Theoretical Capacity and Limitation of Localizing Gross Error by Robust Adjustment

Wang Renxiang

Xian Research Institute of Surveying and Mapping, China

Abstract

The basic rules of the increment relationship between the weight matrix P and matrix G, where G=Qvv.P, are given, from which the capacity and some limitation of localizing gross errors by Robust adjustment can be discussed theoretically. In order to overcome the discussed limitation, a proposal for improving Robust adjustment is given by author. To show the discussions, an example for calculating the parameters of relative orientation by Robust adjustment is made.

KEY WORD: localizing gross error, capacity, limitation, checking residuals

Introduction

In recent years, gross errors detection and location is very attractive topic in photogrammetry. More and more photogrammetrists research on Robust-least-squares-adjustment (hereafter simplied Robust adj.). The method of Robust adj. is using iterations computed by conventional least-squares-adjustment with weight function. After convergen, the gross error revealed in the corresponding residual will be increased gradually and from the magnitude of the residual taken as evidence of the gross error directly.

A lot of published papers have stated that Robust adj. is less sensitive against gross errors. Up to now, however, the adjustment is still lacking in estimating the capacity of localizing (or eliminating) gross errors theoretically. From the point-review of localizing gross errors, the variation of matrix G is an essential defference in adjusted resuts between Robust adj. and conventional least squares adjustment. The paper investigates the variational behaviour of matrix G, whilst matrix P was variated, which would be as a key for further study the problems about Robust adj., such as the capacity of localizing gross error; the limitation of localizing gross errors; further improving the adjustment, etc.

Variational Rules of Increment Relationship Between Matrix P and Matrix G

The relationship between the vector of residual V computed after least squares adjustment and vector of observational error E is given by the formula

$$V = -G \cdot E \qquad (1)$$

$$G = Q_{vv} \cdot P$$

where V=the vector of residuals; E=the vector of observational error with distribution $N(0, \sigma_0^2)$; Qvv=cofactor matrix of residual; P=weight coefficient matrix of observations. Matrix G can also be expressed as

$$G = I - A(A^T A)^{-1} A^T P \qquad (2)$$

where A=design matrix; I=unite matrix

Let $N = A^T P A$. $\quad R = A N^{-1} A^T \quad T = R \cdot P \qquad (3)$

then $G = I - T \qquad (4)$

If weight matrix P get increment ΔP , then the variated weight matrix $\dot{P}$ will be

$$\dot{P}=P+\Delta P \tag{5}$$

where $P=\begin{pmatrix} P_1 & & & \\ & P_i & & \\ & & & P_m \end{pmatrix}$, $\Delta P=\sum_{i=1}^{m}\Delta P_i=\begin{pmatrix} \delta_1 & & & \\ & \delta_i & & \\ & & & \delta_m \end{pmatrix}$, $\Delta P_i=\begin{pmatrix} 0 & & \\ & \delta_i & \\ & & 0 \end{pmatrix}$

δ_i =increment of P_i , i=1,m

using weight matrix $\dot{P}$ for least-squares adjustment, one get

$$\dot{T}=\dot{R}\cdot\dot{P}=A\dot{N}^{-1}A^T\dot{P}$$

where $\dot{N}=A^T\dot{P}A=N+\Delta N$

and $\Delta N=A^T\Delta PA=$ incernent of matrix N

$$\dot{N}^{-1}=(N+\Delta N)^{-1}=N^{-1}(I+\Delta N\cdot N^{-1})^{-1}$$

whereas N^{-1} can be regarded as an approximation of $\dot{N}^{-1}$ and $\dot{N}^{-1}$ can therefore be expanded in Tylor's series as fellow

$$\dot{N}^{-1}=N^{-1}(I-\Delta N\cdot N^{-1}+(\Delta N\cdot N^{-1})^2-(\Delta N\cdot N^{-1})^3+\cdots)$$

$$\dot{T}=(I-R\cdot\Delta P+(R\cdot\Delta P)^2-\cdots)R(P+\Delta P)$$

$$\Delta G=R\cdot P-\dot{T}=\sum_{n=1}^{\infty}(-1)^n(R\cdot\Delta P)^n\cdot G \tag{6}$$

