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## ABSTRACT

A wild A8 was used to observe 16 sets of 28 grid points using different rotation angles. The coordinates of the perspective center (P.C.) were calculated using different number of the grid points. The effect of the rotation angles on the stability of the P.C. as well as the ultimate number of grid points required for the determination of the coordinates have been investigated.

Moreover, fifty aerial triangulation tests were performed using strip of eleven models. In each test different value of error ranging from 0.01 to 0.05 mm was introduced in either $x, y$, or $z$ coordinates of P.C. It has been found that errors in $x$ and $y$ coordinates of $P$. C. have more influence on the results than $z$. Also the planimetric ground coordinates are less affected than heights by the errors in the coordinates of P.C. points.

## Introduction

In semi-analytical aerial triangulation, the coordinates of perspective center points are required to strengthen the lateral geometrical transformation of models. Although, it is well known that the coordinates of the P.C. must be determined, there is no agreement on the required accuracy for their determination. The author saw one organization using a Wild Al0 for their aerial triangulation. They set base components by and bz to their zero values and bx to the required base. Then, without any measurements at all, they use the index readings of the $A 10$ as the coordinates of P.C. Another organization uses $\triangle Z$ method and they only observe two points along $Y$ axis of the plotter. Then they use the simple formula given on the manual of Wild Al0 or A8 to calculate the coordinates of the P.C. A third organization also uses $\Delta Z$ method, but they measure nine grid points and a person has to spend have day with calculating machine filling the forms supplied by ITC institution to calculate the coordinates of P.C.

Normally, for many of the known stereoplotters, the coordinates of the P.C. are determined when all rotation elements are set to zeroes. But for semi analytical aerial triangulation, model coordinates are measured after relative orientation. If there is any maladjustment in the instrument, then after the relative orientation, one expects that the calculated coordinates of the P.C. will change for different rotation angles. Then, one factor which should be investigated is the effect of rotation angles on the stability of P.C. coordinates. Number of measured grid points is an important factor which influence the accuracy of determination of P.C. coordinates and should also be investigated.

Accordingly, A research program has been conducted to study the effect of the two above factors. The description of the tests as well as the results are reported in the first section of this paper.

Although, the important role which the coordinates of P.c. play in the process of aerial triangulation, few authors have investigated the problem. In most of these investigations, for example Dowman (1973), the coordinates of $P . C$. were determined using different methods.

Aerial triangulation tests were carried out using the coordinates of P.C. determined by each method, and the results of the different aerial triangulation tests were compared.

In this investigation,a different approach was used. Measured coordinates including the coordinates of P.C. of 11 models of one strip were available. Different values of errors were introduced into $x, y$ or $z$ coordinates of either one P.C. or the two P.C.'s for each model. Using these new coordinates, aerial triangulation tests were carried out. The results of the tests and their analysis are given in the second section of this paper.

## Measurements for First Section

A wild A8 owned by Aerial Survey of Egypt was used for the study. Unfortunately the EK5 coordinate recording system unit was not operational. Accordingly the $x$ and $y$ coordinates of all measurements were read from the counter drum connected to the $x$ and $y$ spindles of the A8.

Twenty eight points were marked on a grid plate. Fig. 1 is a diagram showing the relative locations of these points. Then the grid plate was inserted on the left projector of the A8, the focal length and the bx were set to 152 and 150.0 mm respectively. All rotation angles of the projector were set to their initial values. Then, the coordinates of the 28 points were measured on the two levels $Z=230$ and 330 mm .

The above measurements were repeated 15 times using different rotation angles. Due to limitation of the range of model area of the A8, it was only possible to measure 24 points in cases where $\omega=1,2$ and 4 grades and where $K=4$ grades. Table 1 shows the information details of the 16 set of grid measurements. A computer program (COPC) has been developed to calculate the coordinates of P.C. To check the measurements and to find out any mistakes on the observations, the program was run for the 16 set of measurements shown on table lusing the coordinates of all the measured grid points. It was decided to use variance of unit weight, variance of the measured $x, y$ and $z$ grid coordinates equal $20,20,20$ and $30 \mu$ respectively.

