BLUNDER DETECTION IN CLOSE-RANGE PHOTOGRAMMETRY USING ROBUST ESTIMATION

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ABSTRACT

Both systematic and gross errors have been acclaimed as part of the problems facing phototriangulation today. Two independent algorithms for treating the two types of errors have been combined and developed to process the data in a single run. This paper investigates the effectiveness of robust algorithms in treating blunder-infested photogrammetric data set that requires photo-variant bundle adjustment solution.

1. INTRODUCTION

The totality of errors occurring in photogrammetric measurements and in any measurable observation for that matter, can be effectively grouped into three types: random, systematic and gross errors. Traditionally, gross errors have been detected and eliminated through efficient observational techniques and pre- and post-processing data screening. Systematic errors have been mathematically modelled and computationally accounted for; and more recently, additional parameters have been included in the observation equations to account for systematic errors.

The use of traditional least squares adjustment to process the data is an offshoot of the treatment of the random errors. It should be observed that a set of raw measurements undergoes these three processes sequentially before the desired parameters are obtained. Recently, advantages in computational savings have been reaped and improvement in accuracy has been achieved by combining the simultaneous treatment of systematic and random errors into one process through the use of additional parameters. Still, gross error treatment remains a pre- and a post-adjustment process.

Robust estimation methods are capable of simultaneous parameter estimation and outlier elimination during the estimation process. If our observation equations contain additional parameters to model the effect of systematic errors, then the use of iteratively reweighted least squares with an appropriately chosen M-estimation ρ -function gives us a tool for simultaneous treatment of all errors.

At present, research is continuing at the University of New Brunswick in the development of robust algorithm and software for the simultaneous treatment of all errors in a bundle adjustment. Preliminary results are encouraging and are presented in this paper.

2. ROBUST ESTIMATION METHODS

The poor performance of least squares estimators in the presence of outliers or of minor deviations from the assumptions of the error distribution, led statisticians to search for an alternative method to least squares. This led to the development of robust estimation methods (see [Tukey, 1960; Huber, 1972]). Studies were initially concentrated on the location case, culminating in the famous Princeton Robustness Study [Andrews et al., 1972]. The satisfactory result obtained for robust estimation of location parameters encouraged the natural

generalization of the technique to the more complicated regression case and to other more structured data such as surveying data.

To get an idea of how robust estimation can simultaneously eliminate and decline outliers in the parameter estimation process, a simple example is illustrated for the location parameter case in Kubik and Merchant [1986]. There, the measurement sample 10, 11, 11, 12, 100 has one obvious spurious value, 100. The least squares estimate of the population mean from which this sample is assumed to be drawn is 28.18, whereas a robust estimation method produced the actual mean of 11.0 which would have been obtained with the least squares method in two steps after eliminating the value 100. A similar example is given for the robust regression case in Andrews [1974] using the famous stack loss data. Four outliers were detected in four steps with standard statistical tests, whereas Andrews' sine wave robust estimator detected all four blunders in one step.

Thus, robust estimation procedures can conceptually be grouped into two major parts: (i) robust estimation of location parameters and (ii) robust regression. The first part has direct applications for repeated single variable measurements and can be utilized in specialised applications at the input stage of a bundle adjustment software in order to eliminate simple blunders such as those due to misidentification of points. On the other hand, robust regression has direct application at the adjustment stage and is the method considered in this paper. Huber [1964] classifies robust estimation methods into three categories: (i) M-estimation methods, which are related to the maximum likelihood estimation method, (ii) L-estimation methods, which are linear combinations of the ordered statistics and (iii) R-estimation methods in the location problem have shown that the M-estimation is easier, more flexible and has better statistical properties than L- and R-estimation methods. Moreover, only the M-estimation method has a clear and flexible generalization to the regression case. Hence, it is the only method considered in this study.

2.1 Robust M-Estimation Methods

The classical least squares method minimizes the weighted sum of squares of the residuals given by:

$$\rho(\mathbf{v}) = \mathbf{\hat{v}}^{\mathrm{T}} \mathbf{P} \mathbf{\hat{v}} = \min$$
 (1)

where P is the weight matrix of the observations.