1. Variational Rules of Elements of matrix G, due to increment of one Main-diagonal element of weight matrix P

If matrix P is a diagonal one and only the element P_i get an increment δ_i, then the increment of matrix G is that

$$\Delta G=-(R\cdot\Delta P_i-(R\cdot\Delta P_i)^2+\cdots)G \tag{7}$$

From the characteristics of matrix G, we know that $0\leq g_{ii}\leq 1$, and therefore $0\leq r_{ii}\cdot P_i\leq 1$. If we take $|\delta_i|<P_i$, then the series of G will be convergent and the higher-order terms in Eq.(7) can be negelected. By omitting the terms which are higher than $(R\cdot\Delta P)^3$,we obtain

$$\Delta G=-S_i\cdot R\cdot\Delta P_i\cdot G \tag{8}$$

where $S_i=1-r_{ii}\cdot\delta_i$, sign $(S_i)=+$ and

$$R\cdot\Delta P_i\cdot G=\frac{\delta_i}{P_i}\begin{pmatrix} g_{1i}g_{i1} & \cdots & g_{1i}g_{ik} & \cdots & g_{1i}g_{ii} & \cdots & g_{1i}g_{im} \\ \vdots & & \vdots & & \vdots & & \vdots \\ g_{ki}g_{i1} & \cdots & g_{ki}g_{ik} & \cdots & g_{ki}g_{ii} & \cdots & g_{ki}g_{im} \\ \vdots & & \vdots & & \vdots & & \vdots \\ g_{i1}(g_{ii}-1) & \cdots & g_{ik}(g_{ii}-1) & \cdots & g_{ii}(g_{ii}-1) & \cdots & g_{im}(g_{ii}-1) \\ \vdots & & \vdots & & \vdots & & \vdots \\ g_{mi}g_{i1} & \cdots & g_{mi}g_{ik} & \cdots & g_{mi}g_{ii} & \cdots & g_{mi}g_{im} \end{pmatrix} \tag{9}$$

Using Eq.(8),Eq.(9) and taking $\mathrm{sign}(\delta_i)=-$, the variational relues of elements of matrix G in comparison with the original matrix G can be stated at fellows

--The ith element of main- diagonal will be increased proportional to $g_{ik}\cdot g_{ki}$ while the off-diagonal elements will be reduced, and the increased value just equal to the sum of all the reduced values in absolute;

$$\Delta g_{ii}=s_i(g_{ii}-g_{ii}^2)\frac{|\delta_i|}{P_i},\ \Delta g_{kk}=-s_i g_{ik}\cdot g_{ki}\frac{|\delta_i|}{P_i} \tag{10,a}$$

According to the characteristics of matrix Qvv, we have

$$g_{ii}-g_{ii}^2=\sum_{\substack{k=1\\(k\neq i)}}^{m}g_{ik}\cdot g_{ki} \quad \text{and so} \quad \Delta g_{ii}=-\sum_{\substack{k=1\\(k\neq i)}}^{m}\Delta g_{kk}$$

--The ith row element g_{ji} will be reduced;

$$\Delta g_{ji}=-s_i\, g_{ji} g_{ii} \frac{|\delta_i|}{p_i}, (j=1,m, j\neq i) \qquad (10,b)$$

-- The ith column element will be increased

$$\Delta g_{ij}=s_i \cdot g_{ij}(1-g_{ii})\frac{|\delta_i|}{p_i}, (j=1,m, j\neq i) \qquad (10,c)$$

--The elements with the exception of mentioned above will be either increased or reduced;

$$\Delta g_{k\ell}=-s_i\, g_{ki} \cdot g_{i\ell}\frac{|\delta_i|}{p_i}, (\ell=1,m, k=1,m, k\neq\ell\neq i) \qquad (10,d)$$

Because $s_i gn(g_{ki} g_{i\ell})$ is uncertainty.

The Eq.(10,a-10,d) are provided that $p_i \neq 0$

2. Mathematical formula of increment of matrix P and G when matrix P is united one (P=I)

When all observations are assumed to be of equal weight and correlation free, i.e., P=I, then

$$\Delta G=\Delta Q_{vv}=\sum_{n=1}^{\infty}(-1)^n (R\cdot\Delta P)^n \cdot Q_{vv}$$

The norm of matrix R. ΔP is satified as $\|R\cdot\Delta P\|_1 \leq \|R\|_1 \cdot \|\Delta P\|_1 < 1$, because where $R=I-Q_{vv}$, $0\leq r_{ij}\leq 0.5$, (i=1,m.j=1,m j$\neq$i) and $0\leq r_{ii}\leq 1$, (i=1,m) as well as we take $|\delta_i|<1$, (i=1,m) therefore $\|\Delta P\|_1 < 1$, consequently the series of Qvv must be convergent one and the high -order terms can be negelected. To simplify the subsequent discussions, we take first-order terms of ΔQ_{vv}, then we have

$$\Delta Q_{vv}=-(R\cdot\Delta P_1\cdot Q_{vv}+R\cdot\Delta P_2\cdot Q_{vv}+\cdots+R\cdot\Delta P\cdot Q_{vv})$$

