A New Holographic Inverse Filtering Method And Its Application To Restoration Of Degraded Remote Sensing Images

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# I. Introduction

The sharpness and resolution of the remote sensing images are limited by the image motion. In recent years, it was improved by the forward motion compensation. But in many cases, there are also many linear motion blur images. The a posteriori methods of degraded pictures can be categorized under two heads: (a) digital, (b) optical(analog). Digital methods, which are mainly computer assisted, have produced some excellent results and are commended for their ability to realize nonlinear transformations. But when the image to be processed has a large space-bandwidth product, digital techniques become difficult and expensive. This poses a serious limitation to the application of digital techniques. On the other hand, optical image processing is inexpensive and simple in comparision. But there are a serious limitations on the coherent optical image processing techniques: the low efficiencies, the small range of linear filters and the coherent noise. To overcome response of the this disadvantages, a new holographic inverse filtering system has been developed by the authors. The inverse filter is made by recording a hologram in the back of an attenuate mask 1/ [H] and then the filtering operation is done in multi-channel. The theoretical analysis and the experimental result are given.



# II. Description of the Optical System

Fig. 1: The experimental system of filter generating and image processing The optical experimental system is shown in Fig.1. The 65mw He-Ne laser is used as the coherent source: One bundle of beam splited by the beam spliter BS is collimated by lens L<sub>o</sub> and illuminates the object plane OP, the other beam splited is collimated by L<sub>R</sub> and illuminates the filter plane FP at the angle  $\Theta$  as the reference beam. The intensity of the reference beam is controlled by a pair of polarizers  $g_1$  and  $g_2$ . The cameras L<sub>1</sub> and L<sub>2</sub> with focal length equal to 600mm are used as the Fourier transform lenses. The front focal plane of L<sub>1</sub> is the object (input) plane OP, the back focal plane is the filter plane FP, which is also the front focal plane of the lens L<sub>2</sub>.

In the image processing step, the reference beam is blocked. The deblurred aerial image is formed in the image plane IP (the back focal plane of  $L_2$ ). The frequency of the grating used in this system is 40 lps/mm. The recording material is the Tianjing holographic I plate, its sensitivity is  $3.0 \times 10^{-5} \text{J/cm}^2$ .

#### III. Theory

The theoretical background of optical filtering is well known. However, for the sake of completeness, a brief outline of the mathematics involved is given as follows.

The formation of a blurred photograph (for that matter any photographs, blurred or sharp) can be described in general by the superposition integral

$$b(x,y) = \iint_{-\infty}^{+\infty} f(x',y')h(x,y;x',y')dx'dy'$$
 (1)

where f(x, y) and b(x, y) are the image and object intensity distribution respectively, and h(x, y) is the impulse response of the optical system. For certain types of blurs, which include linear motion of the camera under the assumption that the picture occupies only a limited region of the field. Eq.(1) can be simplified to a convolution integral

$$b(x,y) = \iint_{-\infty}^{+\infty} f(x',y')h(x-x',y-y')dx'dy'$$
(2)

In the Fourier plane, Eq.(2) can be written as

$$B(\xi, \eta) = F(\xi, \eta) \quad H(\xi, \eta) \tag{3}$$

where  $\xi$ ,  $\eta$  are spatial frequencies in the  $x_t$  and  $y_t$  directions, and the B, F and H are the Fourier transforms of b, f and h respectively. By passing Eq.(3) through a filter with transmittance proportional to H<sup>-1</sup>( $\xi$ ,  $\eta$ ), we can recover F( $\xi$ ,  $\eta$ ) completely, provided H never goes to zero within the region where F( $\xi$ ,  $\eta$ )≠0. The operations of multi-channel inverse filtering system are as follows.