It was realized that the measurements nos. 1,4 and 6 from table 1 have to be remeasured. Also, few points on some tests show large residuals and were eliminated. The main reason for these mistakes is errors in reading the coordinates from the counter drum of the A8. Before remeasuring the 3 sets, Aerial Survey of Egypt bought a Wild EK22 data-acquisition unit and a Wild RAP system with TA digital plotting table. The new instruments were connected to the same A8 used for the grid measurements. Due to the major work on the A8 to install the above items, the coordinates of the P.C. shifted and this is clear from results shown on table 1 .

After, all mistakes on the measurements were corrected or eliminated, the computer program was run several times for each set of measurements using $4,6,8,12,20,24$ or 28 points. Fig. 1 shows the pattern of the measured 28 grid points and table 2 shows the arrangement of grid points used for each run.

To be able to compare the results and to observe the effects of different factors on the values of the calculated P.C.; the differences (discrepancies) between the calculated coordinates of the P.C. when all rotation angles are zeroes and the 28 grid points were used; and the calculated coordinates of the P.C. from any computer run (i). Table 3,4 and 5 show the discrepancies of the coordinates of the P.C. for different values of rotation angles $\omega, \phi$ and $K$ when $4,6,8,10,12,14,20$ and 28 grid points were used in the calculations.

From tables 3,4 and 5 , it can be seen that:
1-The coordinates of the $P . C$. change when either the number of points used in the calculation or the rotation angles are changed. The maximum change can be seen on $z$ values when $\omega$ rotations equal $1.0,2.0$ and 4.0 grades. These are the observations taken after the installation of EK22 unit and RAP system on the A8 instrument. The change in $z$ exceed $700 \mu$ This indicates that taking a fixed values for the coordinates of P.C. is not correct. Instead, new set of observations to calculate the new coordinates of P.C. after calibration or adjustment of the instrument is necessary.

2-Excluding the results when $\omega$ equal $1.0,2.0$ and 4.0 grads, one can draw the following conclusions
$i-$ The discrepancies in $x$ reach a maximum value of $100 \mu$ when $\omega=0.1 \mathrm{~g}$ and six grid points were used; and in 75 cases from total of the 94 determinations the discrepancies in $x$ were less than $40 \mu$.
ii- The discrepancies in $y$ reach a maximum value of $70 \mu$ when $\phi=4.0 \mathrm{~g}$ and 4 grid points were used; and in 76 cases from total of the 94 determinations the discrepancies in $y$ were less than $40 \mu$.
iii- The discrepancies in $Z$ reach a maximum of $110 \mu$ when $\phi=0.5 \mathrm{~g}$ and 8 grid points were used; and in 59 cases from total of the 94 determinations the discrepancies were less than $40 \mu$.

3- For the used A8, it seems that rotation angles do not affect significantly the values of the calculated coordinates of P.C. Also,it seems that number of points used in calculation of the coordinates of P.C. is not the main factor which affect the accuracy. In the author opinion, according to the experience he gained from this investigation, 10 points will be most satisfactory for determination of the coordinates of P.C. as well as it will make it easier to find errors in the observations.

Tests and Results for the Second Section
Measured coordinates of 11 models from strip at scale $1: 10,000$ covering an area near the city of Al-Mansora, Egypt were available to the author from previous investigation, [Ali (1983)]. The model coordinates were measured using a WILD A8 stereoplotter owned by Aerial Survey of Egypt

A simple computer program for Block Adjustment With Independent Models (BAWIM) developed by International Institute for Aerial Survey and Earth Science (ITC) was also available, [ Amer (1978)]. The BAWIM program reads the measured model coordinates as well as the ground coordinates of the control points. Then the program calculates the ground coordinates for all points on the model. If any point appears in more than one model, the program calculates the arithmetic mean of its coordinates and the deviations (residuals) of the coordinates from the mean. A Root Mean Square of Errors (RMSE) computed for these residuals is termed relative error which is used by many organization as an indicator to the quality of aerial triangulation adjustment.

