AN UNCONVENTIONAL APPLICATION OF ANALYTICAL PLOTTERS TO ARCHITECTURAL PHOTOGRAMMETRY: PROJECTION, PLOTTING AND DIGITIZING ON NON PLANE SURFACES

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ABSTRACT

As known, the classic photogrammetric plotting is carried out in the form of a graphic (or digital) orthogonal projection of the points of the object on a X,Y reference plane.
In some cases it would be better to obtain the projection on a different surface: for instance, a bas-relief sculptured on a column would be better represented on a cylinder and subsequently expanded on a plane.
This result has been obtained using an analytical plotter. In particular:
- cylindrical coordinates are displayed and can be recorded;
- the plane expansion of the cylinder is drawn in real time;
- the floating mark moves automatically on a cylindrical surface if the operator does not use the Z control, and a "cylindrical contour line" can be plotted.
Similar procedures are applicable to spherical and conical reference surfaces.
A practical application is described: a portion of the exterior of the Baptistry in Pisa has been plotted in graphical and digital form, obtaining a direct projection on a cylindrical reference surface.

INTRODUCTION

Actually the photogrammetric surveying of an object (ground, monument, industrial machinery, small art object, etc.) gives a graphic orthogonal projection of the object points on an X,Y reference plane.
The result is then a drawing obtained on the coordinatograph. The third dimension Z is either recorded as "spot elevation" or, in some cases, can be represented by "contour lines".
Purely digital approaches, giving as output a file of recorded points, are still based on the same concepts and results.
This plane orthogonal projection is not always what we would like to obtain. For instance, let's examine the case of a bas-relief sculptured on a column, a painting on a cylindrical voult, a monument round-shaped: the orthogonal projection described above will give a deformed view of the real shapes, excepted for a narrow area corresponding to the tangent of the cylinder and the projection plane.

A different type of representation would be suitable.
It would be better to carry out the projection on a cylindrical reference surface, which is then expanded on a plane at the time of drawing.
This means that we would obtain a plotting in cylindrical coordinates, so that X and Y represent the coordinates of the points projected on the cylinder subsequently expanded on a

plane, and Z represents the radial displacement of the point from the reference cylinder.

The contour lines should describe "equal displacements" from the said reference surface.

In this report we shall illustrate the procedures used to obtain this sequence of operation on the DIGICART analytical stereoplotter. The main features are:
- cylindrical coordinates are displayed in real time;
- the plane expansion of the reference cylinder is drawn by the plotter;
- cylindrical contour lines can be followed. In other words, the Z elevation control of the instrument gives a variation of "cylindrical elevation": if the operator does not use this control but only the X,Y motion of the instrument carriage, the floating mark moves on a cylindrical surface, parallel to the reference one.

Similar procedures can be applied to any geometrically defined surface, as cone, sphere, ellipsoide, etc.

1. ANALYTICAL BASES FOR THE CYLINDRICAL PROJECTION

Let us examine the particular case of a convex right circular cylinder, having its axis coincident with the Y axis and radius R (see fig. 1). Extension to other surfaces (oblique cylinder, cone, sphere, etc.) can be easily obtained.
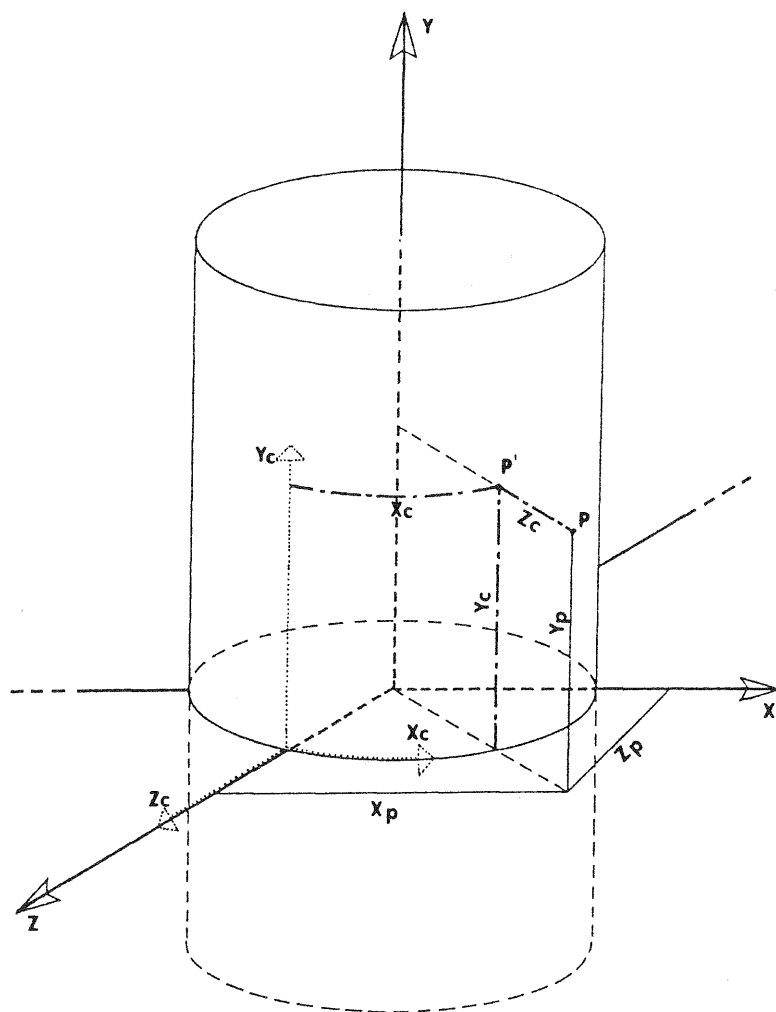