# A. The basic relations

In the filter-generating and image-processing steps, a grating, with spatial frequency  $p_o$ , is superposed on the transparency representing the PSF (point spread function) and on the transparency representing the blurred picture. The amplitude transmittance of the grating with lines running parallel to the  $y_o$  axis (perpendicular to the blurring direction) is described by a periodic function  $g(x_o)$ , which can be expressed as a Fourier series

$$g(x_0) = \sum_{n=\infty}^{\infty} C_n \exp \left\{ j 2\pi n p_0 x_0 \right\} \qquad n=0, \pm 1, \pm 2,$$
(4)

where the Fourier coefficients are given by

$$C_{n} = p_{0} \int_{0}^{1/p_{0}} g(x_{0}) e^{-j2\pi n p_{0} x_{0}} dx_{0}$$
(5)

The amplitude transmittance of the grating and PSF transparency in contact is

$$h^{(p)}(x_0, y_0) = h(x_0, y_0) \cdot g(x_0)$$

its Fourier transform is

$$H^{(p)}(\xi, \eta) = \sum_{n=-\infty}^{\infty} C_n \cdot H(\xi - np_0, \eta)$$
(6)

Similarly, the Fourier transform of the transmittance of the blurred picture in contact with the grating is

$$B^{(p)}(\xi, \eta) = \sum_{n=-\infty}^{\infty} C_n F(\xi - np_0, \eta) \cdot H(\xi - np_0, \eta)$$
(7)

where we have applied the convolution theorem to Eq.(2). Corresponding to the various diffraction orders of the modulating grating, the spectra of the PSF and blurred picture appears in a series of equally spaced islands in the filter plane, the amplitude distributions across each islands vary in proportional to the Fourier transform H and B respectively. The amplitudes of various islands, if compared at points corresponding to the same spatial frequency, are proportional to the Fourier coefficients of the amplitude transmittance of the grating.

In the following, we will assume that the functions h(x,y) and f(x,y) are band-limited in the  $x_0$  direction, as expressed by

$$\frac{H(\xi, \eta) = 0}{F(\xi, \eta) = 0}$$
 for  $|\xi| \ge p_0/2$  (8)

It is equivalent to requiring that various islands of the spectrum do not overlap.

## B. The attenuating mask

Two parts are involved in the filter-generating step. The first is to get an absorptive mask with the amplitude transparency in proportional to  $|H^{(p)}(\xi,\eta)|^{-1}$ , by recording the intensity distribution  $|H^{(p)}(\xi,\eta)|^2$ . And then to make the filter by recording a hologram in the back of the absorptive mask  $|H^{(p)}(\xi,\eta)|^{-1}$ , with the interference between a weak reference beam and the Fourier spectrum of  $h^{(p)}(x,y)$ .

In the object plane, the collimated beam transilluminates a transparency representing the function h(x,y) which is in contact with the transmission grating, the distribution of the Fourier spectrum plane is

$$\mathbf{H}^{(\mathbf{p})}(\xi,\eta) = \frac{\mathbf{A}_0}{\mathbf{j}\lambda \mathbf{f}} \sum_{n=-\infty}^{\infty} \mathbf{C}_n \cdot \mathbf{H}(\xi - n\mathbf{p}_0,\eta)$$
(9)

where H is a normalized function,  $A_0$  is the peak value of its zero order and  $\lambda$  is the optical wavelength. As a consequence of the band-limited condition (8), the terms in Eq.(9) do not over-

lap, hence can be squared individually and summed to yield  $|A^{(\,p\,)}\,(\,\xi,\,\eta)\,|^{\,2}\,$  .The exposure for the mask  $|\,H^{(\,p\,)}\,|^{-1}$  can thus be written as

$$E_{a}(\xi, \eta) = \sum_{n=-N}^{N} E'_{an}(\xi - np_{0}, \eta)$$

where N is the highest order used in this system. The exposure of the n-th islands is

$$E_{an}(\xi - np_{0}, \eta) = \frac{\Delta t_{1} \cdot A_{0}}{\lambda^{2} f^{2}} \cdot |C_{n}|^{2} \cdot |H(\xi - np_{0}, \eta)|^{2}$$
(10)

The amplitude transmittance of the n-th island of the mask, for the case of the ideal recording medium with a Gamma of 1 can be expressed

$$\tau_{n} = \frac{\mathbf{k}_{n} \lambda \mathbf{f}}{\sqrt{\Delta t_{1} \cdot \mathbf{A}_{0}}} \cdot \left| \mathbf{C}_{n} \cdot \mathbf{H} \left( \boldsymbol{\xi} - n \boldsymbol{p}_{0}, \boldsymbol{\eta} \right) \right|^{-1}$$
(11)

where  $k_n$  is a constant which is related to the photographic techniques. Now we have got the attenuating mask  $|H^{(p)}|^{-1}$ .

