Transformation Matrix for study and Evaluation of Projection Aspects of the Earth Observation Area

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Abstract

A simplified transformation matrix which is tentatively called "Q-Matrix" has a function to make a perspective projection onto a quadratic surface including sphere.

1. Introduction

Graphical explanation for a kind of the earth observation area by satellite onboard sensor on map is very important on study and evaluation of the earth observation system design. In many cases the graphic technology is applied for this purpose using a personal computer and a visual display terminal including XY-plotter.

In this case the simplified and easily understandable mathematic procedure is required to make the graphical implementation program. A simplified transformation matrix tentatively called "Q-Matrix" was made on thinking the algorism.

It is noticed that this convenient matrix has a function to make a perspective projection onto a quadratic surface including sphere in three dimentional coordinates.

2. Making the matrix

Making process of the matrix is explained as follows. In case of drawing for instance the satellite observation areas on map, we prepare a sphere which means the earth as three dimentional object and a line which means projection line.

The sphere which is one of quadratic surface in three dimentional coordinates is expressed by next equation.

$$(x^* - x_0)^2 + (y^* - y_0)^2 + (z^* - z_0)^2 = R^2 \quad (1)$$

Where x^*, y^* and z^* are transformed vector points on the sphere. x_0, y_0, z_0 are center of the sphere. R is radius of the sphere.

The projection line in three dimentional coordinates is expressed by next form.

$$\frac{x^* - x_1}{x_2 - x_1} = \frac{y^* - y_1}{y_2 - y_1} = \frac{z^* - z_1}{z_2 - z_1} \qquad (2)$$

Where x^*, y^*, z^* are vector point on the line. x_1, y_1, z_1 are position of viewing point. x_2, y_2, z_2 are position of conducting point.

When the projection line crosses the object (sphere in here) in three dimentional space,an intersection point can be made on the surface of the object (sphere). This intersection point is in common with the line and sphere. The point is expressed by vector x^*, y^* and z^*. This point is obtained by finding the common root on two equations (1) and (2). Parametric symbol"t" is considered for equation (2) in this case. Later, this parameter"t" becomes a root of quadratic equation (7).

$$\frac{x^* - x_1}{x_2 - x_1} = \frac{y^* - y_1}{y_2 - y_1} = \frac{z^* - z_1}{z_2 - z_1} = t \qquad (3)$$

This equation(3) is deformed as follows.

$$\begin{aligned} x^* &= x_2 t + x_1 (1-t) \\ y^* &= y_2 t + y_1 (1-t) \\ z^* &= z_2 t + z_1 (1-t) \end{aligned} \qquad (4)$$

As a result, the above relation is made as four rows and four columns matrix form as the row matrix.[3]

$$\begin{bmatrix} x^* & y^* & z^* & 1 \end{bmatrix} = \begin{bmatrix} x_2 & y_2 & z_2 & 1 \end{bmatrix} \begin{bmatrix} t & 0 & 0 & 0 \\ 0 & t & 0 & 0 \\ 0 & 0 & t & 0 \\ x1(1-t), & y1(1-t), & z1(1-t), & 1 \end{bmatrix}$$

This"T" is called a transformation matrix. (5)

$$T = \begin{bmatrix} t & 0 & 0 & 0 \\ 0 & t & 0 & 0 \\ 0 & 0 & t & 0 \\ x_1 (1-t), & y_1 (1-t), & z_1 (1-t), & 1 \end{bmatrix} \qquad (6)$$

Procedure is to substitute the vector point P(x^*, y^*, z^*) of the line equation (3) to the sphere equation (1). After the algebraic procedure we get a new quadratic equation (7).[3]

$$A\cdot t^2 + B\cdot t + C = 0 \tag{7}$$

Where $A=(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$

$B=2(x_2 - x_1)(x_1 - x_0)+2(y_2 - y_1)(y_1 - y_0)$
$+2(z_2 - z_1)(z_1 - z_0)$

$C=(x_1 - x_0)^2 +(y_1 - y_0)^2 +(y_1 - y_0)^2 - R^2$

The root"t" as a solution of this quadratic equation is a well-known following expression.

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \tag{8}$$

If the viewpoint P_1 (x_1, y_1, z_1) is at origin in three dimentional coordinates, following simple matrix is obtained.

$$T = \begin{bmatrix} t & 0 & 0 & 0 \\ 0 & t & 0 & 0 \\ 0 & 0 & t & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

This type's matrix has a local scaling effect.[1] However there are three diagonal same elements which enables to have following overall scaling matrix (10) using relationship of (11) and (12).

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/t \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} X & Y & Z & Q \end{bmatrix} = \begin{bmatrix} x & y & z & 1 \end{bmatrix} \cdot T \tag{11}$$

$$\begin{bmatrix} x^* & y^* & z^* & 1 \end{bmatrix} = \begin{bmatrix} X/Q & Y/Q & Z/Q & 1 \end{bmatrix} \tag{12}$$

This method is a special utilization of fourth element"Q" as homogeneous coordinates.