The ρ function in equation (1) can be made more general by replacing the weighted sum of squares of the residuals by a less rapidly increasing function [Huber, 1977]. The objective then boils down to minimizing

$$\sum \rho(v_i / \sigma_i)$$

or equivalently solving the system of equations

$$\sum \psi \left(v_i / \sigma_j \right) \frac{\delta f}{\delta x_j} = 0, \ j = 1, 2, \dots, u$$
(2)

in which the previous objection function

$$\rho(\mathbf{v}) = \widehat{\mathbf{v}}^{\mathrm{T}} \mathbf{P} \widehat{\mathbf{v}}$$

for the least squares method is a subclass. To make the estimator scale invariant (see Huber 1973; Hogg, 1979), ψ is set equal to ρ' in equation (2) and the expression is divided by σ_v , the standard deviation of v. The ψ functions and their associated tuning constants further divide the M-estimators into subclasses. Thus, there are Huber's M-estimator, Hampel's M-estimator, Andrews' M-estimator, etc. Also Huber's M-estimator with a tuning constant equal to infinity gives the usual least squares estimator. A collection of presently available ψ functions is given in Faig and Owolabi [1988a].

Quite apart from the possibility of nonlinearity of the functional model f, most ψ functions are available in nonlinear form, requiring that the solution of equation (2) be iterative in nature. Of the three approaches available to solve equation (2) [Holland and Welsch, 1977], the iteratively reweighted least squares method is the most favoured and widely used, because of its flexibility. Furthermore, it only requires computing a weight function as a function of the scaled residuals, that is, $w(v/\sigma_v) = \psi(v/\sigma_v)/(v/\sigma_v)$; and then using an existing weighted least squares algorithm.

3. PHOTO-VARIANT SOLUTION FOR CLOSE-RANGE DATA

The systematic errors affecting photogrammetric measurements include film deformation, lens distortion, refraction and other anomalous distortions. Usually, these distortions are modelled and their values obtained from calibration reports. Recent advances in data processing techniques favour the idea of including the distortions as additional parameters in the solution, which are recovered simultaneously with the exterior orientation elements and the object space coordinates.

Since metric cameras have stable interior geometry over a period of time, the additional parameters are usually carried as invariant from photo to photo. A more sophisticated data processing algorithm allows for the distortions to vary from photo to photo. This is known as photo-variant solution [Moniwa, 1981]. Thus, the suitability of non-metric camera for close-range data acquisition is enhanced [Karara and Faig, 1980]. Significant contributions in the modelling of the systematic errors, the classification and performance of various additional parameter models in photo-variant and photo-invariant bundle adjustment are reviewed in Faig and Owolabi [1988b].

4. SIMULTANEOUS SOLUTION FOR ALL ERRORS

Although we do not expect to have data sets as large in close-range applications as in aerial triangulation, the frustration of having to sequentially process data infested with blunders, suggests an alternative "automated" procedure. Already, close-range data acquired with a non-metric camera requires the use of a photo-variant bundle solution. Invariably, the measurements are usually infested with blunders. It then sounds reasonable to take advantage of robust estimation methods in the estimation of the desired parameters. A robustified bundle adjustment procedure has been developed along this direction. It has already been utilised in comparing the effectiveness of using ordinary least squares plus data snooping and Andrews' sine wave robust M-estimator (see Faig and Owolabi, [1988a]). In that study, the robust method revealed the exact amount of the imbedded blunder in the residuals, while the least squares method distributed the blunder to other unperturbed points. In this paper, the study is generalised to include photo-variant self-calibration and comparison of several robust M-estimators for processing close-range data.

5. EMPIRICAL STUDY

Using the procedure and software described in Woolnough [1973], data for four photographs were generated with close-range characteristics.

In the test performed by Faig and Owolabi [1988b] to compare several additional parameters, it was shown that parameter sets that model physical causes or model effects by the use of trigonometric terms performed better than parameter sets that model effects with ordinary polynomials. For this reason, the parameter set by Kilpela [1980] was used for this study.

Two control point patterns (high and low) were used for comparison purposes. Three sizes of blunders: 3 um (small size), 10 um (medium size) and 10 mm (large size) gross errors were added to one coordinate of an image point, and each robust M-estimator tabulated in Faig and Owolabi [1988a] was used to process the data in turn while the photo-variant self-calibration mode was activated in the adjustment.