Several tests were carried out using BAWIM program to study the effect of the error in the coordinates of $P . C$. on the aerial triangulation. The tests can be arranged in five groups as follows:

1- In the first group, only one test was performed to adjust the strip using the calculated coordinates (assumed correct) of P.C.; test no. 1
table 6. The adjusted ground coordinates of model points resulting from this test were used as a reference for the second and third groups of tests which will be explained later. To obtain the maximum possible values of error in aerial triangulation due to errors in coordinates of P.C., minimum ground control points (two horizontal and three vertical) were used for this test and also for the tests of the second and third groups. Three iterations were used for all tests reported in this paper, because this is usual practice in aerial triangulation adjustment.

2- In the second group, 15 tests were performed by adding different values of error ranging from 0.05 to 5.00 mm to either $x, y$ or $z$ coordinates of both left and right $P$.C. points in all the eleven model of the strip. These tests are nos. 2 to 16 given in table 6 .

3- In the third group, 18 tests were performed by adding different values of error ranging from 0.01 to 5.00 mm to either $x, y$ or $z$ coordinates of only the left P.C. points of the eleven models of the strip. These tests are nos. 17 to 34 given in table 7 .

4- In the fourth group, it was noticed that errors in aerial triangulation especially in height reach unacceptable values even when small values of error in coordinates of P.C. were added. It was decided to repeat some of the tests using two vertical ground control points at the beginning, two at the middle and two at the end of the strip. This is similar to the number of vertical control points required by many mapping organizations (two vertical control points for every five models).

Only one test, number 35 given in table 8, was performed in this group. The correct coordinates of P.C's were used. Two horizontal and six vertical control points were used for this test as well as all tests of group 5. The adjusted coordinates of this test were used as reference for the fifth group of tests.

5- In the fifth group, 15 tests were performed by adding different values of error ranging from 0.05 to 5.00 mm to either $x, y$ or $z$ coordinates of only the left P.C. points of the eleven models of the strip. These tests are numbers 36 to 50 given in table 8 .

After performing all the 50 tests, the adjusted ground coordinates of strip points for each test of groups 2 and 3 were compared with the result of the reference test no. l. The differences in the calculated ground coordinates of the same points were calculated. Because the model points and P.C. points serve two different purposes in mapping, they are separated. Then RMSE's for the calculated differences of $X, Y$ and $Z$ coordinates were calculated for the model points and P.C. points for each test and they are given in tables 6 and 7 for groups 2 and 3 respectively. Also, the RMSE's which show the relative accuracy of aerial triangulation as obtained from BAWIM program are given in these tables. Same procedure has been followed for group no. 5 as the results of its tests were compared with the result of the reference test no. 35; and the corresponding RMSE's are given in table 8.

## Conclusions for Second Section

From the results of the 50 aerial triangulation tests which have been performed using the strip of 11 models at scale 1: 10,000, the following conclusions can be drawn:

1- It can be seen that there are linear relations between the values of the introduced errors in the coordinates of P.C. points and the calculated RMSE's of model and P.C. points. For example one can see that :
a- from table 7, when three vertical control points were used,
$\mathrm{d} z=35$ ex
$d z=11$ ey
$\mathrm{d} z=0.3 \mathrm{ez}$
b- from table 8 , when six vertical control points were used,

$$
\mathrm{d} z=3.0 \text { ex } \quad \mathrm{d} z=1.3 \text { ey } \quad \mathrm{d} z=0.4 \mathrm{ez}
$$

where
$d z$ is the RMSE in $Z$ coordinates for model points in $m$. ex, ey and ez are the introduced error in $x, y$ and $z$ coordinates of the left $P$. C. points respectively in mm.

From the above relations, one can see that errors in $x$ coordinates of P.C. points have the greatest influence while errors in $z$ coordinates have the least influence on the adjusted heights of model points.