Using Eq.(9) to above equation, yields

$$\left.\begin{aligned}
\Delta q_{ii}&=(q_{ii}^2-q_{ii})\delta_i+q_{ik}q_{ki}\delta_k+\sum_{j=1}^{m} q_{ji}q_{ij}\delta_j\\
\Delta q_{kk}&=(q_{kk}^2-q_{kk})\delta_k+q_{ki}q_{ik}\delta_i+\sum_{j=1}^{m} q_{jk}q_{kj}\delta_j\\
\Delta q_{ik}&=q_{ik}(q_{ii}-1)\delta_i+q_{ik}q_{kk}\delta_k+\sum_{j=1}^{m} q_{ij}q_{jk}\delta_j\\
\Delta q_{ki}&=q_{ki}q_{ii}\delta_i+q_{ki}(q_{kk}-1)\delta_k+\sum_{j=1}^{m} q_{kj}q_{ji}\delta_j
\end{aligned}\right\} \qquad (11)$$

The Eq.(11) are provided that $j\neq i\neq k$.

Theoretical Capacity of Localizing Gross Error by Robust Adjustment

As well know that gross errors can be distributed to every residual of observations which are taken into the adjustment. In general case , it is hardly to reconized the gross error observation from least-squares residuals directly. Robust adj. is using iterations with weight function in order to make the gross error observations can easy be reconized from Robust residuals. For this purpose , we know that the magnitude of diagonal element of matrix G related to gross error observation, must be rather large after iterations. Hance, the functional essentiality of robust adj. is to increase the magnitude of diagonal elements related to gross error as large as possible. For discussion of the reliability of adjustment, we assume that only one of observations,i.e.,observation i is

with gross error ∇_i. Because $0\le g_{ii}\le 1$, so g_{ii} should be as near as possible to the value 1 after iterations. In order to locate the gross error correctly, the condition $\Delta q_{ii}>0$ must be satified. We would from the condition to discuss how large gross error can be located by weighted iteratoon Least- Squares adjustment. That would be an interesting problem in recent years.
Considering $\Delta q_{ii}>0$ and Eq.(11), we have (let sign δ =-)

$$(q_{ii}-q_{ii}^2)|\delta_i|>\sum_{k=1}^{m} q_{ik}^2|\delta_k| , \quad (k\neq i) \qquad (12)$$

where $|\delta_j|=1-f(\bar{v}_j)$ $j=1, m$ (13)

$f(\bar{v}_j)$ =weight function for Robust adj., $\bar{v}_j=v_j/\sqrt{q_{jj}}$

We know that $\bar{v}_i=\sqrt{q_{ii}}\cdot\nabla_i+\dot{v}_i$

where $\dot{v}_i=(\sum_{k=1}^{m} q_{ik}\cdot\varepsilon_k)/\sqrt{q_{ii}}$, of which standard diviation is $\sqrt{1-q_{ii}}\cdot\sigma_0$, according to the characteristics of normal distribution, the value of $\dot{v}_i$ is able to be taken as $\dot{v}_i=\sqrt{1-q_{ii}}\cdot t\cdot\sigma_0$ associated with probability as

$$Pr\{|\dot{v}_i|>\sqrt{1-q_{ii}}\cdot t\cdot\sigma_0\}=\begin{cases}\frac{\alpha}{2}, & t>0\\ 1-\frac{\alpha}{2}, & \text{eleswhere}\end{cases}$$

where $t=\sqrt{\chi^2_{\alpha,1}}$, α =significant level

and $\bar{v}_i=\sqrt{q_{ii}}\cdot\nabla_i\pm\sqrt{1-q_{ii}}\cdot t\cdot\sigma_0$,

$|\bar{v}_i|_{min}=\sqrt{q_{ii}}\cdot|\nabla_i|-\sqrt{1-q_{ii}}\cdot|t|\cdot\sigma_0$ associated with probability $1-\frac{\alpha}{2}$

On the other hand, we have

$\bar{v}_k=[q_{ki}\cdot\nabla_i+\dot{v}_k]/\sqrt{q_{kk}}$, $(k=1, m, k\neq i)$

where $\dot{v}_k=\sum_{j=1 \, (j\neq i)}^{m} q_{kj}\cdot\varepsilon_j$, which is $N(0,(q_{kk}-q_{ki}^2)\sigma_0^2)$ and can be taken as $\dot{v}_k=\sqrt{(q_{kk}-q_{ki}^2)}\,t\,\sigma_0$ Therefore we have $\bar{v}_k=[q_{ki}\cdot\nabla_i\pm\sqrt{(q_{kk}-q_{ki}^2)}\,t\,\sigma_0]/\sqrt{q_{kk}}$

Most procedures of Robust adj. have been set up a critical value C (1.0- 2.0σ_0) , if $|\bar{v}_j|\le C$, $\delta_j=0$. Consequently, there are only several per-cent weights of observation are reduced, of which $|\bar{v}|$ are greater than C caused by relative large value of $|q_{ki}|$ and/or $|t|$.