6. DISCUSSION AND CONCLUSION

Tables 1 and 4, 2 and 5, and 3 and 6 show the root mean square errors at check points when 3 um, 10 um and 10 mm blunders were introduced using several M-estimators and two different control point patterns, respectively. First, a reference adjustment was carried out using least squares with additional parameters and blunder-free data. The result is tagged LSSA in the tables. Next, the blunder was introduced and the data was adjusted without additional parameters and then with additional parameters. The results are tagged LSAB and LSAC respectively. Thereafter, ten robust estimation methods were used to process the data.

It can be seen that the effect of small-sized blunders on the adjusted coordinates is deceptive. The results appear to be good (see LSAB in Tables 1, 2, 4 and 5); however there was improvement in accuracy when robust estimators were used. Moreover, the imbedded blunders were revealed in the residuals (see Tables 7 and 8). On the other hand, large-sized blunders tend to deterioriate the adjusted coordinates completely (see LSAB in Tables 3 and 6). Usually one would have to do some statistical tests to detect the errors, eliminate them and then perform the adjustment again. However, it can be seen from Tables 3 and 6 that the blunder was deprived from participating in the solution, thereby improving the accuracy of the solution obtained earlier for LSAB and LSAC, provided good geometry is still maintained. The blunder was also revealed in the residual (see Table 9). The trade-off for this improvement is the exhorbitant rise in computational time.

There was no robust method that displayed any consistency from one error size to another and from one control point pattern to another. The Hinich's robust method performed best in plan and height with few iterations when a small-sized blunder was introduced with the high density control point pattern. Huber's method has the worst result in planimetric and height accuracy, although it has fewer iterations. With large-sized blunders, Cauchy's robust method performed best with the low density control point pattern, while Fair's method produced the worst result. Nevertheless, the common characteristic for all the robust methods is their ability to discriminate against blunders by giving them low or zero weight in the solution.

It is remarkable that Andrews', Hinich's, Danish, Huber and Least sum estimators consistently worked with fewer iterations. On economic considerations therefore, they may be favoured for processing large photogrammetric data sets.

TABLE 4 : RMSE Values for Low Density Control with Sum Blunder Using Several M-estimators

NAME	XY (MM)	Z (MM)	ITR	TIME(SEC)
LSAA	0.041	0.217	з	10.289
LSAB	0.059	0.391	3	5.905
LSAC	0.047	0.317	з	10.336
ANDREWS	0.053	0.352	11	35.988
TUKEY	0.041	0.282	46	140.657
HINICH	0.048	0.311	6	20.125
CAUCHY	0.044	0.312	19	60.077
WELSCH	0.041	0.313	53	163.942
HUBER	0.053	0.362	8	26. 379
LOGISTIC	0.051	0.339	15	48.124
FAIR	0.045	0.307	31	97.421
DANISH	0.050	0.320	10	32.613
L-SUM	0.050	0,315	8	26,4 73

TABLE 5 : RMSE Values for Low Density Control with loum Blunder Using Several M-estimators

NAME	XY(MM)	Z(MM)	ITR	TIME(SEC)
LSAA	0.041	0.217	3	10.289
LSAB	0.047	0,302	3	5.842
LSAC	0.043	0.296	з	10.488
ANDREWS	0,053	0.352	10	33.019
TUKEY	0.040	0.260	36	110.875
HINICH	0.043	0.271	6	20.204
CAUCHY	0.044	0.295	20	63.234
WELSCH	0.040	0.279	20	63.106
HUBER	0.053	0.362	8	26. 389
LOGISTIC	0.043	0.270	11	35.891
FAIR	0.044	0.295	27	84.931
DANISH	0.043	0.282	16	41.258
L-SUM	0.048	0.306	12	38.841
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 TABLE 2 : RMSE values for High Density Control

 with 10um Blunder Using Several M-estimators

 NAME
 XY(MM)

 Z(MM)
 ITR TIME(SEC)

TABLE 1 : RMSE Values for High Density Control With Sum Blunder Using Several M-estimators

0.210

0.398

0.293

0.327

0.253

0.250

0.261

0.288

0.340

0.252

0.294

0.256

0.281

0.040

0.054

0.048

0.048

0.043

0.039

0.039

0.043

0.049

0.042

0.046

0.042

0.045

NAME

LSAA

LSAB

LSAC

ANDREWS

TUKEY

HINICH

CAUCHY

WELSCH

LOGISTIC

HUBER

FAIR

DANISH

LSAA

LSAB

LSAC

ANDREWS

TUKEY

HINICH

CAUCHY

WELSCH

LOGISTIC

HUBER

FAIR

DANISH

L-SUM

L-SUM

XY(MM) Z(MM) ITR TIME(SEC)