2- Let us assume that in practice the acceptable accuracy for planimetric and height aerial triangulation adjustment to be $40 \mu$ of photo scale, then for the photographs used in this study 40 cm is the acceptable limit. Accordingly, it is possible to conclude that :-
i- If the errors in the measured coordinates of left and right P.C. points have the same values, then they have no influence on the adjusted planimetric coordinates of model points. Their influences in the adjusted heights of model points are small and do not exceed the practical limit except when the introduced error in either $x$ or $y$ coordinates of $P$.C. points reach 5.0 mm . The introduced error in $x$, $y$ or $z$ coordinates of $P . C$. points are transformed to their calculated ground coordinates.
ii- If sufficient number of vertical control points is used, the introduced errors in the coordinates of either the left or right P.C. points have practically no effect on the adjusted planimetric coordinates (table 8).
iii- If minimum number of vertical control points is used, an error of $10 \mu$ in the $x, 50 \mu$ in $y$ or 1.0 mm in $z$ coordinates of left or right P.C. points cause the results of aerial triangulation to approach the allowable limit in height (see table no. 7).
iv- Even if sufficient number of vertical control points is used, errors of $0.10,0.25$ or 1.00 mm in either $x, y$ or $z$ coordinates of P.C. points respectively will cause the results of aerial triangulation to exceed the allowable limit in height (see table 8).
v- The relative accuracy as normally calculated and given by aerial triangulation computer programs does not give any clue about the existence of errors in the measured coordinates of P.C. points or about the absolute error on the adjusted ground coordinates which may exist due to these errors.

## References

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Amer F, 1978."Adjustment of Aerial Triangulation Part 1 ", ITC Lecture Notes.

Dowman I.J.; " A Working Method for the Calibration of Plotting Instruments Using computers", Photogrammetric Record, October 1973.


Fig 1- Pattern of the Measured 28 Grid Points.

Table 1- Information for Grid Measurements.

| Serial | Vaues of Rotation Angles |  | NO. of <br> Measured <br> Grid Points |  |
| :---: | :--- | :--- | :--- | :--- |
| 1 | OMG (g) | PHI (g) | KAP (g) |  |
|  | 100.0 | 100.0 | 100.0 | 28 |
| 2 | 100.1 | 100.0 | 100.0 | 28 |
| 4 | 100.5 | 100.0 | 100.0 | 28 |
| 4 | 101.0 | 100.0 | 100.0 | 24 |
| 5 | 102.0 | 100.0 | 100.0 | 24 |
| 6 | 104.0 | 100.0 | 100.0 | 24 |
| 7 | 100.0 | 100.1 | 100.0 | 28 |
| 8 | 100.0 | 100.5 | 100.0 | 28 |
| 9 | 100.0 | 101.0 | 100.0 | 28 |
| 10 | 100.0 | 102.0 | 100.0 | 28 |
| 11 | 100.0 | 104.0 | 100.0 | 28 |
| 12 | 100.0 | 100.0 | 100.1 | 28 |
| 13 | 100.0 | 100.0 | 100.5 | 28 |
| 14 | 100.0 | 100.0 | 101.0 | 28 |
| 15 | 100.0 | 100.0 | 102.0 | 28 |
| 16 | 100.0 | 100.0 | 104.0 | 24 |

Table 2- Arrangement of Grid Points Used in Calculation.

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Ser. No. of Numbers (names) of Grid Points Used in
    No. Used Computer Runs ( see Fig. 1)
of Run Points
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I- Runs with two points in each raw (see Fig. 1)
$\begin{array}{llll}1 * & 4 & \text { points nos. } 1,4,25, & 28 \\ 1 * & 4 & \text { points nos. } 5,8,21, & 24\end{array}$
$2_{2 *}^{2} 6$ points of run no. $1+$ points nos. 13,16
points of run no. $1^{*}+$ points nos. 13,16
310 points of run no. $2+$ points nos. $5,8,21,24$
3* points of run no. $2^{*}+$ points nos. $9,12,17,20$
414 points of run no. $3+$ points nos. $9,12,17,20$
II- Runs with four points in each row (see Fig. 1)
5* 8 points nos $1,2,3,4,25,26,27,28$
points nos. $5,6,7,8,21,22,23,24$
612 points of run no. $5+$ points nos. $13,14,15,16$
6* points of run no. $5^{*}+$ points nos. $13,14,15,16$
720 points of run no. $6+$ points nos. $5,6,7,8,21,22,23,24$
7* points of run no. 6tpoints nos. $9,10,11,12,17,18,19,20^{*}$ poin
828 all measured grid points.
*Runs when it was possible to only measure 24 grid points (when $\omega=1,2,4 \mathrm{~g}$ and when $\mathrm{K}=4 \mathrm{~g}$ ).

