $F_{ij}(\alpha)$ are determined. Thus there is no point in a very elaborate root finding procedure.

Step(iii): intersection of three spheres.

The perspective centres corresponding to a set of S_i from step(ii) are given by the two points of intersection of three spheres radius S_1, S_2 and S_3 with centres at $\underline{X}_1$, $\underline{X}_2$ and $\underline{X}_3$. Routines exist for solving this problem in the air survey case but they are not all sufficiently general for close range work.

The equations for these spheres may be written in terms of the variable $\underline{X}$ as:

$$(\underline{X}-\underline{X}_1)^2 = S_1^2 \qquad ---(21)$$

$$(\underline{X}-\underline{X}_2)^2 = S_2^2 \qquad ---(22)$$

$$(\underline{X}-\underline{X}_3)^2 = S_3^2 \qquad ---(23)$$

Taking (22) from (21) gives the plane containing the circle of intersection of the corresponding spheres as:

$$\underline{X}.(\underline{X}_1-\underline{X}_2) = 1/2\left(\underline{X}_1^2-\underline{X}_2^2-(S_1^2-S_2^2)\right) \qquad ---(24)$$

similarly for the other planes of intersection

$$\underline{X}.(\underline{X}_2-\underline{X}_3) = 1/2\left(\underline{X}_2^2-\underline{X}_3^2-(S_2^2-S_3^2)\right) \qquad ---(25)$$

These planes meet in a line which is perpendicular to the plane of the triangle with vertices at $\underline{X}_1$, $\underline{X}_2$ and $\underline{X}_3$ and which contains the two points of intersection of the spheres. The normal $\underline{K}$ to this triangle is given by:

$$\underline{K} = \underline{X}_1\wedge\underline{X}_2+\underline{X}_2\wedge\underline{X}_3+\underline{X}_3\wedge\underline{X}_1$$

Hence the equation of the plane containing this triangle is:

$$\underline{X}.\underline{K} = \underline{X}_1.(\underline{X}_2\wedge\underline{X}_3) \qquad ---(26)$$

Solution of (26) with (24) and (25) gives $\underline{X}_p$ the point at which the line joining the intersections of the three spheres crosses the plane of the triangle.

The plane of the triangle is the perpendicular bisector of the line joining the two points of intersection of the three spheres. Thus the estimates of $\underline{X}_o$ may be written:

$$\underline{X}_o = \underline{X}_p \pm t\,\underline{K} \qquad \text{---(27)}$$

Substitution of (27) in (21) gives:

$$t^2 = (S_1^2 - (\underline{X}_p - \underline{X}_1)^2)/\underline{K}^2$$

Step(iv): choice of the best solution.

From (6) $\cos\theta_{ij} = (\underline{X}_i - \underline{X}_o).(\underline{X}_j - \underline{X}_o)/S_i S_j$

$= (\underline{u}_i.\underline{u}_j + f^2)/(\sqrt{(\underline{u}_i^2+f^2)}\sqrt{(\underline{u}_j^2+f^2)})$

for all pairs of points imaged on the photograph. Thus the 'best' estimate of the perspective centre $\underline{X}_o$ is taken as that which gives the minimum value of the sum:

$$T(\underline{X}_o) = \sum_{i>j} \left\{ \frac{(\underline{X}_i-\underline{X}_o).(\underline{X}_j-\underline{X}_o)}{S_i S_j} - \frac{\underline{u}_i.\underline{u}_j + f^2}{\sqrt{(\underline{u}_i^2+f^2)}\sqrt{(\underline{u}_j^2+f^2)}} \right\}$$

4. A Method of Finding the R Matrix

The orientation of the 'internal' co-ordinate system of the camera in the act of taking the pth photograph may be referred to as the orientation of that photograph. This orientation is described in terms of the rotation matrix $\underline{\underline{R}}^{(p)}$ in (1).
From (2) and (5)

$$\hat{\underline{G}}_i = -\underline{\underline{R}}\,\hat{\underline{P}}_i \qquad \text{---(28)}$$

where $\hat{\underline{G}}_i = \underline{G}_i / |\underline{G}_i|$ depends on $\underline{X}_i$ and $\underline{X}_o$
and $\hat{\underline{P}}_i = \underline{P}_i / |\underline{P}_i|$ depends on $\underline{u}_i$ and f.
Thus they may be computed for any point imaged on the photograph for which initial co-ordinates are known using the best estimate of $\underline{X}_o$ from section(3). To illustrate the method of solution the indices on $\hat{\underline{G}}_i$, $\hat{\underline{P}}_i$ and $\underline{\underline{R}}$ may be rearranged to give:

$$y_i(j) = (\hat{\underline{G}}_i)_j\,,\quad A_{ik} = (\hat{\underline{P}}_i)_k \quad \text{and} \quad r_k(j) = R_{jk}$$

then (28) becomes:

$$\underline{y}(j) = \underline{\underline{A}}\,\underline{r}(j)$$

where the $\underline{A}$ matrix is the same in each case. Thus provided i runs at least from 1 to 3 it is possible to give a least squares solution for the $\underline{r}(j)$ and hence the R_{ij} as though they were independent variables. This estimate of $\underline{\underline{R}}$

i.e. $\underline{\underline{R}}^T = (\underline{r}(1), \underline{r}(2), \underline{r}(3))$

is very nearly orthogonal because (28) implies that it leaves the length of a unit vector unchanged. This is exactly the same method of finding $\underline{\underline{R}}$ normally used in air survey work. The only refinement before using it as an initial $\underline{\underline{R}}$ for the 'bundle adjustment' program is to ensure that it is orthogonal to the accuracy of the computer by using the Schmidt procedure. This ensures that $\underline{\underline{R}}$ is a true rotation matrix.

5. An Intersection Method to Find the Unknown Points

An intersection may be performed with any number of cameras greater than one as follows:

from (28) $\underline{X}_i = \underline{X}_o^{(p)} - \underline{\underline{R}}^{(p)}\underline{\hat{P}}_i^{(p)} |\underline{X}_i - \underline{X}_o^{(p)}|$

let $d_i^{(p)} = |\underline{X}_i - \underline{X}_o^{(p)}|$

then $\underline{X}_o^{(p)} = \underline{X}_i + d_i^{(p)} \underline{\underline{R}}^{(p)} \underline{\hat{P}}_i^{(p)}$ ---(29)

At this stage in the calculation $\underline{X}_o^{(p)}$, $\underline{\underline{R}}^{(p)}$ and $\underline{\hat{P}}_i^{(p)}$ have been estimated therefore only $\underline{X}_i$ and $d_i^{(p)}$ are unknown. If a solution is sought which minimises V_i where:

$$V_i = \sum_p V_i^{(p)} = \sum_p (\underline{X}_o^{(p)} - \underline{X}_i - d_i^{(p)} \underline{\underline{R}}^{(p)} \underline{\hat{P}}_i^{(p)})^2$$

the sum being taken only over photographs containing images of the ith point, then (29) looks like a linear least squares problem with $3n_p^{(i)}$ equations (where $n_p^{(i)}$ is the number of photographs with images of the ith point) in $3 + n_p^{(i)}$ unknowns. Thus (29) may be solved in a least squares sense provided that the ith point is imaged by at least two independent cameras.

This method is somewhat different from that which is usually employed for two photographs but has the advantage that it is easily generalised to any number of photographs greater than two and gives an easily soluble linear problem.

6. Practical data input

While we have been running photogrammetric programs at NPL it has been found that a major source of effort is the preparation of a good set of data. There are three additions to the 'front end' software which can greatly increase the efficiency of data preparation.

First: each photograph is checked for repeated occurences of the same point, as identified by its serial number. Any repeated occurences are listed and execution of the program terminated.

Second: an entry to the package is provided in an interactive mode. This ensures that a data file may easily be created in a suitable format to be read correctly by the main program. This provision is particularly useful for large data sets and users who are not familiar with the layout of the data.

Third: some form of gross error detection is necessary. This is provided by examining the virtual displacements necessary to obtain a perfect intersection as described in section(5). At NPL these virtual displacements $V_i^{(p)}$ have proved remarkably good indicators of points with gross data errors and in many cases show which photograph is at fault. The advantage of inspecting for gross errors at this stage is that they have not been 'spread out' over the rest of the data by a global least squares adjustment.

Ideally all data checking and correction should be done as the points are being measured so that they can be remeasured immediately if there is any reason to suspect an error. In order to implement this philosophy a version of the above analysis has been used to produce a resection/intersection package for a 32k microcomputer (BBC Model B) which will be used to control an automated version of a Zeiss ZKM measuring machine.

7. Conclusion

Despite extensive discussion of the general 'bundle' solution in the literature there seems to be little written on reliable methods of finding initial estimates of the photogrammetric variables. These initial estimates are required to linearise the equations giving the image co-ordinates in terms of the object co-ordinates etc.

