DIGITAL IMAGE RECTIFICATION ON MICROCOMPUTERS
FOR ORTHOPHOTO PRODUCTION

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1. Introduction

In Poland, we use Ortoprojectors with optical transfer of elementary images however without correction of lateral slope (Orthophot B end C of Zeiss Jena, PPO 8 of Wild), for the production of photographic maps by differential rectification.

In a hilly terrain these orthoprojectors can't give good results.

Lack of orthoprojectors equipped with lateral slope correctors on the one hand and accessibility of use of a film scanning and writing unit (OPTRONICS) on the other, induced us in 1981 to begin work on rectification of digital aerial photographs with the use of a popular Polish computer ODRA 1325 (28k words of 24 bits).

If some approximate solutions were not introduced, the processing of huge amount of information, which digital image comprises, would lead to time consuming computations.

That's why instead of applying a classic solution, which, for an indirect method, consists in the assignment of optical density for each rectified point, first by projection of these points onto a plane of original photos and then by interpolation of the density between neighbouring points, an approximate solution was used.

The approximations consist in:

1) using the rigid elementary image being composed of a group of 9 (3x3) pixels. To the central pixel, the optical density is assigned by "the nearest neighbourhood method". The pixel to be found by this method, and 8 pixels of original photos surrounding it, form the elementary image, which is transfersed to the rectified photos as a whole, without any change.

2) strict computation of a central pixel position of elementary image on the plane of an original photo (by a central projection) for only let's say every fourth elementary image. The position of remaining elementary images is determined by linear interpolation.

The assumption that an elementary image is rigid has of course some disadvantages. Such an elementary image is transferred from an original photo to an orthophoto without any additional corrections (e.g. scale or rotation). The only correction that is applied is positional correction of the center. Such solution is worse than the one applied in even simple strip rectifiers (Orthophot B) where each elementary image is corrected due to the scale and position of it's central point.

However, we should realize that pixel size for a digital image varies from 0,025 mm to 0,1 mm, which corresponds to the 0,075 x 0,075 - 0,3 x 0,3 mm of the elementary image size. However the typical width of a slit is equal to 2 or 4 mm, even up to 8mm. Dimensions of elementary image, which are assumed in our solution are then several times smaller than dimensions of elementary images in strip rectifiers so that is why the proposed solution can give good results in spite of simple calculations.

The digital terrain model (DTM), which is essential to analytical rectification of digital images can be obtained either from observation on a stereoplotter or from the central bank of terrain relief data.

2. Terrain relief information

In this method of an image rectification the most suitable form of terrain relief representation is the DTM. It is used for calculation (interpolation) of hights of points at given positional coordinates. The DTM was assumed in the form of a regular grid of squares or rectangles. Such DTM could be handled by ODRA 1325 computer.

For a better representation of terrain relief such a model could be complementd by the hights at characteristic points of the terrain features and by points located on characteristic terrain lines (e.g. ridge or trickle lines).

Another problem lies in choosing the dimension of the DTM grid. In the method of digital rectification there is no interpolation of the optical density between the center of pixels, so the grid dimension should be the multiple of the pixel size.

The dimension of the DTM grid should also be matched to the average scale of a rectified part of a photo according to the formula:

$$a = \frac{W}{Z_p} n p = \frac{W}{c_k} n p \qquad (1)$$

where: a - dimension of the grid side;

Z_p - the vertical component of the vector from the projection center to the gravity center of a photo part to be rectified;

W - height of the projection center above an average terrain elevation

n - the multiple of the grid side dimension

p - the pixel size

For a geometric correction of a photo (i.e. projection of a regular terrain grid of suitable density onto a photo) it is necessary to densify the original DTM grid. The heights in nods of such a dense grid are calculated by bilinear interpolation according to formula:

$$h = a_{11} + a_{21} x + a_{12} y + a_{22} xy \qquad (2)$$

This polynom fits the heights in the four nods and makes possible the linear interpolation along grid sides as well as along all cross-sections parallel to them.