Fig. 1

If XP, YP, ZP are the traditional orthogonal coordinates of a generic point P, the relationships with the cylindrical coordinates XC, YC, ZC of the same point are the following:

$$XC = R * ATAN (XP/ZP)$$
$$YC = YP$$
$$ZC = SQRT (XP*XP + ZP*ZP) - R$$

We suppose that the hardware and software design of the DIGICART 20 analytical stereoplotter, manufactured by Officine Galileo, is well known. The first prototype was named DS (Digital Stereocartograph) and was presented at the ISP Congress in Ottawa 1972 (see (1),(2),(3)).
Just remember that only two differential movements DX and DY are controlled by the computer, while the main carriage of the instrument is directly moved by the operator.
The real-time program, modified for cylinder projection, must carry out the following elementary functions.

1.1 Single point survey, i.e. y-parallaxe elimination

A previous phase of simultaneous exterior orientation has given the orientation parameters of the two photograms, i.e.

XC1, YC1, ZC1
XC2, YC2, ZC2 ground coordinates of the perspective centers

B (φ1,ω1, K1) exterior orientation matrices of the two came-
C (φ2,ω2, K2) ras. We will call Bx, By, Bz and Cx, Cy, Cz the
3 columns of the matrices B and C respectively.

The problem is: given the plate coordinates x1, y1, x2, calculate y2 and eliminate the y-parallaxe by means of the DY motor.
The solution is found, for each real-time cycle, by computing:
- the internal direction tangents tx1, ty1, tx2
- the external d.t.s, obtained transforming the internal ones:

$$TX1 = \frac{|tx1 \quad ty1 \quad 1 \ |\cdot Bx}{|tx1 \quad ty1 \quad 1 \ |\cdot Bz}$$

$$TY1 = \frac{|tx1 \quad ty1 \quad 1 \ |\cdot By}{|tx1 \quad ty1 \quad 1 \ |\cdot Bz}$$

- the d.t. ty2, using the parallaxe equation
- and finally y2 = - p * ty2 (p is the principal distance).
No change in this standard procedure is required for the cylinder projection: obviously the exploration of the stereoscopic model is not influenced by the shape of the object present in the model.

1.2 Contouring

In traditional orthogonal projection, contouring is obtained as follows.
Problem: given   x1, y1
        pre-fixed   Qc = elevation of the contour line,
we want to calculate x2, y2 and determine the DX,DY movements.
Solution: for each real-time cycle, the following is calculated
- the d.t.s tx1, ty1, TX1, TY1 as in "single point mode"
- the difference DZ1 = Qc - ZC1
- the ground coordinates of the collimated point P:

$$XP = XC1 + TX1 * DZ1$$
$$YP = YC1 + TY1 * DZ1$$

- this ground point can be "projected" to the right plate;
  in fact:

$$TX2 = \frac{XP - XC2}{Qc - ZC2}$$

$$TY2 = \frac{YP - YC2}{Qc - ZC2}$$

and, calling c the transposed orientation matrix c^T and cx, cy, cz its columns, it results

$$x2 = -p \frac{|TX2 \quad TY2 \quad 1|\cdot cx}{|TX2 \quad TY2 \quad 1|\cdot cz}$$

$$y2 = -p \frac{|TX2 \quad TY2 \quad 1|\cdot cy}{|TX2 \quad TY2 \quad 1|\cdot cz}$$