3

3

з

8

31

6

24

21

6

13

29

10

8

10.430

5 932

10 546

26.819

93.888

20.538

76.752

67.344

20.521

42.549

92.588

33.105

26.807

LSAA 0.040 0.210 3 10.430 LSAB 0.050 0.295 3 5.951 LSAC 0.048 0.286 3 10.465 ANDREWS 0.048 0.327 8 25.819 TUKEY 0.040 0.254 37 116.177 HINICH 0.039 0.231 8 26.733 CAUCHY 0.045 0.285 32 101.316 WELSCH 0.044 0.305 29 92.005 HUBER 0.043 0.340 6 20.305 LOGISTIC 0.041 0.249 9 30.057 FAIR 0.043 0.264 14 45.445 DANISH 0.042 0.236 12 35.346 L-SUM 0.044 0.279 9 29.818	NAME	A : (1017			
LSAA 0.040 0.110 0 10.040 LSAB 0.050 0.295 3 5.951 LSAC 0.048 0.286 3 10.465 ANDREWS 0.048 0.327 8 25.819 TUKEY 0.040 0.254 37 116.177 HINICH 0.039 0.231 8 26.733 CAUCHY 0.045 0.285 32 101.316 WELSCH 0.044 0.305 29 92.005 HUBER 0.049 0.340 6 20.305 LOGISTIC 0.041 0.249 9 30.057 FAIR 0.043 0.264 14 45.445 DANISH 0.042 0.236 12 35.346 L-SUM 0.044 0.279 9 29.818		0.040	0 210	а	10.430
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LSAC 0.048 0.286 3 10.465 ANDREWS 0.048 0.327 8 25.819 TUKEY 0.040 0.254 37 116.177 HINICH 0.039 0.231 8 25.733 CAUCHY 0.045 0.285 32 101.316 WELSCH 0.044 0.305 29 92.005 HUBER 0.049 0.340 6 20.305 LOGISTIC 0.041 0.249 P 30.057 FAIR 0.043 0.264 14 45.445 DANISH 0.042 0.236 12 35.346 L-SUM 0.044 0.279 9 29.818	LSAB	0.050	0.295	э	0.801
ANDREWS 0.048 0.327 8 25.819 TUKEY 0.040 0.254 37 116.177 HINICH 0.039 0.231 8 25.733 CAUCHY 0.045 0.285 32 101.316 WELSCH 0.044 0.305 29 92.005 HUBER 0.049 0.340 6 20.305 LOGISTIC 0.041 0.249 P 30.057 FAIR 0.043 0.264 14 45.445 DANISH 0.042 0.236 12 35.346 L-SUM 0.044 0.279 9 29.818	LSAC	0.048	0.286	з	10,465
TUKEY 0.040 0.254 37 116.177 HINICH 0.039 0.231 B 26.733 CAUCHY 0.045 0.285 32 101.316 WELSCH 0.044 0.305 29 92.005 HUBER 0.049 0.340 6 20.305 LOGISTIC 0.041 0.249 P 30.057 FAIR 0.043 0.264 14 45.445 DANISH 0.044 0.279 9 29.818	ANDREWS	0.048	0.327	8	25.819
HINICH 0.039 0.231 8 26.733 CAUCHY 0.045 0.285 32 101.316 WELSCH 0.044 0.305 29 92.005 HUBER 0.049 0.340 6 20.305 LOGISTIC 0.041 0.249 P 30.057 FAIR 0.043 0.264 14 45.445 DANISH 0.042 0.236 12 35.346 L-SUM 0.044 0.279 9 29.818	TUKEY	0.040	0.254	37	116.177
CAUCHY 0.045 0.285 32 101.316 WELSCH 0.044 0.305 29 92.005 HUBER 0.049 0.340 6 20.305 LOGISTIC 0.041 0.249 P 30.057 FAIR 0.043 0.264 14 45.445 DANISH 0.042 0.236 12 35.346 L-SUM 0.044 0.279 9 29.818	HINICH	0.039	0.231	8	26.733
WELSCH 0.044 0.305 29 92.005 HUBER 0.049 0.340 6 20.305 LOGISTIC 0.041 0.249 9 30.057 FAIR 0.043 0.264 14 45.445 DANISH 0.042 0.236 12 35.346 L-SUM 0.044 0.279 9 29.818	CAUCHY	0.045	0.285	32	101.316
HUBER 0.049 0.340 6 20.305 LOGISTIC 0.041 0.249 9 30.057 FAIR 0.043 0.264 14 45.445 DANISH 0.042 0.236 12 35.346 L-SUM 0.044 0.279 9 29.818	WELSCH	0.044	0.305	29	92.005
LDGISTIC 0.041 0.249 P 30.057 FAIR 0.043 0.264 14 45.445 DANISH 0.042 0.236 12 35.346 L-SUM 0.044 0.279 P 29.818	HUBER	0.049	0,340	6	20.305
FAIR 0.043 0.264 14 45.445 DANISH 0.042 0.236 12 35.346 L-SUM 0.044 0.279 9 29.818	LOGISTIC	0.041	0.249	ę	30.057
DANISH 0.042 0.236 12 35.346 L-SUM 0.044 0.279 9 29.818	FAIR	0.043	0.264	14	45.445
L-SUM 0.044 0.279 9 29.818	DANISH	0.042	0.236	12	35.346
	L-SUM	0.044	0.279	9	29.818