Taking into account the actual dimensions of the grid (a

-length, b - width) and the actual scale of the coordinates the formula (2) can be writen in the following form :

$$h=h_D + (h_C - h_D)\,\frac{X}{a} + (h_A - h_D)\,\frac{Y}{b} + (h_B - h_C - h_A - h_D)\,\frac{XY}{ab} \qquad (3)$$

3. Projection of the DTM points on the image plane of an original photo.

In our solution the indirect method of digital aerial photos rectification was used, because in such a method an interpolation can be done within regular grid defined by pixel grid of an orthophoto. Terrain points and their images should fulfil the colinearity equation. But this equation can not be used directly because we should :

- transform the digital image coordinate system (the so called pixel system) to the image coordinate system, and
- transform the DTM coordinate system to the ground coordinate system

The necessity of applying these additional transformations is caused by the facts that (fig.1) :

a) the aerial photo is scanned in the random coordinate system that is only approximatelly parallel to the image coordinate system, and
b) the DTM coordinate system differs from the state coordinate system

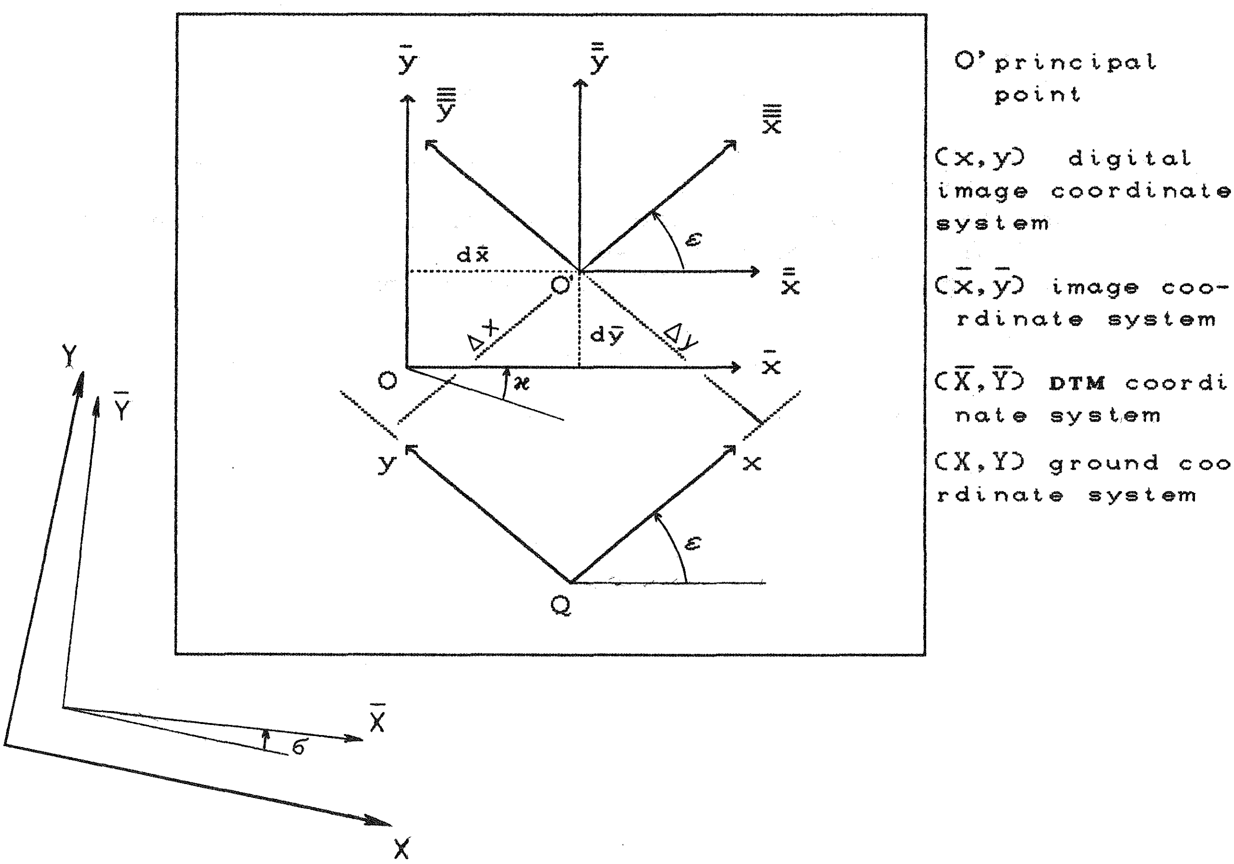


Fig.1 The coordinate systems

In both cases the transformation consists of a rotation and a translation. The dependence between the coordinates of a digital photograph and the orthophoto in the DTM coordinate system can he writen as:

$$\begin{bmatrix} x - \Delta x \\ y - \Delta y \\ -c_k \end{bmatrix} = \frac{1}{\lambda} D \begin{bmatrix} \bar{X} - \bar{X}_o \\ \bar{Y} - \bar{Y}_o \\ \bar{Z} - \bar{Z}_o \end{bmatrix} \qquad (4)$$

where:

x, y – the coordinates in the pixel coordinate system which results from the method of photo scanning ; x corresponds to the number of line while y corresponds to the position of the pixel in a line,

$\Delta x, \Delta y$ – the coordinates of the principal point in the pixel coordinate system,