All this refers to the conventional contours. For the cylindrical projection, we don't want a constant Qc, but a constant value for ZC, where ZC is the "cylindrical elevation", i.e. the distance between the point and the reference cylinder.
The ZP coordinate is a simple function of ZC (see fig. 2):
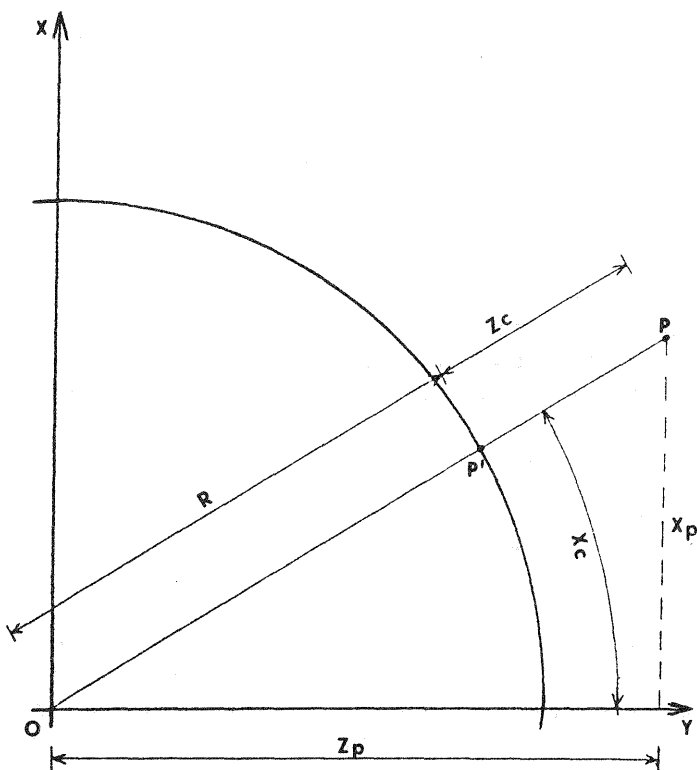
$$ZP = SQRT \; ((R + ZC)**2 - XP**2)$$

Fig. 2

Therefore the only change in the flow of calculations is that the value DZ1 = ZP-ZC1 cannot be constant along the "cylindrical contour", but must be computed at each real-time cycle, because ZP contains XP, and XP changes. For calculating DZ1, at each cycle we must utilize the value of XP as computed during the previous cycle.

1.3 Cylindrical ground coordinates
The cylindrical ground coordinates XC, YC, ZC can be easily computed, by using the formulas already seen in par. 1.
They are displayed on the screen and, of course, ZC will not change when the operator moves only the main carriage of the instrument, without moving the Z control.
The plotting table, controlled by XC, YC, will plot cylindrical contour lines.
When required, the cylindrical coordinates can be recorded automatically on the hard-disk, using the available sampling procedures (time, space, or space + angle modes) and the usual digital mapping techniques for coding points and lines.

2. DESCRIPTION OF THE DIGICY PACKAGE

A software package, called DIGICY (DIGIcart for CYlindrical projection), has been developed on the DIGICART system.

The sequence of the operations necessary for plotting a stereo-pair in a cylindrical projection is the following:

- at least 3 points of known coordinates (5 or more if a self calibration is made, in case of use of non metric cameras) are collimated stereoscopically by the operator;
- the program carries out a simultaneous exterior orientation (and self calibration, if required). The values of the orientation parameters and the residuals on the known points are printed and/or displayed;
- at this stage a traditional surveying, in the usual orthogonal system, could be performed. In particular the operator can explore the model, check the control points, verify the absence of parallaxe in the whole model space, etc. The y-parallaxe is automatically eliminated by the computer, the X,Y,Z coordinates are displayed on the screen, the plotting table is able to draw a conventional X,Y projection of the object.
- the operator collimates a few (at least 3) points on the surface that will be interpolated by the reference cylinder. The axis and radius of the cylinder are computed by a rigorous mean square routine. Results and residuals (i.e. radial discrepancies) are printed. In alternative, the X,Y coordinates of the axis and the radius can be typed on the keyboard.
- the procedure for cylindrical projection is then activated: the display shows the XC, YC, ZC cylindrical coordinates. In particular ZC represents the radial distance of the floating mark from the reference cylinder.
- when the operator moves the main carriage, the floating mark follows automatically a path parallel to the reference cylindrical surface. So he can follow a "cylindrical contour" in the same way as in the traditional survey. Moving the Z control, the ZC changes and the floating mark moves on a different cylindrical surface, thus allowing the usual collimation of any point in the model.