# C. The holographic inverse filter

As mentioned in section B, to make the filter, it is necessary to replace the attenuating mask  $|H^{(p)}|^{-1}$  into its recording position, and it is illuminated by the fields A  $^{(p)}(\xi,\eta)$  and the reference beam Rexp  $\{-j2\pi\xi,x_f\}$ ,  $\xi_r$  is the spatial frequency of the reference beam. The mask attenuates the A<sup>(p)</sup> and reference beam equally, the ratios between the two beams remain unchanged. Now we record the hologram in the back of the mask, the exposure to which the holographic plate in the plane( $x_f, y_f$ ) is subjected is

$$\mathbf{E}_{f}(\boldsymbol{\xi},\boldsymbol{\eta}) = \left[ (\mathbf{H}^{(p)}(\boldsymbol{\xi},\boldsymbol{\eta}) + \mathbf{R} \cdot \exp\{-j_{2}\pi\boldsymbol{\xi}_{r}\boldsymbol{x}_{f}\} ) \cdot \boldsymbol{\tau}(\boldsymbol{\xi},\boldsymbol{\eta}) \right]^{r}$$
(12)

 $E_f$  can be arranged and seperated into two terms

$$\begin{split} \mathbf{E}_{\mathbf{f}} &= \mathbf{\bar{E}} + \mathbf{\widetilde{E}} = (\sum_{\mathbf{\bar{E}},\mathbf{n}} + \sum_{\mathbf{\bar{E}},\mathbf{n}}) \\ \mathbf{\bar{E}}_{\mathbf{n}} &= 1 + \lambda^{2} \mathbf{f}^{2} \cdot (\mathbf{R} / \mathbf{A})^{2} \cdot |\mathbf{C}_{\mathbf{n}}|^{-2} \cdot |\mathbf{H}(\xi - \mathbf{n}\mathbf{p}_{0}, \eta)|^{-2} \\ \mathbf{\widetilde{E}}_{\mathbf{n}} &= \frac{\lambda f \mathbf{R} / \mathbf{A}_{0}}{|\mathbf{C}_{\mathbf{n}}\mathbf{H}|} \sin(2\pi\xi_{\mathbf{r}}\mathbf{x}_{\mathbf{f}} + \Psi_{\mathbf{H}\mathbf{n}} + \delta_{\mathbf{n}}) \end{split}$$
(13b)

where  $\tilde{E}$  is the d-c part of Eq.(12),  $\tilde{E}$  is its a-c part,  $\varphi_{H_n}$  and  $\delta_n$  are defined as the phases of  $H(\xi - np_0, \eta)$  and  $C_n$  respectively.  $R/A_0$  is the ratio of the reference beam to object beam. Eq.(13b) is the term of interest for the inverse filter. If the hologram with a perfectly linear recording medium (amplitude transmittance proportional to exposure), the ideal amplitude transmittance corresponding to this term is

$$\tau_{fn} = \frac{\lambda \cdot f \cdot R / A_0}{C_n \cdot H} \exp\{-j2\pi\xi_r x_f\}$$
(14)

## D. The deblurring processing

During the image processing step, the blurred image and the grating are placed in contact, which yields an amplitude

$$B^{(p)}(\xi, \eta) = \frac{1}{j\lambda f} \sum_{n=-N}^{N} C_n \cdot F(\xi - np_0, \eta) \cdot H(\xi - np_0, \eta)$$
(15)

incident on the filter plane. The filter is placed precisely to the position where it is recorded. The amplitude component of interest transmitted by the filter is the term-by-term product of Eq.(14) and (15)

$$F_{c}(\xi, \eta) = \sum_{n=-N}^{N} R / A_{0} F(\xi - np_{0}, \eta) \cdot exp\{-j_{2}\pi\xi_{r}x_{f}\}$$
(16)