$\bar{X}, \bar{Y}, \bar{Z}$ – the coordinates in the DTM coordinate system

$\mathbf{D}$ – the transformation matrix (3 x 3)

The matrix $\mathbf{D}$ is an orthogonal one only in the case when the constituant transformation matrices (particularly $\mathbf{D}_\varepsilon$ and $\mathbf{D}_\sigma$) do not include affine change of the scale. It results from superposition of all elementary transformations occurring on the way from the map to the digital image. It can be writen as:

$$\mathbf{D} = \mathbf{D}_\varepsilon * \mathbf{D}_\varkappa * \mathbf{D}_\varphi * \mathbf{D}_\omega * \mathbf{D}_\sigma \qquad (5)$$

where :

$\mathbf{D}_\varepsilon$ – the transformation matrix, which includes the angle ε between the pixel and image coordinate systems or possibly the affinity of the scale of the digital image

$\mathbf{D}_\omega$, $\mathbf{D}_\varphi$, $\mathbf{D}_\varkappa$ – the matrices, which include tilt, tip and swing of an original aerial photos

$\mathbf{D}_\sigma$ – the matrix, which includes the angle between the ground and DTM coordinate systems.

From (4) we can obtain the following equations :

$$x = -c_k \frac{(X - X_o)d_{11} + (Y - Y_o)d_{12} + (Z - Z_o)d_{13}}{(X - X_o)d_{31} + (Y - Y_o)d_{32} + (Z - Z_o)d_{33}} + \Delta x$$

$$(6)$$

$$y = -c_k \frac{(X - X_o)d_{21} + (Y - Y_o)d_{22} + (Z - Z_o)d_{23}}{(X - X_o)d_{31} + (Y - Y_o)d_{32} + (Z - Z_o)d_{33}} + \Delta y$$

By these equations the DTM points are projected onto the image plane. In such a case the orthophoto is formed in the DTM coordinate system but not in the ground coordinate system (fig.1)

4. Formation of the rectified digital image

4.1. Determination of the position of centers of elementary images on an original photo.

The nods of the DTM grid are projected onto the image plane (see &3). They form an irregular grid of tetragons. This grid is then densified by dividing each tetragon's side in to equal parts (fig 2)