- the plotting table is controlled by the XC, YC coordinates. The consequence is that it draws directly the plane expansion of the projection on the reference cylinder.
- note that a function for interpolating horizontal circular sections is always available. It is very useful for the survey of columns: sections can be determined at different heights, thus allowing the correct projection of the diameters on the reference surface (even in the usual plane case).

3. FURTHER DEVELOPMENTS

Quite similar plotting procedures can be obtained for different types of reference surfaces. In practice, two other cases of simple geometrical surface have been taken into account: sphere and cone.

Surveying of spherical shapes is very common:not only for architectural applications, where dome voults are often spherical, but also in the small scale mapping from space photographs the shape of the ground cannot be considered referred to a plane, and a correct contouring requires an

instrument able of following "spherical contours".
When a sphere and, in general, a surface not expandable on a
plane is considered , the problem is a little more complex.
In fact, in order to obtain the final plane representation, the
reference surface must be projected in turn on an expandable
surface (cone or cylinder), using the well known traditional
methods for cartography.
We are developing a new package suitable for such applications,
named DIGISP (DIGIcart for SPherical projection), which carries
out the following functions:
- the floating mark moves on a surface parallel to the
  reference sphere, when the Z control is not activated;
- the geographic coordinates φ, λ are displayed. In addition,
  the correct ZS is shown, which represents the displacement
  from the reference surface;
- a cartographic projection, conical (e.g., Lambert) or
  cylindrical (e.g., U.T.M.), is calculated. The cartographic
  X,Y coordinates can be displayed and they are used for the
  plotting table control, in order to plot directly the
  rigorous map.
This can be carried out also for architectural applications,
where a conical projection of the sphere is suitable for
representing a strip around the tangent area between sphere and
cone, much larger then the narrow zone usually acceptable when
considering a tangent plane.

4. AN APPLICATION: THE PHOTOGRAMMETRIC SURVEY OF THE PISA BAPTISTRY

At the "International Conference and Workshop on analytical
instrumentation" - Phoenix Arizona - Nov. 87, the author
presented (4) a first application of the DIGICY procedure,
showing a stereomodel of a Coca Cola can.
The two peculiar aspects of the method appeared evident:
- the floating mark moves automatically on the cylindrical
  surface of the can, if the Z control is not utilized
- a plotting in scale 1/1 of the writing "Coca Cola" can be
  superimposed on the one drawn on the original Coke can.
Both results are impossible to obtain with a conventional
surveying and plotting, based on orthogonal projection.
A more interesting and stimulating application is in progress
in these last months on a very famous test area: the St John
Baptistry in Pisa.
The monument, built from 1152 to the end of the 14th century,
is a wonderful example of romanic architecture, with gothic
decorations in its upper part.
The plan is circular, with an exterior diameter of about 36
meters. The height is more than 55 m, including the St John
statue on the top of the dome.
The basic control network is an octogonal closed traverse,
measured with a final accuracy on the points of +- 5 mm. The
accuracy of the many control points, determined on the monument
without any target, is +- 10 mm.
The taking stations are at about 40 m from the exterior wall.
The camera used has been a JENOPTIC UMK, focal length 99 mm,
130 x 180 mm^2 format. The average scale is than about 1/400.
The axis of the camera has been oriented in a direction roughly
perpendicular to the base, with a tilt of +15 degrees. The base

length is about 10 m. The film used was a panchromatic b/w, 50 ASA speed.

The same images have been taken also with a HASSELBLAD 6x6 non metric camera, 50 mm f.l., 100 ASA film for color slides, image scale 1/800, for testing the self calibration procedures and determining the achievable accuracy.

At present, we can show the first results, obtained from one UMK stereomodel. The orientation phase gave discrepancies, on the 9 control points used, of 9 mm as mean value, maximum 16 mm in X,Y and 10 mm, max 19 in Z.

A plotting in scale 1/50 of a quarter of the cylinder has been obtained from this single model, using the DIGICY procedure. The drawing size, of about 80 x 60 cm^2, exceeds widely this A4 size, and cannot be reproduced in this paper. I will show some details by means of slides during the presentation. The whole drawing is shown at the exhibition in the GALILEO stand.

A digital recording of the same plotting has originated a graphic data base, managed by an OLIVETTI workstation with a package named GART, particularly suitable for this type of applications.

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