We take $|\bar{v}_j/\sigma_0|^{-2}$, (j=1,m) as weight function for discussion, then we get

$$|\bar{v}_i|>\sqrt{(q_{ii}-q_{ii}^2)\left(\sum_{k=1}^{m} q_{ik}^2/\bar{v}_k^2\right)^{-1}}=\sqrt{(q_{ii}-q_{ii}^2)[\sum_{k=1\,(k\neq i)}^{m} q_{kk}/(\nabla_i^2+\Delta_k)]^{-1}} \qquad (14)$$

where $\Delta_k=(\frac{q_{kk}}{q_{ki}^2}-1)\,t^2\sigma_0^2$ of which magmitude , in general case, is more small as compare with ∇_i^2 in the denominator, and alowable to be replaced by a `mean value $\bar{\Delta}$ ' for overcome the difficulty of algebric deduction, to which we assum that $\bar{q}_{kk}=\frac{r}{m}$, $\bar{q}_{ki}^2\doteq\frac{q_{ii}}{m}\doteq\frac{r}{m^2}$ $\frac{r}{m}\ge 0.35$, r=3, $\bar{t}$=0.7979 and computed $\bar{\Delta}=5\sigma_0^2$

where $r=\sum_{k=1}^{m} q_{kk}$ and $\bar{q}_{kk}$, $\bar{q}_{ki}^2$, $\bar{t}$ is the average value of q_{kk} , q_{ki}^2 , t respectively.

Therefore we have

$$|\bar{v}_i|>\sqrt{(q_{ii}-q_{ii}^2)[\sum_{k=1\,(k\neq i)}^{m} q_{kk}/(\nabla_i^2+5\sigma_0^2)]^{-1}}=\sqrt{(q_{ii}-q_{ii}^2)(\nabla_i^2+5\sigma_0^2)/(r-q_{ii})}$$

In order to estimate the capacity of localizing gross error with high probability, we take that

$$|\bar{v}_i|_{min}>\sqrt{(q_{ii}-q_{ii}^2)(\nabla_i^2+5\sigma_0^2)/(r-q_{ii})}$$

Then $$\sqrt{q_{ii}}\,|\nabla_i|-\sqrt{(q_{ii}-q_{ii}^2)(\nabla_i^2+5\sigma_0^2)/(r-q_{ii})}>\sqrt{1-q_{ii}}\cdot|t|\cdot\sigma_0 \qquad (15)$$

associated with probability $1-\frac{\alpha}{2}$

In this paper, the value of ∇ computed with Eq.(15) and associated with probability $1-\frac{\alpha}{2}$ listed in the table(1) are taken to discribe the capacity of localizing gross error.

In Robust adj. the observations of which residual is greater than the 'critical value', have opportunites to be revalued. If the main-diagonal elements related to gross error is increased in first iteration, then the residual will be converged to the corrected value. It should be point out, however, that the suitability of the capacity of localizing gross error as table(1) are limited in the observations of which residual are not heavity correlation.

Table(1) The capacity of localizing gross error

$\lvert\nabla\rvert$ (σ_0) \ t / $1-\frac{\alpha}{2}$ / q_{ii}	1.28	1.64	1.96	3.29	
	90%	95%	97.5%	99.95%	
0.6	2.5	2.9	3.3	5.0	r=3
0.5	3.1	3.6	4.1	6.4	
0.4	3.7	4.5	5.3	8.2	
0.2	7.6	9.0	10.5	17.0	

The limitation of Localizing Gross Errors By Roubust Adjustment

This problem is concerned with many factors, such as geometris strength of system, redundant number of observation, number of gross errors, magnitude of every gross error and their distribution in the system. It is diffcult to dedue a sophistiated error analysis of the limitation in considerations of all the factors mentioned above, we have already know that the most serious factor for localizing gross errors is the correlation of residuals. In this paper, we would restrict the discussions in observation i and observation k of which residuals are heavity correlated.