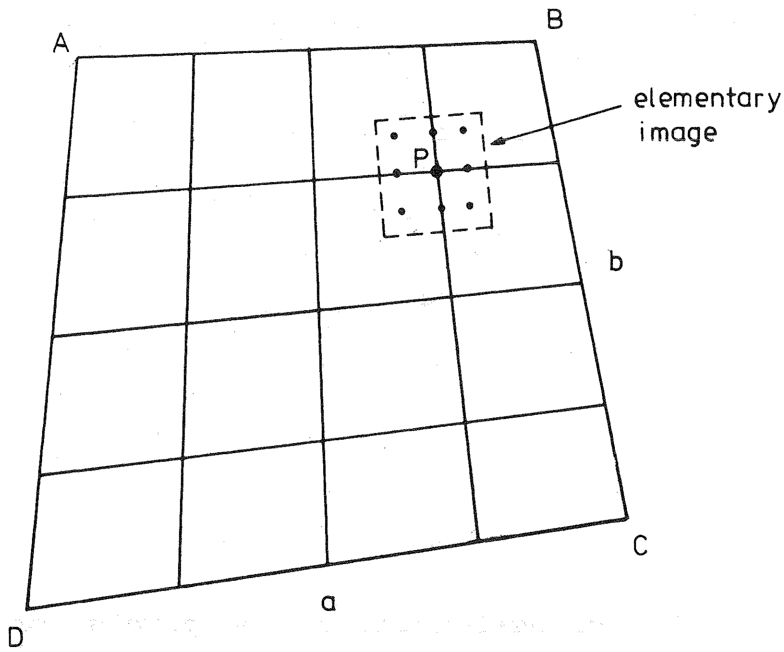


Fig. 2 Determination of the position of elementary image

The number of sections on which each side is divided must be chosen in such a way that the nods of this new grid should coincide with the centers of the elementary images.
It is done by the following linear interpolation :

$$X_p = X_D + (X_A - X_D)\frac{lb}{k} + (X_C - X_D)\frac{la}{k} + (X_B - X_C - X_A + X_D)\frac{la}{k}\,\frac{lb}{k}$$

$$Y_p = Y_D + (Y_A - Y_D)\frac{lb}{k} + (Y_C - Y_D)\frac{la}{k} + (Y_B - Y_C - Y_A + Y_D)\frac{la}{k}\,\frac{lb}{k} \qquad (7)$$

where :

X_p, Y_p - the coordinates of the centers of the elementary images,

$X_{A,B,C,D}$, $Y_{A,B,C.D}$ - the image coordinates of the dense grid of the DTM

$\frac{la}{k}$, $\frac{lb}{k}$ - the partition ratio of the grid sides

4.2 The optical density assignment to the pixels of the rectified digital image.

"The nearest neighbourhood method" was used for the density assignment of the central pixel of an elementary image. The same densities, which have the pixels of the original photos surrunding the central pixel, are assigned to the remaining pixels (fig.3). Such a method transfers the elementary images without deformation. However, we should realize that some pixels of the original photos can be taken twice, which causes their duplication on the rectified image, while the others can be neglected, which in turn can cause image discontunity.

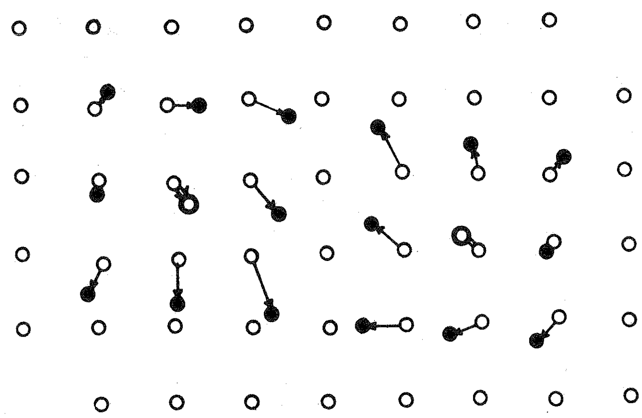


Fig.3 Method of assignment of the pixels forming elementary images

o — the position of pixels on the original photo
• — the position of pixels on the rectified photo
o — the position of the center of an elementary image