Table 3- Discrepancies for different Values of $\omega$ and Different Numbers of Points ( $\mathrm{PHI}=100.0 \mathrm{~g}$ and $\mathrm{KAP}=100.0 \mathrm{~g}$ ).

| No. of | Vx | Vy | Vz | No. of | Vx | Vy | Vz | No. of | Vx | Vy | Vz |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pts. | mm | mm | mm | Pts. | mm | mm | mm | Pts. | mm | mm | mm |
|  | OMG | 100.1 | g |  | OMG | 100.5 | g |  | OMG | 101.0 | g |
| 4 | 0.09 | 0.04 | 0.06 | 4 | 0.02 | 0.02 | -0.02 | 4 | 0.05 | 0.14 | 0.67 |
| 6 | 0.10 | 0.04 | 0.02 | 6 | 0.02 | 0.04 | -0.04 | 6 | 0.08 | 0.14 | 0.66 |
| 10 | 0.08 | 0.03 | 0.02 | 10 | 0.01 | 0.05 | -0.01 | 10 | 0.05 | 0.15 | 0.64 |
| 14 | 0.07 | 0.04 | 0.00 | 14 | 0.03 | 0.05 | -0.01 | 8 | 0.03 | 0.13 | 0.72 |
| 8 | 0.07 | 0.05 | 0.07 | 8 | 0.06 | 0.00 | 0.02 | 12 | 0.06 | 0.13 | 0.72 |
| 12 | 0.09 | 0.04 | 0.03 | 12 | 0.05 | 0.03 | 0.01 | 20 | 0.05 | 0.13 | 0.67 |
| 20 | 0.07 | 0.03 | 0.02 | 20 | 0.04 | 0.05 | 0.01 | 24 | 0.05 | 0.13 | 0.66 |
| 28 | 0.06 | 0.05 | 0.01 | 26 | 0.05 | 0.05 | 0.02 |  |  |  |  |
|  | OMG | 102.0 | g |  | OMG | 104.0 | g |  |  |  |  |
| 4 | 0.03 | 0.14 | 0.69 | 4 | -0.12 | $-0.07$ | 0.76 |  |  |  |  |
| 6 | 0.03 | 0.13 | 0.73 | 6 | -0.04 | -0.03 | 0.70 |  |  |  |  |
| 10 | 0.05 | 0.12 | 0.72 | 10 | 0.00 | -0.05 | 0.69 |  |  |  |  |
| 8 | 0.03 | 0.10 | 0.67 | 8 | -0.02 | $-0.02$ | 0.70 |  |  |  |  |
| 12 | 0.07 | 0.10 | 0.69 | 12 | 0.05 | 0.01 | 0.70 |  |  |  |  |
| 20 | 0.04 | 0.10 | 0.68 | 20 | 0.03 | -0.03 | 0.68 |  |  |  |  |
| 24 | -0.01 | 0.10 | 0.66 | 24 | 0.03 | -0.02 | 0.70 |  |  |  |  |

Table 4- Discrepancies for different Values of $\phi$ and Different Numbers of Points ( $O M G=100.0 \mathrm{~g}$ and $\mathrm{KAP}=100.0 \mathrm{~g}$ ).

| No. of | Vx | Vy | Vz | No. of | Vx | Vy | Vz | No. of | Vx | Vy | Vz |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pts. | mm | mm | mm | Pts. | mm | mm | mm | Pts. | mm | mm | mm |