As there are a great number of different types of photogrammetric problem it is difficult, if not impossible, to give a prescription for doing all of them. Yet it has been possible to structure the 'front end' program used at NPL in such a way that various forms of constraint are written in terms of initial point positions. In particular if some photographs fail to image four initial points a preliminary intersection is carried out to provide additional 'initial' values. Various forms of iterative refinement may be used but these are largely inappropriate for a 'front end' package which is to be followed by a full 'bundle adjustment'.

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Figure (1)

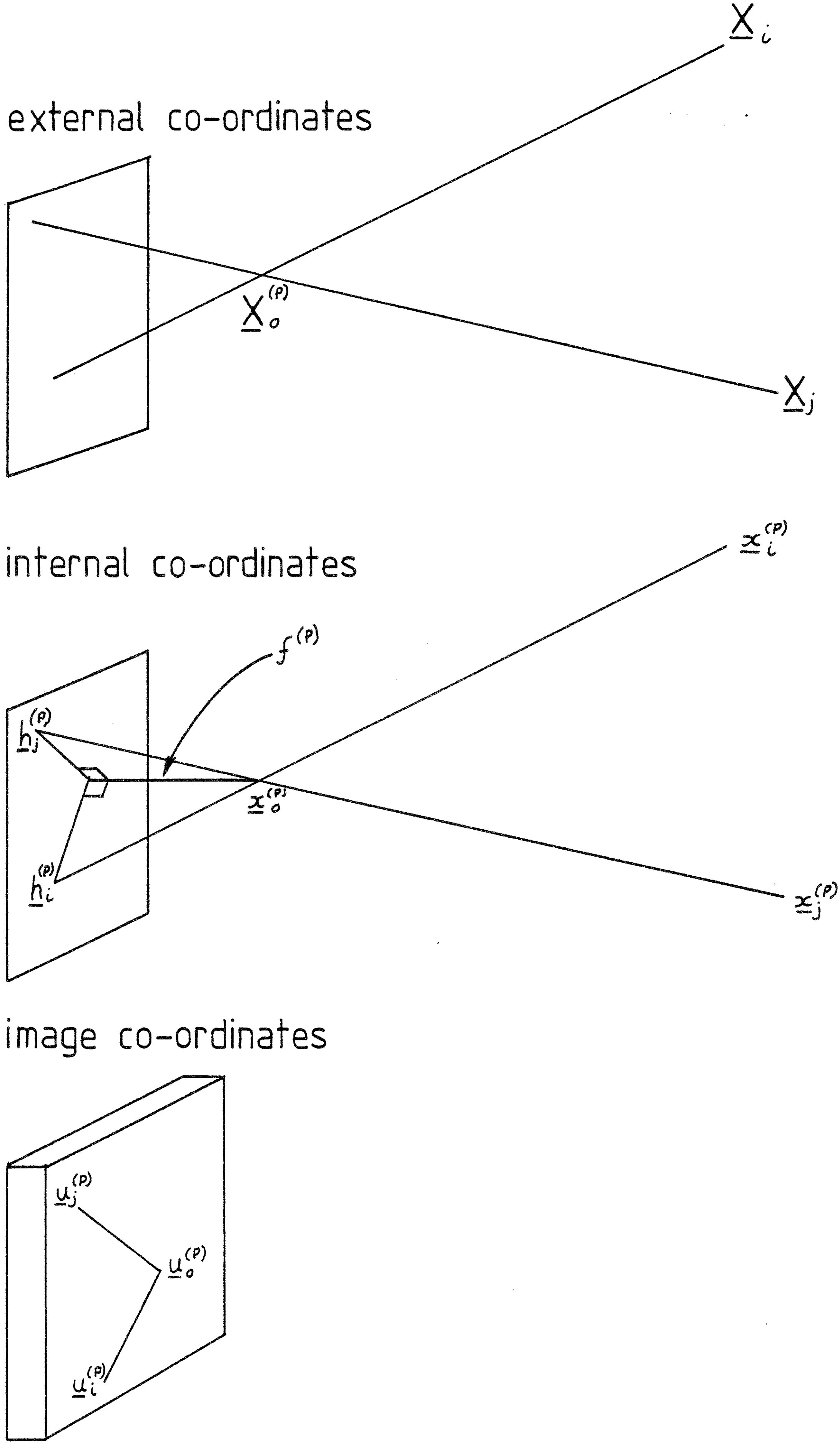
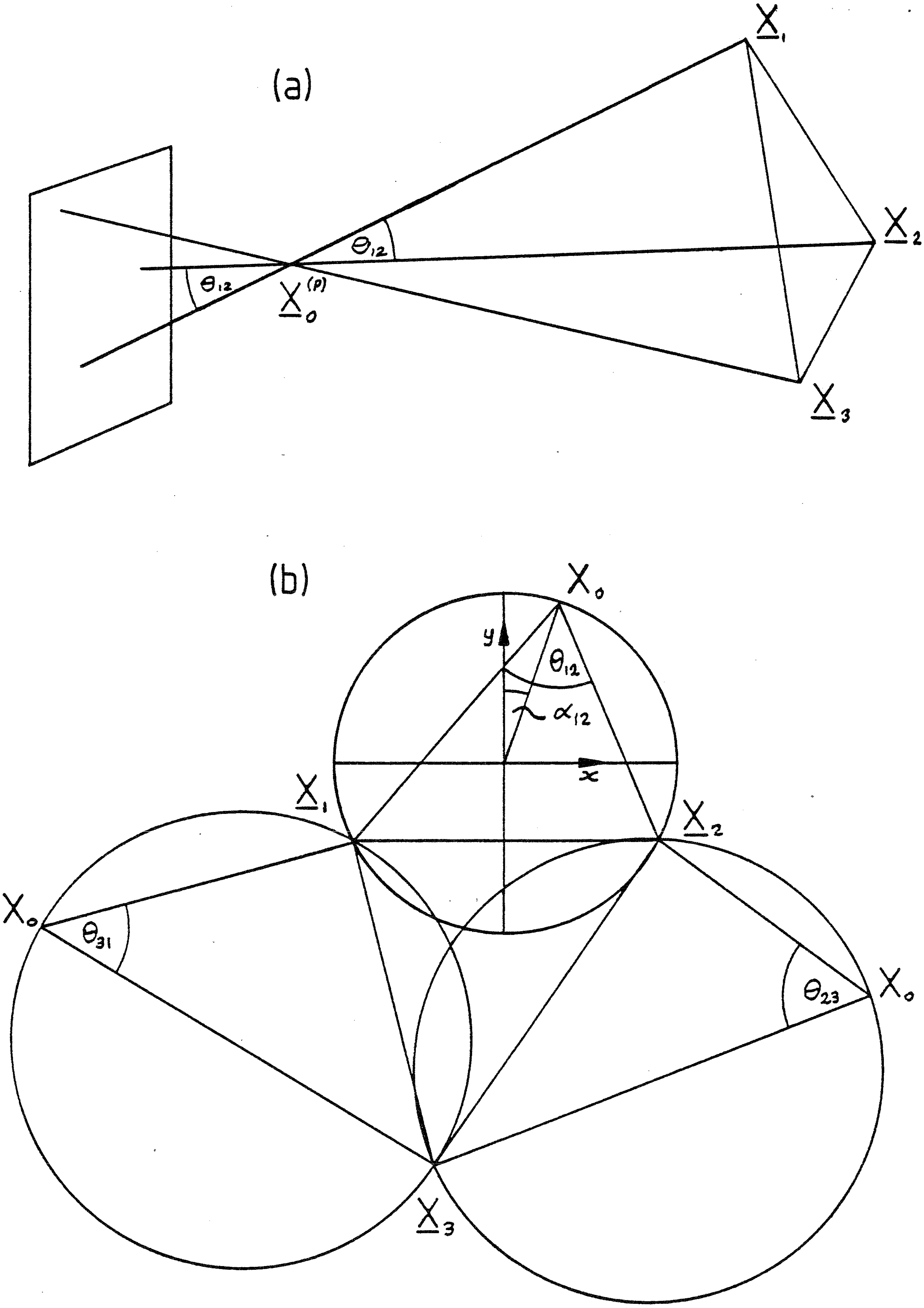

$\underline{X}_i$
external co-ordinates
$\underline{X}_o^{(p)}$
$\underline{X}_j$
internal co-ordinates
$\underline{x}_i^{(p)}$
$f^{(p)}$
$\underline{h}_j^{(p)}$
$\underline{x}_o^{(p)}$
$\underline{h}_i^{(p)}$
$\underline{x}_j^{(p)}$
image co-ordinates
$\underline{u}_j^{(p)}$
$\underline{u}_o^{(p)}$
$\underline{u}_i^{(p)}$

Figure (2)


(a)
$\underline{X}_1$
$\underline{X}_2$
$\underline{X}_3$
θ_{12}
θ_{12}
$\underline{X}_0^{(p)}$
(b)
X_0
y
θ_{12}
α_{12}
x
$\underline{X}_1$
$\underline{X}_2$
X_0
θ_{31}
θ_{23}
X_0
$\underline{X}_3$