5. Calculations

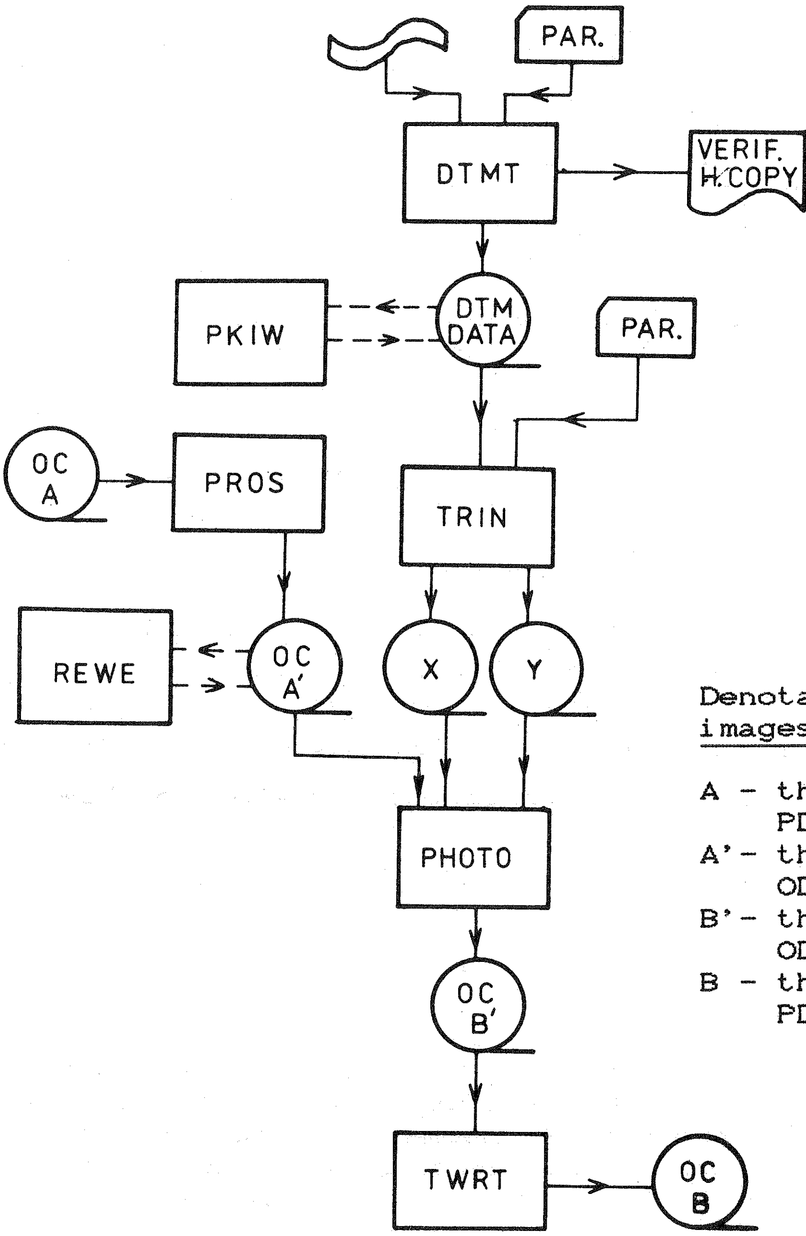
All calculations were done by ODRA 1325 computer. The digital image was obtained by scanning the original photo in the OPTRONICS instrument and by recording it on a magnetic tape with the density of 800 Bpi (A fig.4). The DTM was recorded on a perforated tape and then rearranged and recorded on a magnetic tape (DTM, fig.4).

