PHOTOGRAMMETRY AS AN AID FOR THE ANALYSIS OF THE PERSPECTIVE IN MONUMENTAL BUILDINGS

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Deformating buildings by purpose in order to obtain perspective effects is an old and well-known practice. Artists of different times have looked for solutions able to deceive the observer about the perception of the reality. The deformation of the natural proportion depends on the point of view from which the object is observed and from its location. A further difference is obtained according to whether the object has three or two dimensions and, in this last case, on which surface it is lying.

A typical case can be a statue on a column or on a building, where a big difference exists between the elevation of the object and that one of the observer, the same is true for frescos on curved surfaces.

Laws of perspective are known: once the point of view is fixed, and being known the object and its location, one can design the deformation more suitable to give the illusion of natural proportions.

It happens often, to whom is involved in art, to meet cases where the object is no more located, in relation to the observer, in the original position. For example, a statue might have been moved, as in the famous cases of the Horses of St. Marco (Venice) or the equestrian statue of Marco Aurelio (Rome).

Even if no move has taken place, sometime can be useful for historical purpose to get a knowledge of the perspective intention of the artist: i.e. to determine the point of view a-posteriori.

These two problems should be faced by different methodology, eventhough they share the same theory of perspective. The second case above mentioned, which is frequent in building decoration and in some peculiar architectures, has been analyzed and exemplified here below. Subsequently some pieces of information are given about the methodology that has been accepted to solve the first case too.

The present example deals with the analysis of the perspective, made by Donato Bramante (Urbino 1444 - Rome 1514) for the apse of the church of St. Maria by St. Satiro in Milan. This analysis has been carried out by algorithms applied to photogrammetric measurements.

Takings have been made by Wild P31 f10, with a focalization ring for 7 m, by Agfa Avipan 100 panchromatic plates. The control for the absolute orientation of the photograms was obtained, by multiple intersection, on artificial signals.

The plotting has been performed by Planicomp C100 and the result is shown in fig. 1. The plotting has been particularly difficult for the apse, because of its characteristic faulse and receding perspective which is the reason of its renown: it is also called "faulse choir".

On this scenographic perspective, which simulates in about one meter depth a much longer barrel vault (fig. 2), a lot has been written and hypothesized. Even leaving out the various reconstructions made by the historians of art, the subject is so stimulating that it has been thought it right to go deep into the question.

As a matter of fact, photogrammetry allows to have at one's disposal the spatial coordinates of a large number of points, all determined with high precision. Starting from these points, by using the rules of perspective under certain hypotheses of condition, it is possible to determine the location of the point of view.

