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GEOMETRIC-OPTICAL SITUATION
In photographic flights, the pilot keeps the aeroplane at a constant hight and flies along a predetermined route at a constant speed, the influence of crosswind being largely eliminated by setting the longitudinal axis of the aircraft at an angle relative to the flying direction. This angle of attack is set by the pilot according to verbal feedback with the phom graphy navigator. This procedure cannot fully eliminate deviations, which have to be compensated by rotating the camera. The situation is shown on the diagram (Fig. 1).
The measuring system required for such rotation consists of two parallel CCD rows arranged at right angles with the direction of flight. The CCD rows scan the terrain stripwise through an optical system. Row I captures a strip ahead of the plane, row II one behind the plane. After the first cyclic exposure of row I, row II is exposed repeatedly until it recognizes the strip previously registered by row I. Until that recognition a certain time has passed during which the plane may have turned through the angle $\Delta K$ (yaw) so that the two sensor images are shifted relative to one another, both angularly and especially, linearly. The digitized brightness profiles, filed in the memory of the control computer, can then be shifted back realtive to each other until the best possible agreement is achieved, the amount of shift $\mu$ being a measure of the angle $\Delta \mu$. Recognition is impaired primarily by parallel shifts of the plane's longitudinal axis and by the plane's longitudinal axis and by the plane's roll. The error resulting from the angular offset of the registered brightness profiles due to yaw can be kept small by frequent scanning and optical smearing.
The second variable quantity to be registered is the speed--to-height ratio ( $\mathrm{v} / \mathrm{h}$ ). It is required for setting the frame rate so that all photos have the preset degree of overlap and for the correct compensation of forward motion during film exposure. Fig. 2 illustrates the process of determining the $\mathrm{V} / \mathrm{h}$ ratio. Assuming that row II is exposed at short time intervals of $\Delta t, n \cdot \Delta t$ is the time that passes until row II recognizes the brightness profile registered by row I n steps earlier. Then distance covered is $v \cdot n \cdot \Delta t$, and with a simple proportional equation the dependence $v / h=v / h\left(n^{-1}\right)$ can be determined. Given the fixed row spacing $d$, the focal length of the lens and the known scanning time $\Delta t$, we have

$$
\begin{equation*}
V / h=\frac{d}{I \cdot \Delta t} \cdot \frac{1}{n} \tag{1}
\end{equation*}
$$

The measurement is degraded particularly by vibrations of the plane about its cross axis. The error caused by the lack of certainty whether row II actually scans the very strip previously registered by row I can be reduced by optical smearing.

## PROCESSING OF THE OBTAINED DATA

The comparison of the brightness profiles of row I and row II mentioned above means a determination of the degree of correlation between the two rows. Row II has recognized the brightness profile of row I if the correlation measure adopts an extremum. the time available ( $\Delta t=50 \mathrm{~ms}$ ) is too short to allow the correlation function to be determined. As a substitute, the mean of the absolute differences (MAD) of the pixel values is formed. This MAD function has a minimum whereever the correlation function has a maximum $/ 1 /$. Thus, the search for maximum correlation will, hereafter, always be a search for the minimum of the MAD function. As the formation of the mean of absolute differences always extends over the same number of pixel pairs, division by the number $u$ of pixel pairs may be spared. Then, the formula for computing the MAD function is
$m(j)=\sum_{i=0}^{u}\left|a\left(i+\mu^{u}\right)-b(i)\right| \quad \quad \rho^{u}=-j \cdots+j$
Where $\mu$ designates the shift of row II relative to row $I$. The quantities $a$ and $b$ are the digit ed brightnesses of the pixels of row I and II, respectively. Frequently the difference of thefrow means $(\bar{a}-\bar{b})$ is subtracted within the absolute value signs. This measure would be of importance especially if the rows were exposed for different times or if different drifts of the equal shares might ocour in case of different temperatur coefficients due to temperature variation. To solve these problems, exposure times are determined anev prior to every eposure cycle and maint-ained through the respective cycle for both rows. The temperatur problem is further reduced by mounting both CCD rows on a ceramic base.
In the device here described, the rows have been extended at either and by $|j|=7$ pixels, so that altogether 15 shifts are examined. The pixels used for row extension have to be so selected that they do not in any case exhibit a correlation with row II; they only serve the purpose of always having u pairs of values in order to save the division. Fig. 3 shows the resulting representation of functions for five exposures of row II. The graph also indicates that, with an assumed shift of three pixels, the correlation maximum, i.e. the MAD minimum, is found after four correlations. Thus, the $v / h$ ratio is $n=4$ and the drift angle $\mu=3$. It should be added that in our instrument a shift by seven pixels corresponds to the
registration of a dript angle of $10^{\circ}$; this is due to the row geometry and the dimensions of the lens. Problems in ascertaining the MAD minimum arise if disturbing factors prevent the formation of a function "trough" and the MAD function degenerates into an inclined straight line. In that case the algorithm inds a minimum at the boundary, which of course has to be discarded.