For aerial photos rectification, a system of programs called SPPOC was worked out. The main functions of these programs are as follows :

DTMT – reads the DTM data from the perforated tape and then forms the file on a magnetic tape

TRIN – interpolates the heights for the dense grid and then transforms the nods of this grid to the coordinate system of the digital image

PHOTO – reads the elementary images (3 x 3 pixels) from the digital image file (OC-A', fig.4) The coordinates of the centers of these elementary images are calculated by program TRIN and form the files X end Y (fig.4). The groups of nine pixels are then arranged in a new digital image (OC - B',fig.4)



Denotation of the digital images OC

A - the original image in PDP system
A' - the original image in ODRA system
B' - the rectified image in ODRA system
B - the rectified image in PDP system

Fig. 4 The scheme of the system of programs SPPOC

6. An experimental aerial photo digital rectification

The aerial photo in the scale of 1 : 21000 (c_k = 150 mm, size 23 x 23) was digitally rectified. The photo covers the hilly terrain with elevation ranging within 150 m and slopes up 10^o. Only a part of this photo (11 x 11 cm) was scanned on OPTRONICS film scanning unit with the use of the 50μm slit. It are 2200 lines of 2200 pixels each (totally 4.8 mln of pixels).

The heights of the DTM grid nods were measured on the Stereometrograph of Zeiss Jena connected with the Coordimeter F. The height measurement were made at 2400 grid nods and at 100 characteristic terrain relief points. The sides of the DTM grid equal to 50.4 m .

The elements of the photograph's orientation were calculated by a photogrammetric space resection. The angular elements of the photo orientation are as follows:

$$\omega = -0.695^{g}, \quad \phi = -1.881^{g}, \quad \varkappa = -1.876^{g}$$

Then the data were proccessed by the system SPPOC . Finally, the orthophoto in the scale of the original photograph was produced by the use of the OPTRONICS film writer with 50 μm slit (fig.5)

To determine the accuracy of the orthophoto the 144 control points were measured on the orthophoto. The ground coordinates of these points were calculated from image coordinates measured on the STECOMETER and then their coordinates on the orthophoto were calculed.

After the Helmert transformation the differences between thes coordinates were treated as the orthophoto errors. The scale coefficient of the Helmert transformation equals 1.0005, which allows us to treat the coordinate differences as the true errors of the orthophoto.

To seperate the systematic components, which can by seen in the ΔX and ΔY, the affine transformation was also used.

The mean errors after the Helmert (denoted m_x , m_y) and the affine (denoted m'_x , m'_y) transformations were as follows (the corresponding errors in the orthophoto scale are shown in brackets) :

$m_x = \pm 1.2$ m (60 μm)	$m'_x = \pm 0.6$ m (30 μm)
$m_y = \pm 1.5$ m (75 μm)	$m'_y = \pm 0.7$ m (35 μm)
$m_p = \pm 1.9$ m (95 μm)	$m'_p = \pm 0.9$ m (45 μm)

On fig.6 the errors after both transformations are presented in the form of vectors . The affine deformations found in this way confirm the earlier suspicions that film writing device introduces essential geometric deformations.

After taking into account these deformations the positional error is reduced to the $\pm$ 55μm, so it is at the level of the pixel size (50 μm).

7. Conclusions

The presented method of a digital image rectification can be used on a small computer . Due to the introduction of elementary images (3 x 3 pixels) instead of single pixels, the computation time is significantly shortened.

The experimental image rectification shows that the produced orthophoto has a high positional accuracy ($m_p = \pm$ 45 μm) as well as good photographic quality. The orthophoto produced with this method in the scale of an original photo can by magnified 4 times by optical means.