A perspective is determined by a point of view (PV) at a certain elevation on a reference plane, which is called ground plane (PT). Another plane, called perspective picture (QP), perpendicular to the ground plane, shelters the perspective construction. The point of view and the ground plane generally correspond, in architectures, to the height of an observer's eye and to the trampling plane. The central perspective is peculiar to the "faulse choir": the infinite points which make it up belong to different perspective frames, built up as to be parallel to each other. The main point of the perspective, normal projection of the point of view onto the perspective picture, contemporarily belongs to different perspective planes and to the straight line passing through the point of view and normal to these ones.
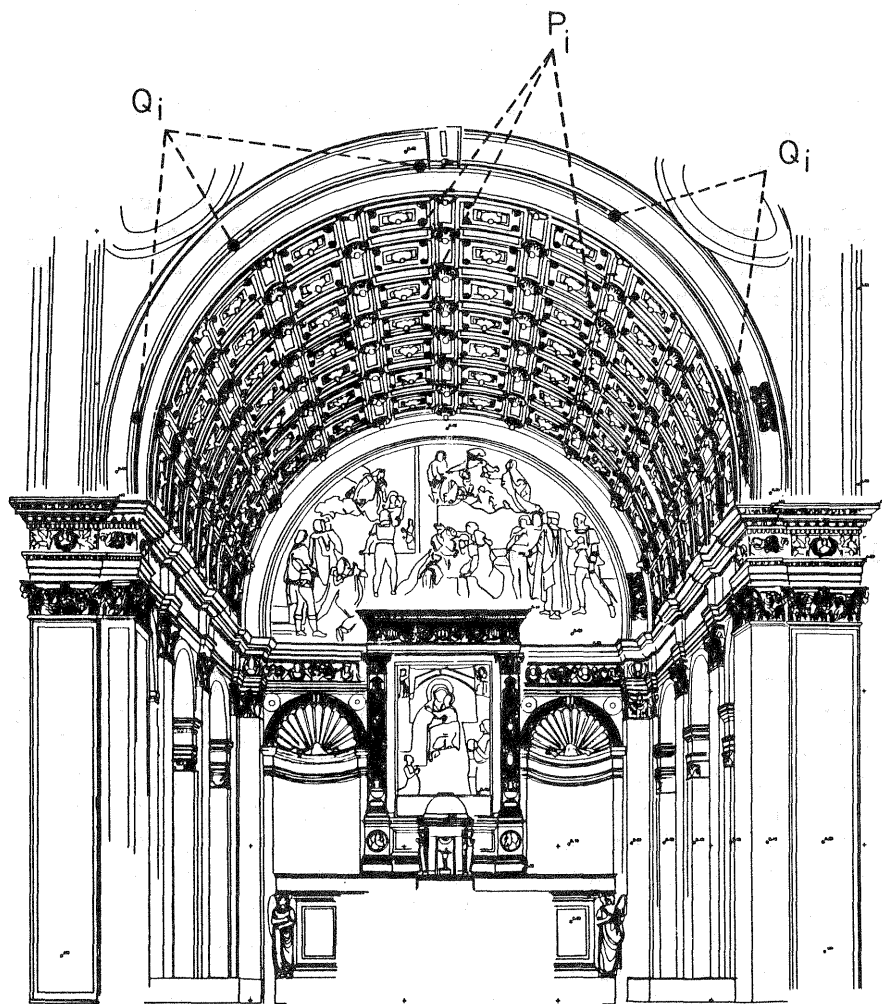
Fig. 1 - St. Maria by St. Satiro, Milan, Italy: plotting of the faulse choir (original 1:20), with the types of point P_i and Q_i measured.

Denoting by α, β, γ the coordinates of a generic point belonging to an imaginary non-perspective choir (fig. 2), by α_o, β_o, H the unknown coordinates of the point of view, by x,y,z the coordinates plotted on the "faulse choir" and referred to a cartesian system having the x axis parallel to the

perspective planes and the y axis which defines the depth, the relations which connect together all these points can be written. They are:

$$x = \frac{(y-\beta_0)(\alpha-\alpha_0)}{\beta-\beta_0} + \alpha_0$$

$$z = \frac{(y-\beta_0)(\gamma-H)}{\beta-\beta_0} + H$$

(1)

where the unknowns are the coordinates α_0, β_0, H of the point of view and the coordinates α, β and γ of the points of the hypothetic choir. The system determinated in this way is always underdetermined, since for each plotted point the two equations (1) (collinearity equations) can be written in the six above mentioned unknowns, three of which, those ones concerning the point of view that gives rise to the projective sheaf, are commun to all points. It is therefore necessary to make hypotheses about the geometry of the hypothetic choir, on the base of evident simmetries and historical investigations. As shown in fig. 2, the hypothesis adopted has been that one of supposing that the perspective of the apse corresponded to a theorical continuation of the first arch to define a barrel vault.

On the "faulse choir" 35 points have been plotted, divided in 5 sections of 7 points each, having simmetries and correspondences (fig. 1). With reference to equations (1), it is therefore possible to write 35x2=70 equations in 35x3+3=108 unknowns. Just using simmetries and correspondences one can see that it is possible to consider α and γ as constants for every section; for every plane on which are lying the first 5 points, the second 5 points, and so on, one can consider constant β.