1. The variational properties of elements of submatrix $\begin{pmatrix} q_{ii} & q_{ik} \\ q_{ki} & q_{kk} \end{pmatrix}$ in matrix Q_{vv}

Assum that $|q_{ik}|=|q_{ki}|>q_{ii}$ or q_{kk}, and q_{ik} or q_{ki} is the largest value in absolute of off- diagonal elements ith or kth row respectively. Furthermore, the observation i and/or k is with gross error and with rather large residual after least-squares- adjustment. We would concentrate on the terms related to δ_i and δ_k, and negelecte the terms in Eq.(11) for discussion. In any case, for δ_i and δ_k, there are only two circumstances, i.e., $\delta_i=\delta_k$ or $\delta_i\neq\delta_k$ to be taken in the discusion. First takes that $\delta_i=\delta_k=\delta$ and sign = - . from Eq.(11), we have

$$\Delta q_{ii}\doteq(q_{ii}-q_{ii}^2-q_{ik}^2)|\delta|, \quad \Delta q_{kk}\doteq(q_{kk}-q_{kk}^2-q_{ki}^2)|\delta| \tag{16}$$

$$\Delta q_{ik}\doteq q_{ik}(1-(q_{ii}+q_{kk}))|\delta|, \quad \Delta q_{ki}\doteq q_{ki}(1-(q_{ii}+q_{kk}))|\delta| \tag{17}$$

From above, we know that the element of the submatrix will be variated as fellows

-- The two main-diagonal elements q_{ii}, q_{kk} will be increased, of which magnitude are not difference too much;

-- The magnitude of off-diagonal elements q_{ik}, q_{ik} increasing or reducing depends on whether $q_{ii}+q_{kk}$ is smaller or greater than 1;

-- If $|q_{ik}|=|q_{ki}|=q_{ii}=q_{kk}$, the results of iterations with reduced weight must be $|g_{ik}|=|g_{ki}|=|g_{ii}|=g_{kk}\leq 0.5$

because $g_{ii}+g_{kk}\doteq q_{ii}+q_{kk}+\Delta q_{ii}+\Delta q_{kk}=2q_{ii}+2(q_{ii}-2q_{ii}^2)|\delta|$, $(q_{ii}+q_{kk})\leq 1$ and $|\delta|<1$ so $g_{ii}+g_{kk}<4(q_{ii}-q_{ii}^2)_{max}=1$. Then we get

$$|g_{ik}|=|g_{ki}|=g_{ii}=g_{kk}\leq 0.5$$

The second case, we takes that $\delta_i\neq\delta_k$. From Eq.(11), we have -- The larger weighted increment the larger the increment will be of related the column elements. The conclusion is approximate for main-diagonal elements, and can be learnt from the formula as fellows

$$\Delta q_{ik}-\Delta q_{ki}\doteq q_{ik}(|\delta_i|-|\delta_k|)$$

$$\Delta q_{ii}-\Delta q_{kk}\doteq q_{ik}^2(|\delta_i|-|\delta_k|)+(q_{ii}-q_{ii}^2)-(q_{kk}-q_{kk}^2)|\delta_k|$$

The variational properties stated above is not exact true because the discussion is base on the first order of the series of ΔQvv, whereas it is enough precision for analysing the limitation of localizing gross errors by Robust adj.

2. Typical mislocalizing gross errors

Under consideration of the conditions, $|q_{ik}|=|q_{ki}|\geq q_{ii}$ or q_{kk}, especially $q_{ii}\doteq q_{kk}$ the gross error whether take place in observation i and/or k, revealed in the resiual i and residual k is not defference too much. In addition, the residual i and k is still consisted observational errors. As result, the relative size between the least squares residual i and residual k is arbitrary at all. With the help of theproperties discussed above,it is not certain that the iterations must be converged to the correct value,i.e., Robust residual with large magnitude taking as evidence gross error observation is unrelibility.

There are three typical mistakes of localizing gross errors as fellows

(a) `Interchanging gross error'

Assum that observation i is with gross error ∇_i, for localizing gross error correctly,the condition,i.e., $\Delta q_{ii}>|\Delta q_{ki}|$, must be satified. Otherwise, the residual k will be greater than the residual i in absolute value, and makes mislocalization of gross error in observation k after iterations. This mistake in gross error location is so-called `Interchanging gross error'.

(b) `Distracting gross error'

So-called `distracting gross error'is that the gorss error ∇_i was distracted to the residual i and residual k after iterations. In the result, gross error location either reduces the capacity or makes mistake. If $q_{ii}<|q_{ik}|$, the mislocalizing as(a),(b)will probably be occured. Nevertheless,after iterations the magnitude of elements of submatrix $\begin{pmatrix} g_{ii}, g_{ik} \\ g_{ki} & g_{kk} \end{pmatrix}$ would necessary be increased. The gross erors, therefore, revealed in residuals will be more prominent in comparssion with conventional least-squares -adjustment, and the capacity of detecting(not locating)gross errors would, of course, be improved.

(c) `Hidden gross error'

If $|\nabla_i|\doteq|\nabla_k|$, $q_{ii}\doteq q_{kk}$, and $Sign(\nabla_i\cdot\nabla_k)\neq Sign(q_{ik})$, the gross errors have small influence on the residuals with the results that there would be hardly any means of detection and locating them. This problem is so-calle `hidden gross error'.

