GEOMETRICAL CALIBRATION AND ONLINE STABILITY CONTROL OF PHOTOGRAMMETRIC SOLID STATE CAMERA STATION

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### Abstract

The paper describes a method for the calibration of close range photogrammetric stations consisting of solid state cameras. Both the calibration of the digitizing of the analog video signal and the determination of the systematic errors, the inner and the outer orientation of each camera of the station are discussed.

The difficulty attached in the latter calibration arises from the problem of overparametrization. The problem is attacked by applying the singular-value decomposition and by considering the principal components.

During the data capture the stability of the calibration must be continuously checked. A simple method using the same calibration software as used in the setup is outlined.

## 1. Introduction

The real-time applications of digital photogrammetry have in recent years gained much interest among the photogrammetric community. This is mainly due to the imposing hardware developments and to the promising market predictions of such disciplines as Machine Vision. Many applications of Machine Vision are used in the manufacturing industry in tasks such as inspection, mensuration and assembly. Vision plays an important role in inspection and metrology, where 2D and 3D data are to be sensed. Some recognition is usually involved in these applications. On the contrary, the recognition problem is minor in assembly. Whereas determination of the position and orientation of parts with very high accuracy is what the vision community wishes to bring to automated assembly /21/.

It is just these kinds of applications demanding high accuracies where the traditional photogrammetric expertise is of use. All the real-time systems developed so far in the photogrammetric community are indeed concentrated on pointwise measurements that usually aim at accurate object coordinates of discrete, easily identifiable points (see e.g. /5/,/11/). At the current stage, the problems of object recognition and interpretation are left to other disciplines. Our task is to bring all the knowledge from photogrammetric network design to the Vision community. This should be regarded as a challenge and it is our responsibility, too.

In this paper a methodology is described for a reliable and accurate geometric calibration of a "photogrammetric" multicamera station consisting of solid state sensors. We hope it will be brought to the Vision markets. The whole calibration system is partially and will be totally installed in a commercial system called Mapvision /11/.

The problem encountered in the calibration usually arises from the fact that these stations are normally calibrated on the job via self-calibration. Inevitably, each camera has it's own inner orientation and it's own additional parameters for describing the systematic deformations. Altogether this produces an overparametrized system. Additionally, in most cases the form of the object is considered most important. To get the best possible precision in this sense the network must be free, so neglecting an absolute coordinate system. This brings up the datum problem. Our solution to all these network problems will be described in chapter 3.

One big problem when using the current image sensors is in the digitization of the analog video signal given as an output by these cameras /3/. Calibration of this source and especially of linejitter is discussed in chapter 2.

Usually after the set up (calibration) of the total station, a long term coordinate measuring phase is entered. In chapter 4 we finally shortly handle the possibilities for checking the stability of the system in online.

### 2. Some aspects about the digitization of the analog video signal

The usage of the analog signal for transferring images from the digital cameras to computers has been found to produce significant problems with respect to the stability of the coordinate system /1,3/. These problems according Dähler /3/ are the following: definition of the horizontal sync pulse, warm up effects, resampling using a phase locked loop (PLL) clock, aliasing because of resampling, response to filters used in analog signal path, and The biggest problem is caused by the warming up (2tailing. 5 pixels), but this can be tolerated by switching the system on at least two hours before starting any measurement. When this is done, previous tests show that in spite of all instabilities, the inner precision of interpolative measurements of symmetric objects is of the order of 0.01 to 0.03 pixels /12/. A big problem arises from inaccuracies in the detection of the horizontal sync pulse /3/, that is of the origo of each line. In the current hardware this is done with an accuracy of 0.25 to 0.5 Also, the usage of a PLL clock causes some fluctuation pixels. within a row /3/. Together these are called the linejittering problem.

A testing procedure was carried out to find out how well this

phenomenon can be calibrated. In this test a line 40 pixels wide in image scale was continuously photographed. In every row the x-position of the line was computed as the gray value weighted center point along the x-axis. Then a least squares line was fitted on these observations. The residuals of this fit were considered as the linejitter. A similar calibration was already used in /20/.To minimize the effect of radial distortion of the camera lens, the line was manually positioned as near as possible to the center of the frame. For one calibration the possible to the center of the frame. For one calibration the line was continuously detected over a period of 20 hours. The jittering was taken to be the mean value of the whole period. This calibration was adopted in a later analysis. The hardware components used in the tests were standard, low-cost equipment.

The analysis of the calibration phase shows some positive and negative aspects with respect to the possibility of calibrating the jittering. The hope for successive adoption comes from the fact that the correlation coefficient between all pairs of the samples range from 0.75 to 0.80. The negative thing is the size of instabilities show standard errors from 0.04 to 0.08 pixels in the position of each individual line.



Figure 1.