Under this hypothesis, one can write 4x7=28 equations of type

$$\beta_i = \beta_j$$

(2)

and 2x(6x5)=60 equations of type

$$\alpha_h = \alpha_k \quad , \quad \gamma_h = \gamma_k$$

(3)

Having moved from underdetermined to overdetermined, the problem seemes to be solved; as a matter of fact, being the perspective frame on different planes very close to each other, it is true that the normal system has a normal defined positive matrix, but in conditions of numerical instability. That means that if the faulse choir was a picture, the relations generating the normal matrix would render it non-positive and therefore non-solvable; the fact that the faulse choir has a depth allows in theory the solution, but such a solution, just for the bad geometrical conditioning, is unstable and consequently unreliable. Some additional relations are therefore needed, inferred from reasonable hypotheses.

Equations (1), (2) and (3), when linearized, take the following shape:

$$a\delta(\alpha) + b\delta(\alpha_0) + c\delta(\beta) + d\delta(\beta_0) + l_1 = V$$

$$e\delta(\gamma) + f\delta(H) + g\delta(\beta) + h\delta(\beta_0) + l_2 = V$$

$$\delta(\alpha_h) - \delta(\alpha_k) + \alpha_h - \alpha_k = V$$

(4)

$$\delta(\beta_i) - \delta(\beta_j) + \beta_i - \beta_j = V$$

$$\delta(\gamma_h) - \delta(\gamma_k) + \gamma_h - \gamma_k = V$$

Solution of system (4) involves some considerations on the relations which lie between the unknowns. A bad determination of the single point $P(x,y,z)$ on the "faulse choir" raises a positioning error on the point $P'(\alpha,\beta,\gamma)$ on the hypothetic faulse choir (fig. 2), leaving practically unchanged the position of the point of view in that it is the emanation of the projective sheaf.
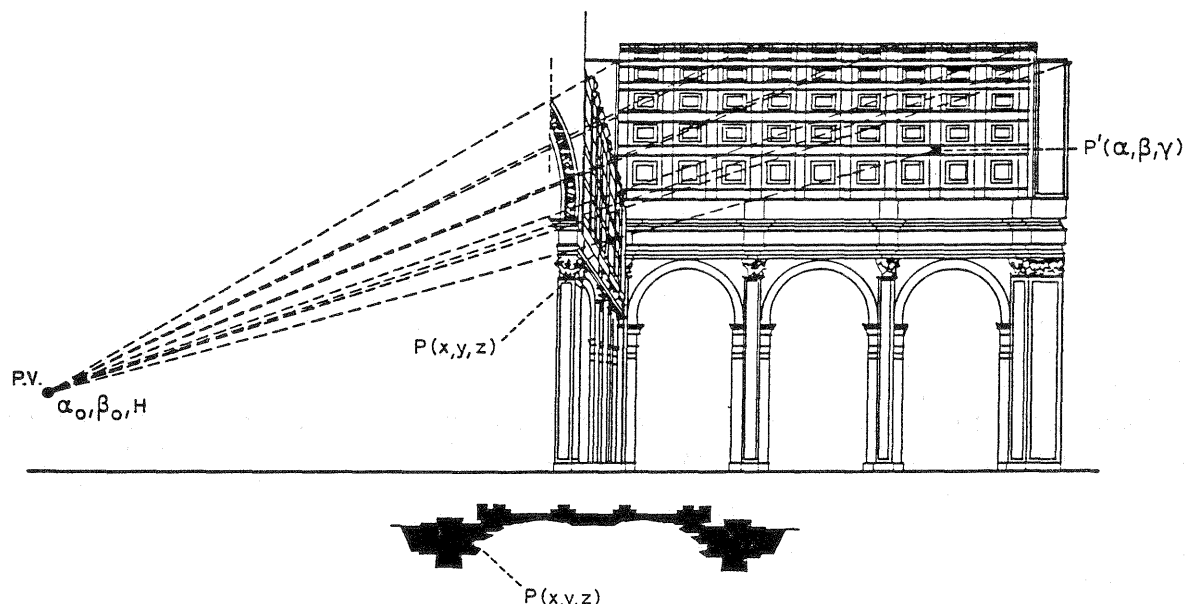

Fig. 2 - Point of view PV (α_0,β_0,H), **faulse choir** (front and section) with points $P(x,y,z)$, hypothetic choir with points $P'(\alpha,\beta,\tau)$.