(d) `Checking residuals'programm

The typical mislocalizing gross errors mentioned above is impossible to be overcome limited in the conditions , $|\delta_i| < 1$, $i = 1, m$. These problems can only be solved by re-adjustment with weighted zero to observation i and observation k simultanersly,i.e., $\delta_1 = \delta_k = -1$, to which we will refer to as 'Checking residuals' programm.

The elements in the submatrix must be as $\begin{pmatrix} g_{11} & g_{1k} \\ g_{k1} & g_{kk} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, and residual i and residual k are no longer correlation after run 'Checking residuals' programm.

For any two observations of which residuals are heavity correlated , and one of which residual has large size, weighted zero must be done to them imperatively , when run 'Checking residuals' programmm.

Examples of Robust Adjustment

1. Simulated data for adjustment

Calculating the parameters of relative orientation are taken as an example of Robust adj. , where includes 10 simulated observations(vertical parallex). The observational error vector is

$$E^T = (-1.0\ \ 0.6\ \ 1.0\ \ -0.7\ \ 0.1\ \ -0.2\ \ -1.0\ \ -0.9\ \ 1.5\ \ 0.9) , \quad \sigma_0 = 0.89$$

Moreover , this example is especial to lay emphasis on the limitations of localizing gross errors stated above.

2. Weight function for example adjustment

The weight function proposed by author has been simplified for this experiment as fellows

$$P_k = \begin{cases} 1, & |\bar{v}_k| \leq c \\ \dfrac{1}{a \cdot |\bar{v}_k|^{2.4}}, & \text{eleswhere} \end{cases} \qquad (18)$$

Where c=2, a=1 for first and second iteration
c=3, a=3 for after iterations

3. Results of adjustment

The results of adjustment with weight function as Eq.(18) are listed in table(2)- table(4).

4. Remarks on the experiments

-- From table(2), we know that gross error can be located with theoertical capacity as table(1), if the residuals are not too heavity correlated. While the correlation coefficient between residual 1 residual 2 equals 1,i.e., $\rho_{1,2} = 1$, the gross error in observation 2 could not be located correctly;

--From table(2), furthermore, know that mislocalizing as (a),(b) and (c) are appeared when observation 1 and/or observation 2 with gross error;

-- From table(3), we know that the mislocalizing gross error as same as table(2) are appeared. In spite of the residuals between observation 3 and 7 is also heavity correlated , however, the gross error in observation 7 can be located correctly, because $q_{77} > |q_{37}|$;

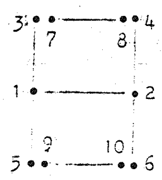
-- From table(4). we know that gross error in observation 5 is mislocalizing because $q_{55} < |q_{59}|$;

Table 2. Experiment in the capacity of localizing gross error

NO	1	2	1	2	3	4	5	6	7	8	9	10
∇	-4	-0.8	-1.0	-9	3	-5	5	-5	-6	-3	3	-3
V	2.3	-2.3	-3.4	3.4	-4.3	4.0	-3.8	4.5	4.0	2.4	-3.3	3.2
∇_0	7.2				6.2				3.8			

Qvv

.37	-.37	-.13	.13	-.13	.13	-.08	.08	-.08	.08
-.37	.37	.13	-.13	.13	-.13	.08	-.08	.08	-.08
-.13	.13	.43	.07	.05	-.05	-.44	-.06	.03	-.03
.13	-.13	.07	.43	-.05	.05	-.06	-.44	-.03	.03
-.13	.13	.05	-.05	.43	.07	.03	-.03	-.44	-.06
.13	-.13	-.05	.05	.07	.43	-.03	.03	-.06	-.44
-.08	.08	-.44	-.06	.03	-.03	.64	-.14	.02	-.02
.08	-.08	-.06	-.44	-.03	.03	-.14	.64	-.02	.02
-.08	.08	.03	-.03	-.44	-.06	.02	-.02	.64	-.14
.08	-.08	-.03	.03	-.06	-.44	-.02	.02	-.14	.64



— Each adjustment with one gross error
— ∇_0=The smallest gross error can be located associated with probability 99.95%
— I=Iterchanging gross error
— D=Distracting gross error
— H=Hidden gross error
— C= Gross error located correctly
— Correlate coefficient $\rho_{1,2}$=1

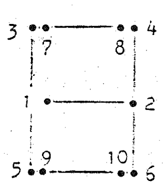
Iterating with weight function					'Checking residuals' programm		
∇_1	∇_2	V_1	V_2	Gross error location	V_1	V_2	Gross error location
10.0	-0.06	-4.7	4.7	D	-9.2	-0.3	C
-1.0	-10.0	-3.9	3.9	D	2.0	9.2	C
-10.0	-10.0	0.45	0.45	H	10.9	10.0	C
-10.0	10.0	10.3	-10.3	C	10.2	-10.5	C