- Typical examples of detected jittering, one sample from each of four types.
  - A) Raw data, correlation between samples 0.78.
  - B) Result from the adoption of the calibrated jittering, correlation between samples 0.02.
  - C) Data smoothed by an operator of size 5, correlation between samples 0.94.
  - D) Result when utilizing "smoothed curve", correlation between samples 0.05.

The clues mentioned above were confirmed when the calibration was utilized later. It was done by compensating the mean curve of the calibration phase from all the measurements. These tests consisted of two sets of 97 samples each taken during a period of two days, two months after the calibration phase. Two different cameras (Hitachi/Pulnix) but the same frame grabber were used. The dependence between these calibrated samples was now very low showing a mean correlation coefficient of 0.02 in both So the instabilities of the line origos seem to be quite sets. The noise level measured as the standard deviation of random. It should be noticed that both all linefits was 0.04 pixels. cameras showed the same results. This could be expected, because the digitization was done by the same frame grabber. It should be emphasized that the calibration phase was carried out only by the Hitachi camera.

In practice, the noise level of 0.04 pixels is a little bit lower because of the interpolative way of detecting objects. This was experimented by recomputing the calibration and the adoption phases by smoothing the successive rows by an operator of size 5. The noise level of the calibrated measurements was 0.02 pixels and the correlation coefficient between the samples raised only to a value of 0.05. On the other hand, the correlation between the samples of the calibration phase raised to 0.94. This gives some optimism concerning the possibility to limit the jittering to a reasonable level. By restricting the tailing effect, too, one can suspect fairly good results until a purely digital transfer from the cameras is commonly available.

## 3. Geometrical calibration of the station

As mentioned, there are two awkward problems which must be jointly solved in such a network we are interested in. These are the datum problem of the usually free network and the problem of overparametrization because of the camera variant self-calibrating parameters. The other problems of network design; the configuration, weighting and densification problems, are not discussed in this paper. The same refers also to the reliability considerations, in this context to the capability of the network to detect gross errors and it's sensitivity to them. The most important problems for a solid state camera station network are the configuration problem and the reliability aspects. One can easily find standard approaches to these (e.g. /8/), if the joint problem is satisfactorily solved.

We have adopted the singular value decomposition as the basic algorithm to attack the problems. That is why it is briefly presented in chapter 3.1. In chapter 3.2 we describe how the problem of overparametrization can be dealt with by analyzing the principal components which are easily computable after a singular value decomposition. In chapter 3.3 the datum problem is combined to the system and in chapter 3.4 the existing software is summarized.

#### 3.1 The singular value decomposition

### 3.1.1 Definition and remarks

Let **A** be an mxn matrix of rank k. Then there exist orthogonal matrices U (mxm) and V (nxn) so that

 $\mathbf{U}^{\mathrm{T}} \mathbf{A} \mathbf{V} = \mathbf{S} = \begin{vmatrix} \mathbf{k} & \mathbf{n} - \mathbf{k} \\ \mathbf{S}_{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{vmatrix} \begin{cases} \mathbf{k} \\ \mathbf{k} \\ \mathbf{m} - \mathbf{k} \end{cases}$ 

where  $\mathbf{S}_1$  is a diagonal matrix with ordered diagonal elements, that is  $\mathbf{s}_1 \geq \mathbf{s}_2 \geq \ldots \geq \mathbf{s}_k > 0$ . The decomposition  $\mathbf{A} = \mathbf{U}\mathbf{S}\mathbf{V}^T$  is called the singular value decomposition of  $\mathbf{A}$ . The numbers  $\mathbf{s}_i$ are singular values and the first k columns of matrices  $\mathbf{U}$  and  $\mathbf{V}$ are the left and right singular vectors of  $\mathbf{A}$  /19/.

Next some useful remarks about the singular value decomposition are given /15/.

- 1) Singular value decomposition is not unique, but the singular values are.
- 2) The eigenvalue decomposition of matrix  $\mathbf{A}^T \mathbf{A}$  is  $\mathbf{VSU}^T \mathbf{USV}^T = \mathbf{VS}^2 \mathbf{V}^T$ . This means that the positive eigenvalues of  $\mathbf{A}^T \mathbf{A}$  are  $\mathbf{s}_i^2$  (i=1,...,k) and the corresponding eigenvectors are the columns of V.
- 3) The pseudoinverse  $\mathbf{A}^{\star}$  of  $\mathbf{A}$  can be defined via singular value decomposition :

$$\mathbf{A}^{+} = \mathbf{V} \begin{vmatrix} \mathbf{S}_{1}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{vmatrix} \mathbf{U}^{\mathrm{T}}$$

4) The condition number that measures the degree of ill-conditioning of A is max(s, )/min(s, )

#### 3.1.2 Computing the singular value decomposition

The singular value decomposition can be computed by many different algorithms, which all have the common properties of iterativeness, quite heavy computational load and complexity. The most powerful of these algorithms is the QR-method, which is also installed here (details can be found in /23/).