 TABLE 3 : RMSE Values for High Density Control

 with 10mm Blunder Using Several M-estimators

 NAME
 XY(MM)
 Z(MM)
 ITR
 TIME(SEC)

0.210

0.335

0.555

0.424

0.327

0.339

0.359

0.329

0.319

0.327

0.300

3

22

23

18

46

10

22

27

11

14

20

14

12

10.430

36.878

67.168

57.915

142.818

32.784

69.861

85.450

35.980

45.095

63.570

45.063

39.184

0.040

0.048

0.048

0.058

0.050

0.050

0.050

0.048

0.051

0.051

0.046

16.336 145.586

18.939 145.048

•	TABLE	6	: R	MSE	Valu	es	for	Low	Dens	sity	Contro	ł.
	with		10mm	BI	under	Us	ing	Seve	ral	M-es	stimato	٢S

F 3					
	NAME	XY(MM)	Z (MM)	ITR	TIME(SEC)
	LSAA	0.041	0.217	3	10.289
	LSAB	18.215	99,439	22	35.412
	LSAC	16.086	96.024	35	100.655
	ANDREWS	0.054	0.357	24	75.304
	TUKEY	0.073	0.573	46	142.713
	HINICH	0.056	0.382	15	47.945
	CAUCHY	0.041	0.217	25	78.755
	WELSCH	0.051	0.410	51	157.5 37
	HUBER	0.054	0.381	11	35.827
	LOGISTIC	0.049	0.217	17	54.1 25
	FAIR	0.059	0.581	22	69.424
	DANISH	0.056	0.400	15	46.219
	L-SUM	0.051	0.321	13	41.939

Note:	LSAA	 Lesst	square		solution	with	additional
		parame	ters,	no	blunder	intro	oduced

LSAB = Least squares solution without additional parameters but blunder introduced