Table 3. Adjustment with three gross errors

Iterating with weight function										'Checking residuals' programm				
∇_1	∇_2	∇_7	V_1	V_2	V_7	Gross error location				V_1	V_2	V_7	Gross error location	
10.0	-0.6	-10.0	-5.2	4.0	10.6	1,2	D	7	C	-9.4	-0.4	10.1	1,2,7	C
-1.0	-10.0	-10.0	-3.8	-3.0	10.6	1,2	D	7	C	1.1	9.0	9.4	1,2,7	C
-10.0	-10.0	-10.0	0.1	-0.1	8.1	1,2	H	7	C	11.4	11.5	10.2	1,2,7	C
-10.0	10.0	-10.0	10.7	-8.3	7.9	1,2,7	C			10.0	-10.1	8.4	1,2,7	C

Qvv

.38	-.35	-.16	.14	-.11	.13	-.10	.08	-.07	.08
-.36	.35	.16	-.13	.11	-.12	.10	-.08	.06	-.08
-.16	.16	.45	.06	.05	-.06	-.43	-.06	.03	-.03
.14	-.13	.06	.43	-.04	.05	-.07	-.44	-.02	.03
-.11	.11	.05	-.04	.42	.08	.03	-.02	-.45	-.05
.13	-.12	-.06	.05	.08	.43	-.03	.03	-.05	-.44
-.10	.10	-.43	-.07	.03	-.03	.64	-.14	.02	-.02
.08	-.08	-.06	-.44	-.02	.03	-.14	.63	-.01	.02
-.07	.06	.03	-.02	-.45	-.05	.02	-.01	.63	-.13
.08	-.08	-.03	.03	-.05	-.44	-.02	.02	-.13	.64



$\rho_{1,2}$=0.987

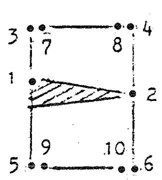
$\rho_{3,7}$=0.801

Table 4. Adjustment with three gross errors

Iterating with weight function											'Checking residuals' programm							
∇_1	∇_2	∇_5	V_1	V_2	V_5	V_9	Gross error location				V_1	V_2	V_5	V_9	Gross error location			
10.0	-0.6	10.0	-6.2	4.8	7.4	-3.6	1,2	D	5,9	D	-10.4	0.1	6.6	-4.5	1,2	C	5,9	D
-1.0	-10.0	10.0	-4.5	3.7	7.7	-3.4	1,2	D	5,9	D	0.9	10.5	4.1	-6.4	1,2	C	5,9	D
-10.0	-10.0	10.	0.3	-0.3	5.8	-4.0	1,2	H	5,9	D	7.8	8.2	5.9	-5.5	1,2	C	5,9	D
-10.0	10.0	10.	9.0	-7.0	0.2	-9.3	1,2	C	5,9	I	7.2	-8.5	2.1	-8.1	1,2	C	5,9	I

Qvv

.40	-.36	-.18	.14	-.11	.13	-.12	.08	-.06	.08
-.36	.33	.16	-.13	.10	-.12	.11	-.07	.05	-.07
-.18	.16	.46	.05	.05	-.06	-.42	-.06	.03	-.04
.14	-.13	.05	.43	-.04	.05	-.07	-.44	-.02	.03
-.11	.10	.05	-.04	.41	.08	.03	-.02	-.45	-.05
.13	-.12	-.06	.05	.08	.42	-.04	.02	-.05	-.44
-.12	.11	-.42	-.07	.03	-.04	.65	-.14	.02	-.02
.08	-.07	-.06	-.44	-.02	.02	-.14	.63	-.01	.02
-.06	.05	.03	-.02	-.45	-.05	.02	-.01	.63	-.13
.08	-.07	-.04	.03	-.05	-.44	-.02	.02	-.13	.63



$\rho_{5,9}$=0.385

$\rho_{1,2}$=0.991

The obervation 5 and 9 have not been weighted zero when run 'Checking residuals' programm.

-- From table(2)-table(4), we know that mislocalizing gross errors as (a),(b) and (c) can only be solved by running 'Checking residuals' programm. Besides, table(4) show that mislocalizing gross error still happened, because the observation 5 and 7 have not been weighted zero, when run 'Checking residuals' programm;

-- The most danger is so-called 'Hidden gross errors'. In this example, gross errors are vainsed in the adjusted results, and impossible to be detected from the residules. Fortunately, 'Hidden gross errors' is not frequency in practical adjustment.

Conclusions

The rules of increment relationship between the weight matrix P and matrix G would be a powerfull tool to study Robust adj., by which an approch to estimating the theoretical capacity of localizing gross errors by Robust adj. has been made and some mislocalizing gross errors in practical adjustment can be explained by the discussed limitation of localizing gross errors and overcome by so-called 'Checking residuals' programm.