8. Bibliography

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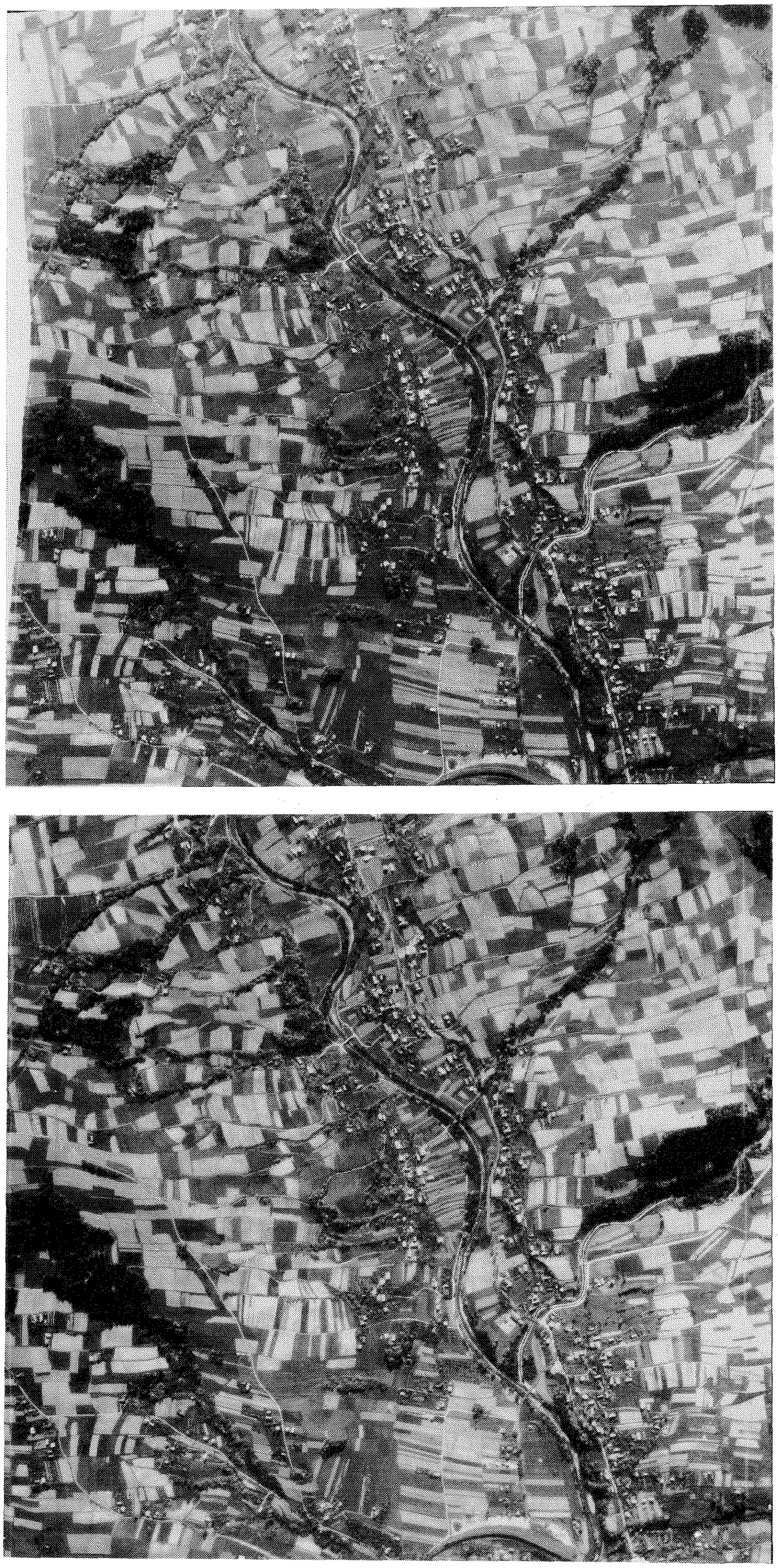