## MEASURES TO EITMINATE DISTURBATOE

From the previous passages it is evident that the state varibles $n$ and a are subjected to a variety of disturbing influences. It suggests itself, therefore, that the values ascertained should be filtered before feeding them to the control elements. Since it is possble to define a linear, though simple, model of the aircraft's motion, to classify the disturbances with regard to their points of attack and to estimate their standerd deviations, an optimum linear filter (KALMAN filter) can be devised, if we assume independence between state variables and disturbances as well as independence of the disturbances from each other. Such a filter has a number of advantages which will be explained later.
The signal generation model corresponds to the system equations of the system "aircraft-camera", with $n(t)$ being the only state variable present. If we assume that in flight along a route at a constant height and with preset wind correction angle and speed, $n(t)$ must be constant, then changes of $n(t)$ can exclusively caused by disturbances (turbulences, thermal currents, crosswind swaying). Hence, we arrive at a very simple motion equation:

$$
\begin{equation*}
n(t)=b \cdot f(t) \tag{3}
\end{equation*}
$$

where $f(t)$ stands for system noise, which may be assumed to be white noise. Further, the observation equation of the process has to be formulated:

$$
\begin{equation*}
y_{j}=n_{j}+N_{j} \tag{4}
\end{equation*}
$$

In which $\mathbb{N}$ designates discrete white disturbances of the measurements with a Gaussian distribution, and $y_{j}$ stands for the $n(t)$ values observed, i.e obtained by the measuring procedure described above. From the above assumptions, the standard deviations of the disturbing signals can be stated to be

$$
\begin{gather*}
\mathbb{E}(b\}(t) \xi(s) b)=R=b^{2} \delta(t-s)  \tag{5}\\
\mathbb{E}\left(\mathbb{N}_{1} \mathbb{N}_{j}\right)=\mathbb{N}_{0} \delta(i, j)  \tag{6}\\
\text { with designating Kronecker' }
\end{gather*}
$$

The KALMAN filter is derived in the way outlined in ref. $/ 2 /$. Adopting the statement derived there from the Fokker-Planck equation for the development of the first two cumulants of the state distribution, we can generally state for linear systems:

$$
\begin{align*}
& \dot{\hat{Z}}(t)=A \cdot \hat{Z}(t)  \tag{7}\\
& \dot{K}(t)=A \cdot K(t)+(A \cdot K(t))^{T}+R \tag{8}
\end{align*}
$$

in which $z(t)$ is the first and $K(t)$ the second cumulant and A the system matrix. Substitution of our model in these evolution equation with
$\hat{Z}(t)=\hat{n}(t)$ and their solution supply the equations for the evolution of the first two cumulants with an unobserved process. If the process is sampled, as it is the case here, the integrals are computed only between two sampling points. The solution is obtained in terms of the transition from the so-called a-posteriori to the a priori value of the cumulants with sampling period I . For the first cumulant we obtain, with $\hat{h}(0)=\hat{n}_{j-1 P}$ and $\hat{n}(T)=A_{J A}$