|  | $\underline{\mathrm{PHI}}=\underline{100.1} \mathrm{~g}$ |  |  |  | $\underline{\mathrm{PHI}}=100.5 \mathrm{~g}$ |  |  |  | $\underline{\mathrm{PHI}}=101.0 \mathrm{~g}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | 0.01 | 0.01 | 0.07 | 4 | 0.06 | -0.02 | 0.10 | 4 | 0.01 | 0.01 | 0.04 |
| 6 | 0.01 | 0.03 | 0.07 | 6 | 0.06 | 0.00 | 0.09 | 6 | -0.04 | 0.00 | 0.00 |
| 10 | 0.02 | 0.02 | 0.07 | 10 | 0.06 | 0.03 | 0.11 | 10 | -0.03 | -0.01 | 0.01 |
| 14 | 0.02 | 0.02 | 0.06 | 14 | 0.05 | 0.02 | 0.09 | 14 | -0.02 | 0.00 | 0.01 |
| 8 | 0.00 | 0.03 | 0.08 | 8 | 0.05 | -0.01 | 0.11 | 8 | 0.01 | 0.01 | 0.06 |
| 12 | 0.01 | 0.04 | 0.08 | 12 | 0.05 | 0.01 | 0.10 | 12 | -0.01 | 0.00 | 0.03 |
| 20 | 0.01 | 0.02 | 0.08 | 20 | 0.04 | 0.02 | 0.10 | 20 | -0.01 | -0.01 | 0.03 |
| 28 | 0.01 | 0.02 | 0.06 | 28 | 0.03 | 0.01 | 0.09 | 28 | 0.00 | -0.02 | 0.02 |
|  | $\underline{\mathrm{PHI}}=$ | 102.0 | g |  | PHI | $=104.0$ |  |  |  |  |  |
| 4 | 0.00 | 0.05 | 0.04 | 4 | -0.02 | -0.07 | 0.04 |  |  |  |  |
| 6 | 0.00 | 0.03 | 0.03 | 6 | -0.03 | -0.06 | 0.02 |  |  |  |  |
| 10 | 0.01 | 0.03 | 0.05 | 10 | -0.03 | -0.05 | -0.03 |  |  |  |  |
| 14 | 0.04 | 0.00 | 0.07 | 14 | -0.02 | -0.05 | -0.01 |  |  |  |  |
| 8 | -0.02 | 0.06 | 0.03 | 8 | 0.05 | -0.04 | 0.10 |  |  |  |  |
| 12 | -0.02 | 0.05 | 0.03 | 12 | 0.02 | -0.03 | 0.08 |  |  |  |  |
| 20 | 0.00 | 0.03 | 0.05 | 20 | 0.02 | $-0.05$ | 0.04 |  |  |  |  |
| 28 | 0.01 | 0.01 | 0.06 | 25 | 0.02 | -0.06 | 0.05 |  |  |  |  |

Table 5- Discrepancies for different Values of $K$ and Different Numbers of Points ( $O M G=100.0 \mathrm{~g}$ and $\mathrm{PHI}=100.0 \mathrm{~g}$ ).

| No. <br> of <br> Pts. | mm | mm | mm | mm | Vm <br> of <br> Pts. | mm | mm | mm | $\mathrm{Vts}$. | mm | mm |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |



Table 6-RMSE's Due to Errors in the Coordinates of Left and Right P.C.'s Using Minimum Number of Ground Control Points.

| Ser. | Value | RMSE* |  | MSE |  | RMS | of |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No | of | Model Point |  | Po |  |  | fro |  |
| of | Error |  |  |  |  |  | M P | ram |
| Test |  | Z** | X | $Y$ | Z | X | Y | Z |
|  | (mm) | (m) | (m) | (m) | (m) | (m) | (m) | (m) |

Group $1-$ No Errors in the Coordinates of P.C.'s
10.00 --- --- $\quad \cdots \quad 0.200 .200 .28$

Group 2.a- Errors in $x$ coordinates

| 2 | 0.05 | 0.00 | 0.19 | 0.23 | 0.00 | 0.20 | 0.20 | 0.28 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 3 | 0.10 | 0.01 | 0.38 | 0.46 | 0.01 | 0.20 | 0.20 | 0.28 |
| 4 | 0.50 | 0.05 | 1.92 | 2.30 | 0.03 | 0.20 | 0.20 | 0.28 |
| 5 | 1.00 | 0.10 | 3.83 | 4.61 | 0.05 | 0.20 | 0.20 | 0.28 |
| 6 | 5.00 | 0.48 | 19.17 | 23.03 | 0.27 | 0.20 | 0.20 | 0.28 |