LSAC = Least squares solution with additional parameters and blunder introduced

SAMPLE OUTPUT

Table 7: Robust Estimator for Outlier Detection Using 3 µm Blunder

 	PHOTO ID #	POINT	I VX I (MM)	WEIGHT	I DUT- I LIER	I VY I (MM)	WEIGHT I	OUT- LIER
ī		1 2	-0.0005	0.000		0.0008	0.0001	1
i	11	3	0.0000	1.000	l	-0.0001	1.0001	1
i	11	4	0.0001	1.000	1	-0.0004	0.0001	1
i	11	1 5	0.0000	1.000	1	0.0001	1.0001	1
i	11	6	-0.0001	1.000	ł	0.0003	1.0001	ł
i	11	1 7	0.0000	1.000	1	0.0007	0.0001	1
i	11	1 8	0.0004	0.000	l	0.0002	1.0001	1
i	11	9	0.0002	1.000		0.0007	0.0001	I
i	11	1 12	0.0000	1.000	1	0.0004	0.0001	1
i	11	13	0.0000	1.000		0.0006	0.0001	1
i	11	14	0.0000	1.000		0.0007	0.0001	1
i	11	1 15	0.0000	1.000	1	I _0.0001	1.000	1
i	11	1 16	0.0000	1.000	I	0.0002	1 1.0001	1
i	11	1 17	0.0000	1.000	1	0.0000	1.0001	1
i	11	1 18	0.0000	1.000	t	0.0001	1.000	1
1	11	1 19	-0.0001	1.000	I	0.0001	1.0001	1
i	11	22	-0.0002	1.000	I	-0.0001	1.000	1
Ì	11	23	0.0000	1.000	1	0.0002	1.0001	1
i	11	1 24	0.0000	1.000	1	0.0003	1 1.000	1
i	11	1 25	-0.0001	1.000	I	0.0000	1 1.0001	1
i	11	26	0.0000	1.000	1	0.0000	1.000	1
Ì	11	27	0.0000	1.000	I	1 -0.0002	1.000	, L
i	11	28	0.0000	1.000	1	0.0001	1.000	I
İ	11	1 29	0.0002	1.000	I	0.0004	0.0001	l I
Ì	11	32	0.0000	1.000	1	-0.0002	1.000	11
i	11	33	0.0000	1.000	I	0.0000	1.000	1
Ì	11	34	0.0000	1.000	1	0.0000	1.000	1
1	11	1 35	0.0000	1.000	1	0.0000	1.000	1
1	11	36	I 0.0000 I	1.000	I	0.0000	1.000	1
1	11	1 37	0.0000	1.000	I	-0.0007	0.0001	
I	11	1 38	0.0000	1.000	1	0.0000	1 1.000	
1	11	39	0.0001	1.000	t	-0.0001	1.000	l
1	11	41	-0.0001	1.000	1	0.0000	1.000	1
1	11	42	0.0001	1.000	1	0.0005	0.000	1
1	11	43	0.0001	1.000	I	0.0005	0.000	
1	11	44	-0.0002	1.000	l	0.0000	1 1.000	I
1	11	45	-0.0003	0.000	I	-0.0001	1.000	
1	11	46	-0.0001	1.000	I	-0.0002	1.000	ļ
1	11	1 47	0.0029	0.000	*	0.0001	1.000	
1	11	1 48	0.0002	1.000	1	0.0000	1.000	ļ
1	11	49	0.0000	1.000	l	0.0002	1.000	
I	11	51	I 0.0001	1.000	1	0.0000	1.000	. !
1	11	1 52	0.0000	1.000	I	0.0002	1.000	
1	11	1 53	_0.0002	1.000	I	0.0001	1.000	ļ
1	11	1 54	0.0003	1.000	I	-0.0002	1.0001	1

SAMPLE OUTPUT

Table 8: Robust Estimator for Outliner Detection Using 10 µm Blunder

I PHOTO I ID #	POINT ID #	I VX I (MM)	WEIGHT	OUT- I LIER I	(MM) A A	WEIGHT	OUT- LIER
1 11	1 2	-0.0004	0.772		0.0007	0.3891	1
11	1 3	0.0000	1.000		-0.0001	0.9681	l I
1 11	1 4	0.0002	0.958		-0.0004	0.791	
I 11	1 5	0.0000	1.000	l 1	0.0000	0.996	1
11	1 6	0.0000	0.999		0.0003	0.882	1
1 11	1 7	0.0000	1.000		0.0004	0.814	
11	1 8	0.0005	0.707		0.0003	0.910	
11	1 9	0.0002	0.936	1	0.0007	0.381	
11	1 12	0.0001	0.990		0.0003	0.866	
11	1 13	0.0000	0.999		0.0003	0.832	
11	14	0.0000	0.999		0.0004	0.805	
11	1 15	0.0000	1.000		-0.0001	0.984	
11	1 16	0.0000	1.000	1	0.0001	0.971	
11	17	0.0000	1.000		0.0000	1.000	
11	1 18	0.0000	1.000		0.0001	0.984	
11	1 19	-0.0001	0.983		0.0001	0.908	
1 11	1 22	-0.0001	0.985		-0.0001		
11	23	0.0000	0.999	1	0.0002	0.957	
1 11	1 24	0.0000	0.998		0.0003	1 0.900	
1 11	1 25	0.0000	1.000		0.0000	1 1.000	
1 11	1 26	0.0000	1.000		0.0000	0.999	i. E
1 11	1 2/	0.0000	0.997		-0.0002		
1 11	1 28	0.0000	0.999		0.0001	0.909	
1 11	1 29	0.0003	0.907		0.0004		l, 1
1 11	1 32	0.0000	1.000	1	-0.0002	1 0.937	i 1
1 11	33	0.0000	1.000		0.0000		
1 11	1 34	0.0000	1.000		0.0000		
1 11	1 35		1.000		0.0000		
1 11	1 30		1.000	1	0.0000		
1 11	1 37		0.977		-0.0004		
1 11	1 30				0.0000	0.993	
	1 39		0.900		-0.0001	0 995	i i
1 11	1 40		0.970		0 0004	0.825	
1 11	1 42		0.997		0.0003	0.873	i i
1 11	1 43		0.034		0.0002	0.926	
1 11	1 77		0.971		-0.0002	0.956	i i
1 11	1 40		0.077		-0.0002	0,906	i i
1 11	1 47		0.000		0.0000	0.998	i i
1 11	1 40		0.847		0.0000	0.998	i i
1 11	1 40		0.047		0 0002	0.956	i i
1 11	1 51	0.0000	0.997		0.0000	1.000	1
1 11	1 52	0.0001	0.992		0.0002	0.944	I I
1 11	1 52	-0.0002	0.948		0.0001	0.980	i i
1 11	1 54	0.0003	0.885		-0.0002	0.954	1
,	, v , v ,						00 Eur (101 vil) vije soje soje vilo