After the Fourier transform produced by  $L_2$ , a filtered image appears in the  $(x_i, y_i)$  coordinates system. The amplitude in the image plane becomes

$$f_{c}^{(p)}(x_{i}, y_{i}) = r_{0} \sum_{n=-N}^{N} f(x_{i}, y_{i}) e^{j 2\pi n p_{0} X_{i}}$$
(17)

where all constant are collected in the complex amplitude  $r_0$ . The deblurred image carries a grating structure, which can be removed by the band-pass filtering process.

Finally, linearity of the deblurring process with respect to irradiance is achieved by photographing the output with  $\gamma = -\frac{1}{2}$ , with this value of  $\gamma$ , the irradiance transmittance of the film is proportional to  $f(x_0, y_0)$ .

## IV. Feature Analysis of the New System

The multi-channel inverse filtering system has many advantages.

# 1°. The large dynamic range

The dynamic range (DR) of a hologram is defined as the largest range in which the amplitude transmittance is proportional to the exposure. The larger the dynamic range is, the higher the deblurring ratio of the inverse filtering system is. The deblurring ratio is the compression ratio of the blur width. For the absorptive filter, its dynamic range is restrained by straight part of the H-D curve.Because holographic filter is attenuating light by diffraction, its dynamic range is restrained in the straight part of the t-E curve (the curve of transmittance vs exposure).

It is well-known that the holographic plate has a big Gamma. Therefore, its dynamic range is small. We find a special photographic processing technique which can make the quasilinear recording range to be 500:1, so the mask  $|H^{(p)}|^{-1}$  does not affect the dynamic range of the inverse filter.

The small DR is a problem which has not been solved properly, because the capacity of the film is far from enough for the realization of linear holographic recording. So the effects of special photographic processing techniques are limited. The method reported here is to compress the exposure range with the aid of the attenuating mask, so the exposure that the film subjected is allmost equal everywhere. It does not require the film to have a large linear region, therefore the filter generated by this system has a large dynamic range than that made by other methods.

# 2°. The high efficiency

If the hologram is used as a inverse filter, the intensity of reference beam must be very small in comparision with that of the object beam. The weaker the reference is, the higher the deblurring ratio is. As general, if we want to get a good deblurring effect, we must choose a smaller ratio of the reference beam to the object beam, the modulations of the fringes are very low, so the efficiency of the filter is small. But in this new system, the inverse filtering function can be got automatically with the help of the mask. We only choose the ratio  $R/A_0$  according to the linear recording condition, so we can get higher modulation interference fringes. The other methods can not compare with this method in efficiency. And it is verified by experiments that there are big differences in efficiency if different photographic condition is used. So it is very important to optimize the recording and processing conditions.

#### 3°. The anti-noise property

The holographic system is damaged by various noise, especially the granularity noise due to random aggregations of the silver grains in the emulsion. Helstrom developed an expression for the optimum filter function, when random zero-mean, additive noise is present. As the criterion for best filtering, the principle of the least mean-squares is used in his derivation. The optimum filter function is

$$t = H^{-1} \frac{\varphi_{0} / \varphi_{n}}{\varphi_{0} / \varphi_{n} + 1 / |H|^{2}}$$
(18)

where  $\phi_n\,,\phi_0$  are the power spectra of the noise distribution and the image respectively. He showed that the smaller the  $\phi_n/\phi_0$  is, the stronger the noise is restrained. From part II we can give an another form of our filter

$$t_{n} = \frac{2\lambda f R / A_{0}}{1 + (\lambda^{2} f^{2} R^{2} / A_{0}^{2}) / |C_{n} H|^{2}} \frac{1}{|C_{n} \cdot H|}$$
(19)

here  $\varphi_n / \varphi_0 = \lambda^2 f^2 (R/A_0)^2 = 0.0014$  (for  $\lambda = 6328A$ , f = 600mm,  $R/A_0 = 1/10$ ), it is very small. So the filter we have got is optimal with respect to the additive noise.