Group 2.b- Errors in y Coordinates

| 7 | 0.05 | 0.01 | 0.23 | 0.20 | 0.01 | 0.20 | 0.20 | 0.28 |
| ---: | ---: | ---: | ---: | ---: | :--- | :--- | :--- | :--- |
| 8 | 0.10 | 0.02 | 0.45 | 0.39 | 0.02 | 0.20 | 0.20 | 0.28 |
| 9 | 0.50 | 0.10 | 2.25 | 1.97 | 0.11 | 0.20 | 0.20 | 0.28 |
| 10 | 1.00 | 0.20 | 4.50 | 3.94 | 0.22 | 0.20 | 0.20 | 0.28 |
| 11 | 5.00 | 1.01 | 22.51 | 19.7 | 1.11 | 0.20 | 0.20 | 0.28 |

2.c- Errors in $z$ Coordinates

| 12 | 0.05 | 0.00 | 0.00 | 0.00 | 0.30 | 0.20 | 0.20 | 0.28 |
| ---: | ---: | ---: | ---: | :--- | :--- | :--- | :--- | :--- |
| 13 | 0.10 | 0.00 | 0.00 | 0.00 | 0.60 | 0.20 | 0.20 | 0.28 |
| 14 | 0.50 | 0.01 | 0.01 | 0.01 | 3.00 | 0.20 | 0.20 | 0.28 |
| 15 | 1.00 | 0.02 | 0.02 | 0.02 | 5.99 | 0.20 | 0.20 | 0.28 |
| 16 | 5.00 | 0.09 | 0.12 | 0.09 | 29.97 | 0.20 | 0.20 | 0.28 |

* For all the tests, numbers of model points and P.C. points=112 and 12 points respectively.
** RMSE's of $X$ and $Y$ coordinates for all the tests $=0.00 \mathrm{~m}$.

Table 7- RMSE's Due to Errors in the Coordinates of Left P.C.'s Using Minimum Number of Ground Control Points.

| Ser. No of | $\begin{aligned} & \text { Value } \\ & \text { of } \\ & \text { Error } \end{aligned}$ | RMSE* |  |  | RMSE* |  |  | RMSE of A.T. from |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Model Points |  |  | P.C. Points |  |  |  |  |  |
|  |  |  |  |  | BAW | Pr | rogr |  |  |  |
| Test |  | X | Y | Z |  |  |  | X | Y | Z | X | Y | Z |
|  | (mm) | (m) | (m) | (m) | (m) | (m) | (m) | (m) | (m) | (m) |

Group 3.a- Errors in $x$ coordinates

| 17 | 0.01 | 0.00 | 0.00 | 0.35 | 0.14 | 0.16 | 0.36 | 0.20 | 0.20 | 0.28 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 18 | 0.05 | 0.00 | 0.00 | 1.75 | 0.68 | 0.79 | 1.79 | 0.20 | 0.20 | 0.28 |
| 19 | 0.10 | 0.01 | 0.00 | 3.49 | 1.35 | 1.58 | 3.58 | 0.20 | 0.20 | 0.28 |
| 20 | 0.50 | 0.05 | 0.02 | 17.45 | 6.75 | 7.89 | 17.87 | 0.20 | 0.20 | 0.28 |
| 21 | 1.00 | 0.11 | 0.06 | 34.90 | 13.47 | 15.77 | 35.72 | 0.20 .0 .21 | 0.29 |  |
| 22 | 5.00 | 1.99 | 1.51 | 174.69 | 66.26 | 78.59 | 177.99 | 0.41 | 0.80 | 0.42 |

table 7- continue
Group 3.b- Errors in y Coordinates

| 23 | 0.01 | 0.00 | 0.00 | 0.11 | 0.12 | 0.09 | 0.07 | 0.20 | 0.20 | 0.28 |
| ---: | ---: | ---: | :--- | :--- | ---: | :--- | ---: | :--- | :--- | :--- |
| 24 | 0.05 | 0.00 | 0.00 | 0.55 | 0.60 | 0.47 | 0.36 | 0.21 | 0.21 | 0.29 |
| 25 | 0.10 | 0.00 | 0.00 | 1.10 | 1.20 | 0.93 | 0.71 | 0.22 | 0.21 | 0.30 |
| 26 | 0.50 | 0.01 | 0.03 | 5.48 | 6.00 | 4.65 | 3.55 | 0.32 | 0.29 | 0.41 |
| 27 | 1.00 | 0.06 | 0.11 | 10.95 | 11.95 | 9.31 | 7.10 | 0.48 | 0.42 | 0.60 |
| 28 | 5.00 | 2.07 | 2.84 | 54.29 | 57.97 | 47.52 | 35.95 | 1.96 | 1.67 | 2.36 |