$$
\begin{equation*}
\hat{n}_{j A}=\hat{n}_{j-1 P} \tag{9}
\end{equation*}
$$

and for the second cumulant, with $K(0)=K_{j-1 P}$ and $K(T)=K_{j A}$,

$$
\begin{equation*}
K_{j A}=K_{j-1 P}+b^{2} T \tag{10}
\end{equation*}
$$

The observation of the process via the linear observation equation (4) permits us to state the function

$$
p\left(y_{i /} n_{j}\right) \text {, which in case of Gaussian observation noise }
$$

is a Gaussian distribution. It is termed a Iikelihood function and makes it possible that the a-priori values computed in the unobserved system motion can be improved in accordance with the information obtained by observation. Ref. / / / contains a dexivation of the transition from a-priori to a-posteriori values. Here if may suffice to state the results for our case:

$$
\begin{align*}
& \hat{h}_{j P}=\hat{n}_{j A}+\frac{K_{j A}}{N_{0}+K_{j A}}\left(y_{j}-\hat{n}_{j A}\right)  \tag{11}\\
& K_{j P}=K_{j A}-\frac{K_{j A}^{2}}{\mathbb{N}_{0}+K_{j A}} \tag{12}
\end{align*}
$$

Thus the recursion for computing the cumulants is closed and the indices $A, P$ may be dropped if (9) is substituted in (11) and )10) in (12):

$$
\begin{align*}
& \hat{n}_{j}=\hat{n}_{j-1}+\frac{K_{j-1}+b^{2} T}{K_{j-1}+N_{0}+b^{2} T}\left(y_{j}-A_{j-1}\right)  \tag{13}\\
& K_{j}=\frac{N_{0}\left(K_{j-1}+b^{2} T\right)}{N_{0}+K_{j-1}+b^{2} T} \tag{14}
\end{align*}
$$

From the recursion equation for the second cumulant (14) it is apparent. That $K_{\text {. }}$ must approach a stationary value, because no process observations are treated. The stationary value of the cumulants is determined only by estimations of the standard deviations of system and observations disturbances. If the iterative computation of the second cumulant is performed in the cyclic computation of the filter, the behaviour of the cumulant approaching its stationary value may be utilized for accelerating the minimization of errors that may occur because of faulty initial values for the first cumulant of the state variables.
In order to make the filtering more lucid, let us introduce the abbreviated notations $q_{j}{ }^{N} e=K_{j}$ and
$y=b^{2} T / N_{0}$. Thus we have, in a Iucid presentation.

$$
\begin{align*}
& q_{j}=\frac{q_{j-1}+\gamma}{q_{j-1}+\gamma^{n}+1}  \tag{15}\\
& \hat{n}_{j}=\hat{n}_{j-1}+q_{j}\left(y_{j}-A_{j-1}\right) \tag{16}
\end{align*}
$$

A numerical simplification is possible by avoiding the division in eq. (15). It is obvious that $\left\{q_{j}\right\}$ approaches a statonary value $a_{\infty}$, which can be computed beforehand. Then, the convergent series (15) can be approximated by another series which converges against the same value $q_{\infty}$ but is easier to compute. In the present case we took recourse to the series

$$
\begin{equation*}
q_{j}=-\frac{1}{2}\left(q_{j-1}+q_{\infty}\right) \tag{17}
\end{equation*}
$$

which can obviously be computed by addition and shifting to the right (especially in a fixed point arithmetic).
As the same mathematical model is employed for filtering the drift deviations, the same filter structure results, but it has different parameters for the standard deviations of errors. The state variable is the shift $\mu(t)$ with the first cumulant $\hat{A}(t)$.