The QR algorithm consists of two basic steps. In the first step the mxn matrix A is reduced by orthogonal transformations H and K to a bidiagonal matrix B, that is

$$\mathbf{H}^{\mathrm{T}} \mathbf{A} \mathbf{K} = \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} \mathbf{n}$$

This reduction is usually done by Housholder transformations. In the second phase the matrix **B** is reduced iteratively by the orthogonal Givens transformations to a diagonal matrix. If  $\bar{H}$  and  $\bar{K}$  are the orthogonal matrices formed in this step, the final singular value decomposition is

$$\mathbf{A} = (\mathbf{H}\overline{\mathbf{H}}) \begin{vmatrix} \mathbf{S}_1 \\ \mathbf{O} \end{vmatrix} (\mathbf{K}\overline{\mathbf{K}}) = \mathbf{U}\mathbf{S}\mathbf{V}^{\mathrm{T}}$$

It can be proved that the iteration really converges /23/. It

can also be accelerated with a proper origo shift of the matrix  $\mathbf{B}^{\mathrm{T}}\mathbf{B}$  /19/. From a software point of view it must be noted that the matrix U need not to be explicitly stored, but the matrix V is usually needed for the analysis of the system.

#### 3.1.3 Solving least squares problems

Let 1 be a m-vector of observations, **A** mxn design matrix and  $\mathbf{A} = \mathbf{USV}^{T}$  the singular value decomposition of **A**. It can be proved /15/ that

by partitioning  $\mathbf{U} = \begin{vmatrix} \mathbf{k} & \mathbf{n} - \mathbf{k} & \mathbf{m} - \mathbf{n} \\ \mathbf{U}_1 & \mathbf{U}_2 & \mathbf{U}_3 \end{vmatrix}$ ,  $\mathbf{S} = \begin{vmatrix} \mathbf{k} & \mathbf{n} - \mathbf{k} \\ \mathbf{S}_1 & \mathbf{0} \end{vmatrix}$   $\mathbf{k}$ ,  $\mathbf{V} = \begin{vmatrix} \mathbf{k} & \mathbf{n} - \mathbf{k} \\ \mathbf{V}_1 & \mathbf{V}_2 \end{vmatrix}$ and by defining  $\mathbf{U}^T \mathbf{1} = \begin{vmatrix} \mathbf{U}_1^T \mathbf{1} \\ \mathbf{U}_2^T \mathbf{1} \\ \mathbf{U}_3^T \mathbf{1} \end{vmatrix}$  and  $\mathbf{Y} = \begin{vmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \end{vmatrix} = \mathbf{V}^T \mathbf{x} = \begin{vmatrix} \mathbf{V}_1^T \mathbf{x} \\ \mathbf{V}_2^T \mathbf{x} \end{vmatrix}$   $\mathbf{k}$ 

a unique solution to the least squares problem  $\hat{e}^T \hat{e}$ =minimum (where  $\hat{e} = A\hat{x}-1$ ) is found by

 $\hat{\mathbf{x}} = \mathbf{V}_1 \mathbf{y}_1$  (choosing  $\mathbf{y}_2 = 0$ ).

The variance covariance matrix of this solution is given by

$$S_{\hat{x}} = V_1 S_{\hat{y}_1} V_1^T$$
, where  $S_{\hat{y}_1} = Diag(1/s_1^2, 1/s_2^2, \dots, 1/s_k^2)$ .

In addition to the least squares condition this solution fulfills the minimum trace condition  $(Tr(S_{\hat{x}})=min)$  and the condition that  $|\hat{x}| = minimum$ . So, it is a so called minimum norm solution /15/. As stated earlier it is a pseudoinverse solution, too.

With respect to the quality of the estimation results and of controlling the quality, the singular value decomposition can be considered to be superb compared to other alternatives because of the following /15/:

- 1) In constructing the solution only orthogonal matrices are used. Because of this it is a numerically very stable solution.
- In connection to the solution one gets numeric information about the stability of the least squares problem in question (remark 4 in chapter 3.1.1).
- 3) A problem with an unknown rank deficiency can be easily solved. The ill-conditioned problems can be stabilized be setting the singular values smaller than a predefined threshold to zero (rejecting principal components). This needs only some extra computations.
- 4) There are versatile means for the statistical analysis of the results.

### 3.2 Solving overparametrized systems

Self-calibration is nowadays a widely adopted method in analytical photogrammetry. However, there are some problems in the use of it and even some theoretically open questions /13/. The main reason for these problems comes from overparametrizing the system. This means that our system often turns out to be ill-

conditioned and even small disturbances in the observations are strongly affecting the variances of the estimated parameters. In worst cases the parameters are undeterminable, meaning a singular normal equation system.