Despite the fact that the limitation of localizing gross errors could not be overcome by weight function reducting weight for observations in general way. Nevertheless, the gross errors revealed in the Robust residuls will be larger remarkly in comparision with least squares residual , and the capacity of detecting (not locating) gross errors will, therefore, be improved. It should be required that all the element in matrix Qvv have to be calculated for further improving Robust adj.. From the point-review of the adjustment with large-scall equations, for instance photogrammmetric block adj., the computional effort will be increased appreciably. It is necessary further studies in the limitation of localizing gross errors, from which one could make intelligent programms by which the results of adjustment would possibility be free from the studied limitation.

Final Remarks

The investigations discussed above are based on the weight matrix P is united one . However, it is easy to be extented to that weight matrix P is doagonal or correlational one by helped the concept of so-called 'Equivalent residual' and 'Cofactor matrix of equivalent residual'. According to the appendix, we have

$$\bar{\bar{Q}}_{VV}=W \cdot G \cdot W^{-1}, \quad \bar{\bar{V}}=W \cdot V$$

Where $W^T \cdot W=P$, W=a square no-singular matrix, $\bar{\bar{Q}}vv$=cofactor matrix of equivalent residual, V=equivalent residual.

If P is diagonal matrix, then $\bar{\bar{Q}}vv$ and $\bar{\bar{V}}$ can be simplied as fellows

$$\bar{\bar{g}}_{ii}=g_{ii}, \quad \bar{\bar{g}}_{ij}=\sqrt{\frac{P_i}{P_j}} \cdot g_{ij}, \quad \bar{\bar{V}}_i=\sqrt{P_i} \cdot V_i \quad (i=1, m; \ j=1, m)$$

All the conclusions discussed above in this paper are suitable for $\bar{\bar{Q}}vv$ and $\bar{\bar{V}}$, because $\bar{\bar{Q}}vv$ and $\bar{\bar{V}}$ have the same characteristics as Qvv and V respectively. Where Qvv and V computed with P=I.

Appendix

Matrix P is symectric positive definite, and always decomposable into a product of a square no-singular matrix W and it's transpose ,i.e.,

$$P=W\cdot W^{T}$$

The vector in Eq.(1),Eq.(2) multiplied by matrix W from left side, we get

$$W\cdot V=-(I-W\cdot A((W\cdot A)^{T}(W\cdot A))^{-1}(W\cdot A)^{T}(W\cdot E) \qquad (A,1)$$

Let $\bar{\bar{V}}=W\cdot V,\ \bar{\bar{A}}=W\cdot A,\ \bar{\bar{E}}=W\cdot E$ (A, 2)

we have $\bar{\bar{V}}=-\bar{\bar{Q}}_{VV}\cdot\bar{\bar{E}}$ (A, 3)

Where $\bar{\bar{Q}}_{VV}=I-\bar{\bar{A}}(\bar{\bar{A}}^{T}\bar{\bar{A}})^{-1}\bar{\bar{A}}^{T}$ (A, 4)

and $Q_{\bar{\bar{E}}\bar{\bar{E}}}=W\cdot P^{-1}\cdot W^{T}=W\cdot W^{-1}(W^{T})^{-1}W^{T}=I$

furthermore , one get

$$\bar{\bar{V}}=-W\cdot G\cdot W^{-1}\,\bar{\bar{E}} \qquad (A,5)$$

$$\bar{\bar{Q}}_{VV}=W\cdot G\cdot W^{-1} \qquad (A,6)$$

Where $\bar{\bar{V}}$ and $\bar{\bar{Q}}vv$ is refered to 'Equivalent residual' and 'Cofactor matrix of equivalent residual' respectively. It goes without saying that $\bar{\bar{V}}$ and $\bar{\bar{Q}}vv$ have exactly the same characteristics as V and Qvv respectively, where V and Qvv are computed after least squares adjustment with P = I. Some problems of adjustment about observations with weight matrix containing unequally accurate and correlated elements can easy be solved by so-called 'Equivalent residual' and 'Cofactor matrix of equivalent residual'.

References

(1) Amer, F : Theoretional Reliability of Elementary Photogrammetric Procedure
ITC Journal, 1981-3

(2) Stefanovic, P.: Blunders and Least Squares
ITC Journal, 1978-1 pp 122-154

(3) Wang Renxiang : Studies on weight function for Robusted iterations
ACTA Geodetica et Cartographica Sinica Vol. 15 no.2 1986 pp 91-101

(4) Wener, H. : Automatic Gross Error Detection by Robust Estimates
ISPRS 1984 Commsion II 3b pp 1101-1108