Fig.5 Aerial photo (left) and orthophoto

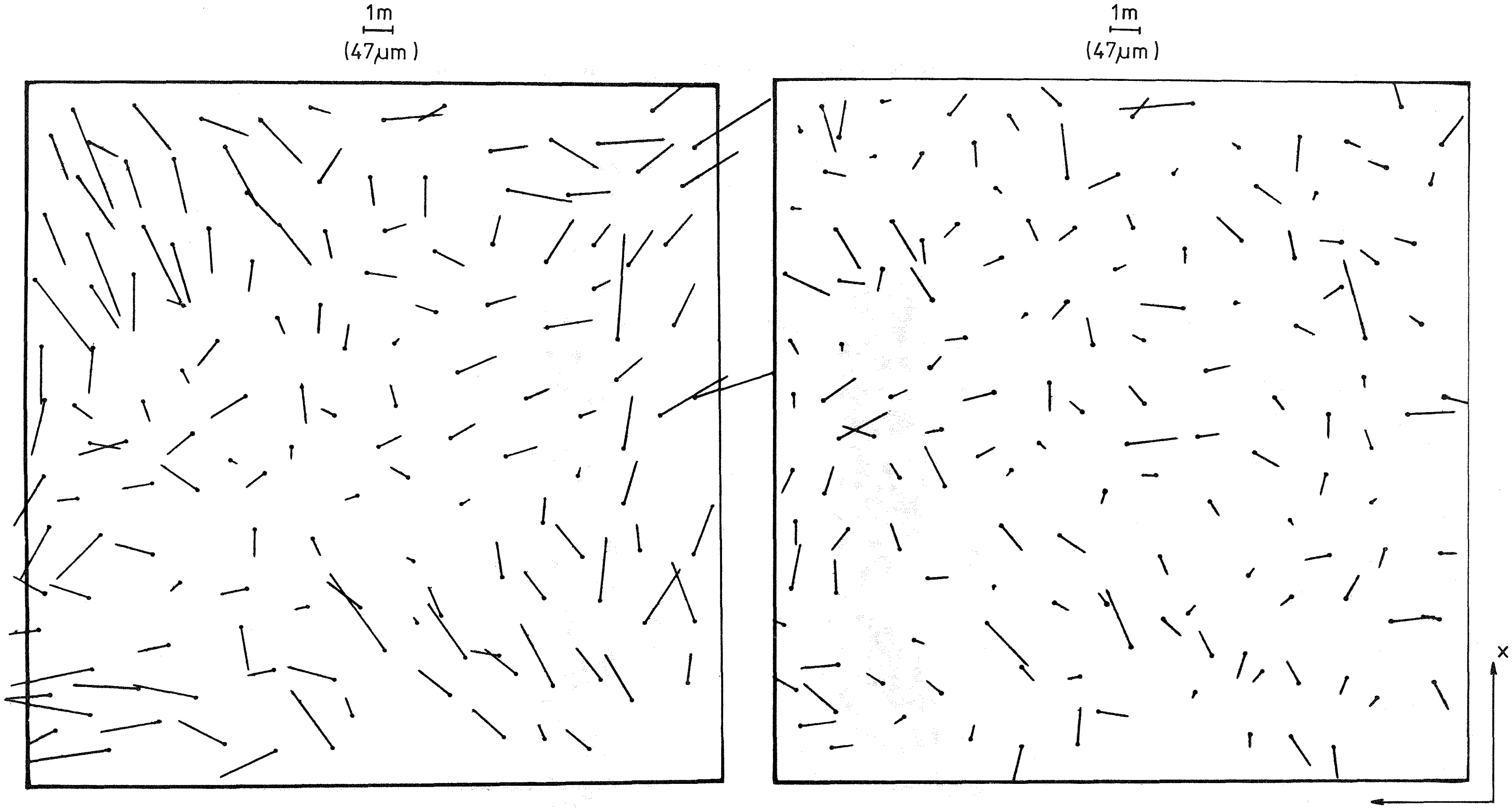

Fig.6 Distribution of the positional errors after transformations

a) Helmert - mean error ± 1.9 m in a terrain (± 90μm in the scale 1 : 21000)

b) affine - mean error ± 0.9m in a terrain (± 42μm in the scale 1 : 21000)