Accordingly the handmade transformation matrix tentatively called Q-Matrix is expressed as follows.

$$T_Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ x_1\ (1-1/Q), & y_1\ (1-1/Q), & z_1\ (1-1/Q), & Q \end{bmatrix} \quad (13)$$

Where

$Q = 1/t$

$t = (-B \pm \sqrt{B^2 - 4AC})/2A$

In case of sphere A,B,C are as follows.

$A = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$

$B = 2(x_2 - x_1)(x_1 - x_0) + 2(y_2 - y_1)(y_1 - y_0)$
$\qquad + 2(z_2 - z_1)(z_1 - z_0)$

$C = (x_1 - x_0)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 - R^2$

Where x_0, y_0, z_0 : Center position of sphere.
y_1, y_1, z_1 : Position of viewpoint.
x_2, y_2, z_2 : Vector of conducting point.

If the viewpoint is at the origin ($x_1 = y_1 = z_1 = 0$) in three dimentional coordinates,the Q-matrix becomes a simple form as follows.

$$T_Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & Q \end{bmatrix} \quad (14)$$

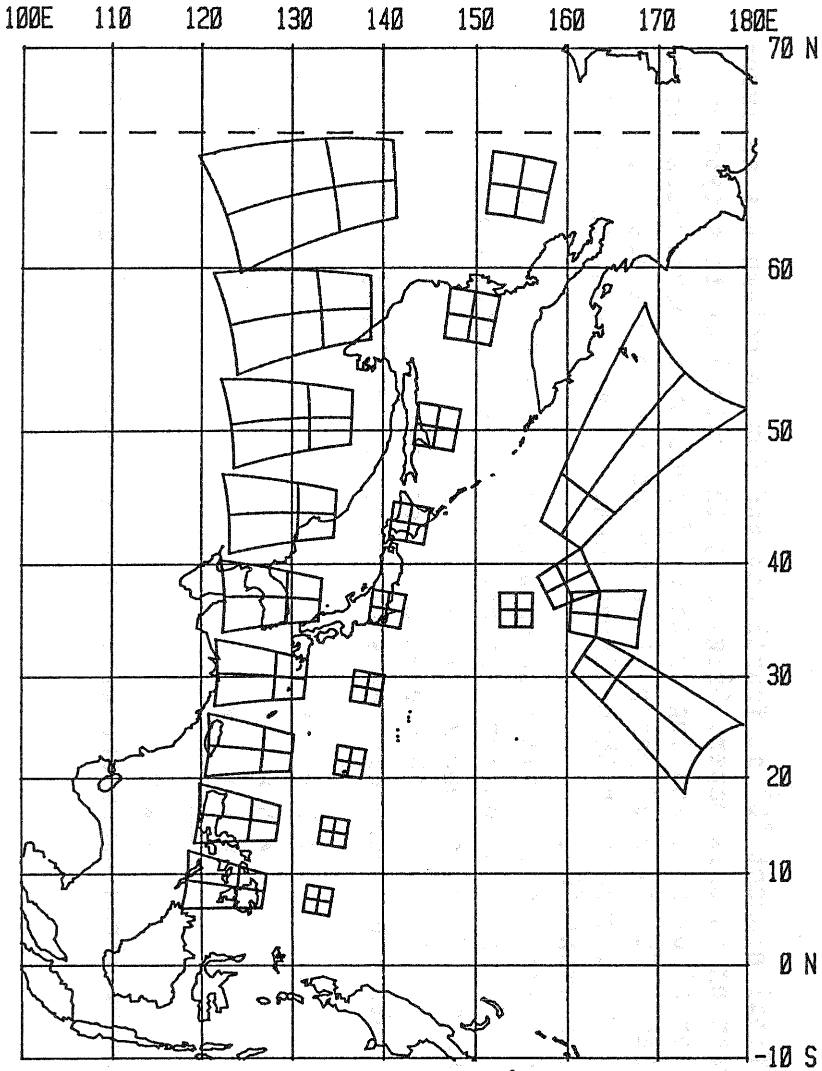
The final element "Q" at fourth row and fourth column has an important meaning in this paper. Originally the matrix of this type has a function of overall scaling in three dimentional coordinates. This matrix has a function of perspective projection onto a quadratic surface like a sphere in three dimentional surface using the final diagonal element "Q".

The Q-matrix,as a result,becomes absolutely different and special function. This is confirmed by reviewing other functions in the four rows and four columns matrix as follows.