Group 3.c- Errors in z Coordinates

| 29 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.03 | 0.20 | 0.20 | 0.28 |
| ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 30 | 0.05 | 0.00 | 0.00 | 0.02 | 0.04 | 0.06 | 0.16 | 0.20 | 0.20 | 0.28 |
| 31 | 0.10 | 0.00 | 0.00 | 0.03 | 0.08 | 0.12 | 0.33 | 0.20 | 0.20 | 0.30 |
| 32 | 0.50 | 0.00 | 0.00 | 0.15 | 0.41 | 0.60 | 1.65 | 0.20 | 0.20 | 0.52 |
| 33 | 1.00 | 0.00 | 0.00 | 0.30 | 0.81 | 1.20 | 3.29 | 0.20 | 0.20 | 0.91 |
| 34 | 5.00 | 0.03 | 0.03 | 1.52 | 4.07 | 6.08 | 16.49 | 0.21 | 0.20 | 4.32 |

```
* For all the tests, numbers of model points and P.C. points= 112
and }12\mathrm{ points respectively.
```

Table 8- RMSE's Due to Errors in the Coordinates of Left P.C.'s Using Six Vertical Ground Control Points.


Group 4- No Errors in the Coordinates

Group 5.a- Errors in $x$ coordinates

| 36 | 0.05 | 0.00 | 0.00 | 0.15 | 0.18 | 0.22 | 0.16 | 0.14 | 0.12 | 0.13 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 37 | 0.10 | 0.00 | 0.00 | 0.31 | 0.37 | 0.44 | 0.31 | 0.13 | 0.13 | 0.13 |
| 38 | 0.50 | 0.00 | 0.00 | 1.53 | 1.83 | 2.21 | 1.56 | 0.25 | 0.32 | 0.15 |
| 39 | 1.00 | 0.01 | 0.01 | 3.05 | 3.66 | 4.42 | 3.11 | 0.50 | 0.60 | 0.21 |
| 40 | 5.00 | 0.05 | 0.09 | 15.11 | 18.21 | 22.01 | 15.31 | 2.67 | 2.88 | 0.80 |

Group 5.b- Errors in $y$ Coordinates

| 41 | 0.05 | 0.00 | 0.00 | 0.07 | 0.16 | 0.15 | 0.02 | 0.17 | 0.13 | 0.13 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 42 | 0.10 | 0.00 | 0.00 | 0.13 | 0.32 | 0.30 | 0.05 | 0.20 | 0.15 | 0.13 |
| 43 | 0.50 | 0.00 | 0.01 | 0.65 | 1.55 | 1.50 | 0.24 | 0.42 | 0.31 | 0.16 |
| 44 | 1.00 | 0.00 | 0.01 | 1.30 | 3.09 | 3.00 | 0.47 | 0.73 | 0.57 | 0.20 |
| 45 | 5.00 | 0.04 | 0.06 | 6.47 | 15.45 | 15.00 | 2.32 | 3.27 | 2.66 | 0.68 |

Group 5.c- Errors in z Coordinates

| 46 | 0.05 | 0.00 | 0.00 | 0.02 | 0.05 | 0.06 | 0.17 | 0.15 | 0.12 | 0.15 |
| ---: | ---: | ---: | ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 47 | 0.10 | 0.00 | 0.00 | 0.04 | 0.10 | 0.12 | 0.34 | 0.15 | 0.12 | 0.19 |
| 48 | 0.50 | 0.00 | 0.00 | 0.20 | 0.48 | 0.59 | 1.69 | 0.15 | 0.12 | 0.47 |
| 49 | 1.00 | 0.00 | 0.00 | 0.40 | 0.96 | 1.19 | 3.37 | 0.15 | 0.12 | 0.89 |
| 50 | 5.00 | 0.03 | 0.02 | 1.99 | 4.86 | 5.99 | 16.86 | 0.18 | 0.12 | 4.32 |

[^0]
[^0]:    * For all the tests, numbers of model points and P.C. points=112 and 12 points respectively.