ESTIMATION OR THE STANDARD DEVIATIONS OF THE DISTURBANCE System disturbances are allowed for by the quantity $b^{2}$ in the filter equation. If one estimates the product $b^{2} T$ instead of the standard deviation $b^{2}$, another simplification results, because $b^{2} T$ can be assumed to be constant at all flying heights, the reason being that standard deviations $b^{2}$ in general decrease with increasing heights, whereas the time required for determining a new state value increases. From the experience of photographic flight navigators one can derive how often per unit time and by what amounts dript corrections have to be made. For example, a statement that along a route of 12 km length and at a flying speed of $250 \mathrm{~km} / \mathrm{h}$ drift is corrected three to four times by $2-3^{\circ}$ each, means that $b^{2}$ must be taken to be $0.225 \mathrm{deg}^{2} / \mathrm{s}$.
For the ascertainment of measurement dist-urbances referred to drift, aviation engineering studies have established the angular velocit-ies about the three main axis as function of time. The measurements revial that roll movements have greatest disturbing influence on driet control, because they are, unfortunately, amplifies by CCD row arrangement. Further, the abovementioned studies clearly show the height dependence of the standard deviations $\mathrm{N}_{0}$. (For estimating $\mathrm{N}_{0}$, we have referred only to disturbances due to roll). To come back to our example: at a height of 3000 m , angular velocities of about $1 \% / s$ may occur, with maximum angular differences being 1. $5^{\circ}$. In the test setup, this measuring error is magnified to $5^{\circ}$, so that a standard deviation of $N_{0}=25 \mathrm{deg}^{2}$ is obtained, which at an angular velocity of $1^{\circ} / \mathrm{s}$ correspond to changes in standard deviation by $N_{0} / t=25 \mathrm{deg}^{2} / \mathrm{s}$. The standard deviation after a time $T$ then is $N_{0} / t \cdot T$. The filter quantity $\%$ is given by

$$
\gamma=\left(b^{2} 。 \mathbb{T}\right) /\left(\left(\mathbb{N}_{0} / t\right) \cdot \mathbb{T}\right)=b^{2} /\left(\mathbb{N}_{0} / t\right)=0.225 / 25=0.01 \text {. }
$$

The calculations demonstrated here have to be made for measurement disturbances at different heights. The results may then be filed as a table in the ROM.
Concerning the state value $n$, the system disturbances can be estimated from a series of aerial photographs taken without followup control of the $\mathrm{V} / \mathrm{h}$ ratio. The greatest error in overlap can be converted into an error of $n$, i.e. the number of steps until the MAD-minimum is reached. The dange rate of standard deviation can be estimated if the time between two exposures is known and the change of the $n$ values can be referxed thereto. The standard deviation of the measurement or observation disturbances can be estimated by the angular velovity of motions about the aeroplane's cross axis, recorded during the aviation engineering studies. The estimations must be made depending on height. The procedure may be largely analogous to the estimations of drift angle disturbances.

## SUMMARY

Aerial cameras for high-accuracy photography are usually operated manually. Automation appears to be cost-saving especially where photographs have to be taken routinely and repeatelly and flying personnel is saved. The principle presented in this article solves the tasks of drift follow-up control and determination of the relative ground speed, which otherwise have to be performed by the operator. Consequently, the automatic controller has to determine continuously the angle between the aircraft's longitudinal axis and the route direction as well as the ratio between flying speed and flying height. These characteristics are obtained passively by optoelectronic scanning of the earth's surface. This principle is much cheaper than radar-based active measuring methods. The measurement data are digitized and processed by two single-chip microcomputers. The article describes the metrological situation and the technical principle, the mathematical treatment of the measured data, and the measures taken for eliminating measuring errors.

RERERRENCES
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Pig. 1 Relation between rotation $\triangle$ Kof the aircraft s longitudinal axis and resulting misalignment $j^{u}$ between the brightness profiles recorded by sows I and II