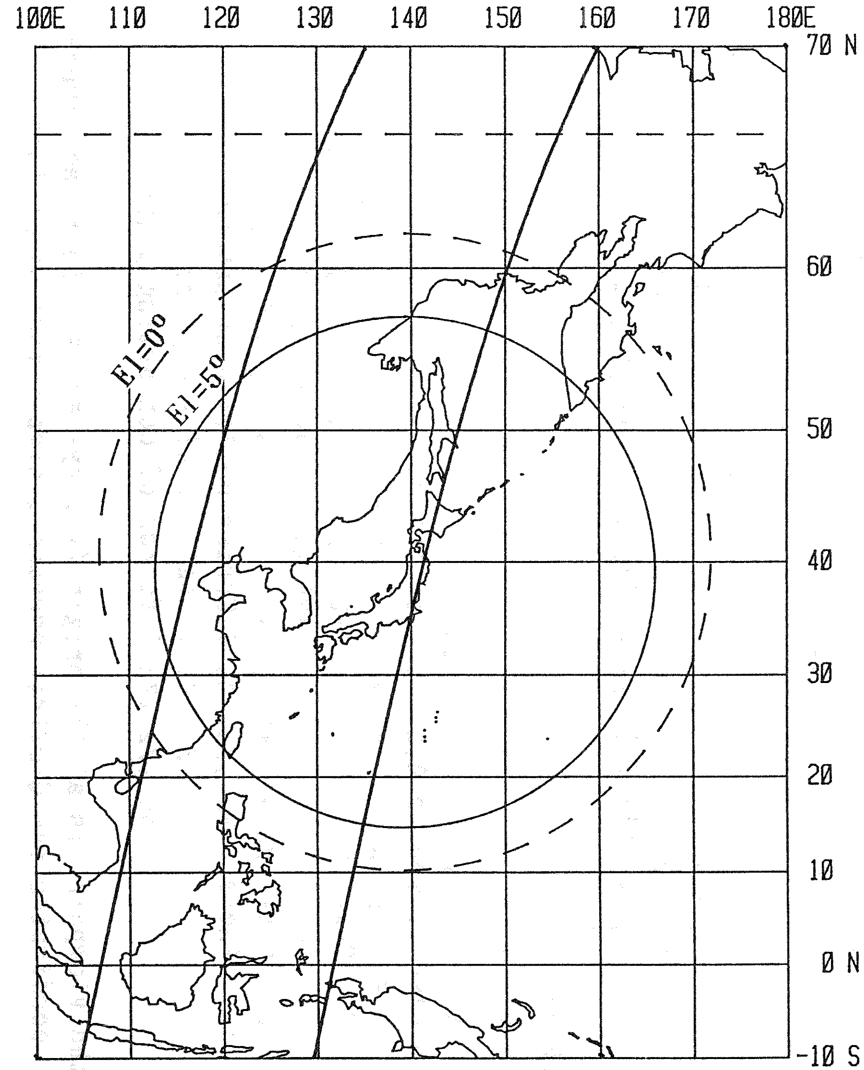
Generally, the next 3x3 elements in rectangular area has a function of rotation around each axis of coordinates.[1][2]

$$T = \begin{bmatrix} \boxed{\begin{matrix} * & * & * \\ * & * & * \\ * & * & * \end{matrix}} & \begin{matrix} * \\ * \\ * \end{matrix} \\ \begin{matrix} * & * & * \end{matrix} & * \end{bmatrix} \quad (15)$$

Following three cases are kinds of homogeneous coordinates.[1][2]



Satellite Onboard Sensor's IFOV
(Nadir viewing & Off-nadir viewing)



Receiving Area and Satellite Paths
(Station: EOC/Hatoyama, Saitama)

Figure: Example of Projection to Quadratic surface

Three elements in under rectangular area has a function of translation which means to move a vector point along the axis of each coordinates in case of row matrix.

$$T = \begin{bmatrix} \ast & \ast & \ast & \ast \\ \ast & \ast & \ast & \ast \\ \ast & \ast & \ast & \ast \\ \boxed{\ast} & \boxed{\ast} & \boxed{\ast} & \ast \end{bmatrix} \tag{16}$$

Three elements in right side rectangular area has a function of perspective transformation onto the two-dimentional viewing plane.

$$T = \begin{bmatrix} \ast & \ast & \ast & \boxed{\ast} \\ \ast & \ast & \ast & \boxed{\ast} \\ \ast & \ast & \ast & \boxed{\ast} \\ \ast & \ast & \ast & \ast \end{bmatrix} \tag{17}$$

Final single element in rectangular area is confirmed to have functions of not only overall scaling but also perspective projection onto a quadratic surface as a special function.

$$T = \begin{bmatrix} \ast & \ast & \ast & \ast \\ \ast & \ast & \ast & \ast \\ \ast & \ast & \ast & \ast \\ \ast & \ast & \ast & \boxed{\ast} \end{bmatrix} \tag{19}$$

3.Conclusion

Matrix,as linear algebra, works well on the mathematical transformation procedure on program implementation, using computer for the analysis of the earth observation area, on the earth surface and projection on a map, including satellite orbit traces.

This normarized type, as a row matrix and four rows and four columns form, is applicable to make linkage with any other type of matrices if they are same form, although the element "Q" is very heavy because of a special function.

References

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[3] Sato H,K.Tsuchiya and T.Igarashi: Projection Method of Sensing and Interference Areas of an Earth Observation Satellite, Proceedings of the thirteenth International Symposium on Space Technology and Science,Tokyo 1982.