If equations (2) and (3) were of type

$$\beta_i = k_i \quad ; \quad \alpha_h = k_h \quad ; \quad \gamma_h = k_h \tag{5}$$

the number of unknowns would decrease (in our case to only 20), but the photogrammetric determination errors of the $P(x,y,z)$ would affect the determination of the point of view $PV(\alpha_0,\beta_0,H)$.
New binding equations of type

$$\beta_i - \beta_{i+1} = \beta_{i+1} - \beta_{i+2} \tag{6}$$

could also been written, and the correspondent ones for the other unknowns α and γ, equations which would impose the equidistance between points belonging to the same section or to the same plane.
This last hypothesis seems to be logical; the previous one is logical and has the advantage to leave to the problem more degrees of freedom. The first one is more free and can allow, with its results,an a-posteriori check of the hypotheses which the other two are based on.
For this reason the system (4) has been solved by imposing a constraint which could serve to stabilize numerically the solution, which is unstable for the above mentioned reasons. This constraint does not bring back to the conditions of (5) and (6), but it just imposes that the 5 points Q_i on the

first crown of the "faulse choir" be the same of the first crown of the hypothetic choir (fig. 1).
The result of the solution gives the three coordinates of the point of view PV:

$$\alpha_0 = 5.058 \pm 0.026 \text{ m}$$

$$\beta_0 = -16.496 \pm 0.432 \text{ m}$$

$$H = 1.155 \pm 0.071 \text{ m}$$

The point of view turns out to be exactly in the middle of the faulse choir and therefore on the axis of the central aisle, in correspondence with the third span and at the height of a sitting observer.
This result, among other things, gives on the hypothetic choir points $P'(\alpha, \beta, \gamma)$ such as to make panels looking like squared and of the same size of those ones of the aisles of the church.

The approach to the determination of the point of view of three-dimensional objects that have been moved from their original location can only be solved by attempts, because of the lack of a reliable and objective comparison with the original. For example, we have the equestrial statue of Marco Aurelio, but we no more have the true emperor and his horse! In this case one cannot use the logical hypotheses on which the mentioned example of the Bramante's perspective are based.
The subject can be dealt with in more than one way: we show here the one that we consider the most suitable to solve the problem with reference to the typical case of the statue; it consists in the philosophy of looking, among all the possible ones, for the perspective representation which better identifies itself with the reality of the object (statue).
One needs, first of all, the digital knowledge of the statue with its three dimensions in a unique reference system. This is a classical problem of numerical photogrammetry. The density of the points is a function of the quality one wants to get in the representations, and whence in the result.
The library programs which deal with the representation of three-dimensional objects in the space, generally do not apply to the graphical representation of a statue.
The bad interpretability by mathematical algorithms of the complex shape of the object, excludes all programs for three-dimensional representations which utilize mathematically expressible surfaces. On the other side, the programs which deal with digital surfaces, no more based on mathematical shapes but generally on grid interpolations, get into a crisis when the object is a close surface.
From what above, the idea came to approach the matter by building up a specific program aiming at solving the problem of the perspective representation.
The first decision concerns the choice of the type of representation to be obtained: either by elementary areas of polygons or by contour lines. Such a choice is disarming for the solution procedures of the problem.
For personal conviction and for objective reasons too, the traditional representation has been chosen by contour lines.
The following procedure has been adopted:
- choice of the point of view;
- choice of the direction of view which can coincide either with a point of the object or to be out of it;
- rigid rototranslation of the coordinates of the points of the photogramme-tric survey in order to make the direction of view and the y axis of the new system coincident;
- scanning of the survey's data bank by a couple of planes, at a fixed distance from each other, normal to the y axis. Drawing out of all points

lying on this slice of space;
- data management of these points in order to locate them on the average plane equidistant from those ones defining the interval;
- polynomial interpolation to obtain the contour lines which correspond to the y of the average plane;
- comparison among the contour lines to eliminate those ones which are hidden from the point of view;
- graphical representation by contour lines.

This program allows to get, as the point of view changes, subsequent sections which are represented by curves of surface. By repeated attempts one looks for that point of view for which the representation achieves those natural proportions the artist wanted to preserve to the observer.
The traditional representation by contour lines is nothing but that one which is described when the point of view is to infinity.

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