Another advantage of this system is that the output image is composed of N pictures. So the SNR of N-channel filtering system is increased by N times than that of a single one. The affection of the system noise is decreased greatly.

## V. Experimental

The linear motion blur image(Fig.2a) simulated in the laboratory and the blurred remote-sensing image (Fig.2b) were processed by this system. The over all Gammas of two blurred images were controlled to be 2 approximately. So that their amplitude transmittances are proportional to the irradiation distributions of the blurred objects.

The image shown in Fig.2b was the deblurred one of Fig.2a using the 3-channel inverse filtering system. The sharpness of the restored picture was improved greatly and most of the details blurred was almost restored.

Fig.2 is the restored remote sensing image. In order to evaluate the capacity of the filtering system, we chose a part of the resolution test marks from an aero-photograph. From the ratio of ground speed to the flight height (V/H) of the aeroplane as well as other references the blur width was figured out, which was  $100\mu m$ . If we observe this photo directly, the











(a) (b)Fig.3. The image of the resolution test target on an aero-photograph

(a) the blurred image (b) the restored image

sharpness of vision is good enough. But if it is measured at a high magnification, the visual effect and the measurement accuracy are very poor. Fig.3 was magnified by 5 times. From it we can find that the azimuth resolution is half of the cross track. In the restored image, the details of azimuth resolution test object can be distinguished, the sharpness is also improved. It can be seen that the restored image is still damaged by the coherent noise.

In many cases, the smear length of the filter is not the same as that of blurred image, our experiments showed that even the match error of the smear length is as high as 20%, the blurred image can still be improved clearly.

# VI. Conclusion

A new method is presented here in which the filtering operations take place in multi-channel at the same time. The filter generated by this system has a large dynamic range and high efficiency than that from other systems. As a by-product, the new filters can be prepared which are sufficiently efficient to significantly restore linear-motion blur images and other degraded images. So the optical image processing is one of the effective methods to improve the quality of the degraded remote sensing images. It is worth while to study further.

- 1. S.I.Ragnarsson, "A New Holographic Method of Generating a High Efficiency, Extended-range Spatial Filter with Application to Restoration of Defocused Images,"Phys.Scr.2,145(1970)
- 2. Y.G.Jiang, Y.R.Xu, "Simple Method for Image Deblurring, "Appl. Opt.22,784(1983)
- 3. C.Zetzsche", Simplified Realization of the Holographic Inverse Filter: A New Method, "Appl.Opt.21,1077(1982)
- 4. J.W.Goodman, "Wave-front Inversion Using a Thin Phase Hologram: A Computer Simulation, "Appl.Opt.23,4575(1984)
- 5. K.Biedermann, "A Founction Charaterizing Photographic Film that Directly Relates to Brightness of Holographic Images," Optik.28,160(1968/69)
- 6. G.L.Cai, "The Problem of Resolution Matching in Aero-photographic System," Acta Geodetic et Cartographica Sinica 16,121(1987)
- 7. Helstrom, C.W., "Image Restoration by the Method of Least Squares, "J.Opt.Soc.Am. 57, 297(1967)

## Abstract

This paper presents a new anti-noise inverse filtering based on the work of predecessors. In this system the system diffraction gratings are superposed on the point spread function (PSF) and blurred images to be restored on the input plane respectively. Then the multi-channel inverse filtering process done on the is Fourier spectrum plane. The SNR of N-channel processing system is increased by N times than that of a single one. In addition, the filter(1/H) is made in the back of an absorptive mask(1/abs(H)), and then the reference beam need not be too low, the modulation of the interference fringes of the filter is increased. Therefore, the filter with high efficiency and extended dynamic range can be achieved. The above mentioned advantages were verified by the theoretical analysis and the experimental results.