SAMPLE OUTPUT

Talbe 9: Robust Estimator for Outlier DectectionUsing 10 mm Blunder

PHOTO ID #	POINT	I VX I (MM)	WEIGHT	I OUT- I VY I LIER I (MM)	I WEIGHT I	OUT- I LIER I
11	2	-0.0005	0.365	1 0.0007	0.2371	!
11	3	0.0000	1.000	_0.0002	1.0001	1
11	4	0.0002	1.000	1 -0.0004	0.3981	1
11	5	0.0000	1.000	1 0.0000	1.000	1
11	6	0.0000	1.000	1 0.0003	1 1.000	1
11	7	0.0000	1.000	1 0.0005	0.318	1
11	8	0.0005	0.336	1 0.0002	1 1.000	1
11	9	0.0002	1.000	1 0.0007	0.230	
11	12	0.0001	1.000	1 0.0003	1.000	ļ
11	13	0.0000	1.000	1 0.0005	0.351	
11	14	0.0000	1.000	1 0.0005	0.3201	ļ
11	15	0.0000	1.000	_0.0001	1 1.000	1
11	16	0.0000	1.000	I 0.0001	1 1.000	I
11	17	0.0000	1.000	I 0.0000	1 1.000	1
11	18	0.0000	1.000	0.0001	1 1.000	1
11	19	-0.0001	1.000	0.0001	1 1.000	1
11	22	1 -0.0001	1.000	_0.0001	1 1.000	1
11 (23	0.0000	1.000	0.0002	1.000	
11	24	0.0000	1.000	I 0.0003	1.000	
11	25	0.0000	1.000	0.0000	1 1.000	1
11	26	0.0000	1.000	1 0.0000	1.000	
11	27	0.0000	1.000	_0.0002	1.000	
11	28	0.0000	1.000	10 I C C 0.0001 -	1.000	
11	29	0.0003	1.000	1. 0.0004	0.419	
11 1	32	0.0000	1.000	0.0002	1 1.000	1
11	33	0.0000	1.000	0.0000	1.000	
11	34	0.0000	1.000	0.0000	1.000	
11	35	0.0000	1.000	0.0000	1.000	
11	36	0.0000	1.000	1 0.0000	1.0001	1
11 (37	-0.0001	1.000	0 0005	0.302	
11	38	0.0000	1.000	0.0000	1.000	
11	39	0.0001	1.000	_0.0001	1.000	
11	41	-0.0001	1.000	0.0000	1.000	
11 1	42	0.0001	1.000	1 0.0004	0.391	
11	43	0.0004	0.441	0.0003	1.000	1
11	44	-0.0001	1.000		1.000	
11	45	-0.0003	1.000	I = 0.0002	1.000	
11	46	-0.0001	1.000	1 -0.0002	1.000	
11	47	-10.0000	0.000	0.0001	1.000	
11	48	0.0003	1.000	-0.0001	1.000	
11 1	49	0.0001	1.000	1 0.0002	1.000	1
11	51	0.0001	1.000	1 0.0000	1.000	
11	52	0.0000	1.000	0.0002	1.000	
11	53	-0.0002	1.000	0.0001	1.000	
11	54	0.0004	0.462	_0.0002	1 1.000	1

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