The problem of self-calibration has been lively discussed in late 70's and in this decade. Several suggestions have been made to overcome the stability problem:

- Orthogonalizing the additional parameters a priori /4,10/. This gives some help, but neglects the other parameters and lacks generality.
- 2) Rejection of parameters correlating with each other. This method also neglects other parameters, but can be extended. Two occurring heuristics that are the selection of a threshold for rejections and the decision which parameter(s) to reject. The threshold selection is difficult, because it is dependent on the network geometry /14/. There might even be a parameter that is not so strongly dependent on the other ones but is still poorly determinable.
- 3) Computing the boundary values of additional parameters and rejecting the instable ones in the sense of Baarda's reliability theory /7/. The detectability of a parameter is tested by the concept of inner reliability, which unfortunately neglects the dependencies between different parameters. This certainly causes difficulties and wrong conclusions (see /14/).
- 4) Huang has reported on the usage of biased estimation for attacking the problem /13/. He also argues about the theoretical aspects of self-calibration. The method of treating additional parameters as fictious observations is shown to confirm the principle of biased estimation. The results are promising, but there is still some lack of generality, because the succesfull adoption of the Ridge estimator presented requires experience and special procedures.

In our case with photovariant parameters and in situations where the best configuration is not always possible to arrange, the ill-conditioned system is often unavoidable. That is why special care has been focused on the problem. Our solution to the After the standard elimination of problem is straightforward. object coordinates and formation of the reduced normal equations, a singular value decomposition is applied. All principal components which are instable, having eigenvalue (see 3.1.1/remark 2) smaller than a given threshold, are set to zero and to be deterministic. The computation of the corresponding  $\hat{x}$ -vector and the variance-covariance matrix  $S_{\hat{x}}$  of the photoparameters is The standard methods can then be now easy (see chapter 3.1.3). applied to solve the object coordinates. In /22/ the same kind of strategy was used, but only the eigenvalues corresponding to the additional parameters were considered.

There are now still two problems to be solved. First one is due to the formation of the reduced normals. This reduces the numerical stability compared to a singular value decomposition of the original design matrix. The only thing to do is to use longer word lengths in computer programs. Secondly, one has to select a threshold for the detection of undeterminable parameters. This is a more serious drawback. We have made a lot of simulation runs to tackle the problem. There are two methods which gave good results. In the first one the boundary value is computed for the effect on the image of the principal component to be tested. The second one looks for the condition number of the remaining part of the reduced normal equations. There seems to be a functional dependence between these two measures. As the second one is much simpler to compute we have installed it to the software.

The statistically insignificant, but stable principal components are not rejected from the system, because in our simulation runs they had no effect, or a small positive effect, to the result.

#### 3.3 The datum problem

A lot of theoretical and practical work has been done with the so called zero-order design, where the optimal reference system for object coordinates is tried to be chosen /6, 8/. The simplest way to define a datum is to fix seven coordinates in a 3D photogrammetric network. A network where this kind of minimum constraint is chosen is called a minimum constraint network. Every different choice of the minimal constraints leads to a different solution, but the form of the object stays the same. On the contrary, the stochastic properties of the solutions can vary a lot. The pseudoinverse solution is regarded as optimal for the problem, because it fullfils the condition of minimum variance  $(TR(S_*)=TR(N^*)=minimum)$ .

As already mentioned, the pseudoinverse solution can be achieved by the singular value decomposition. Anyhow, a solution based on inner constraints is algorithmically most favourable (see e.g. /9/). There we solve a constrained least squares problem

$$v = Ax-1$$
; P and  $G^T x = 0$ .

Here G is a nx7 matrix whose columns are linearly independent and AG=0. The easiest way to find G, which fullfils these conditions is by geometric interpretation. In /9/ such a G matrix for photogrammetric 3D networks is given.

Usually in photogrammetric applications we are interested only in the object space. To achieve an optimal solution only with respect to the object coordinates, the datum must be defined with their help only. In the constrained least squares solution one must simply set the rows of **G** corresponding to the secondary parameters to zero. In case of the pseudoinverse solution, one can eliminate all the secondary parameters from the system.

In our case where we have to attack both the self-calibration and the datum problem, the more general method of pseudoinverse solution is preferable for finding the datum. As stated earlier, the datum problem of any rank deficiency is easily solvable by the singular value decomposition.

The object coordinates are eliminated because of the testing of the photoparameters and that arises a new problem. So, the datum is not optimal with respect to the object space. We cannot recompute the network with a new datum minimizing the trace of  $\mathbf{S_{\hat{x}}}$  corresponding to object coordinates, because of problems aroused by the additional parameters. The only alternative is to use the S-transformation, which can transform any minimum constraint solution to another. If we have two solutions  $\mathbf{x}_1$  and  $\mathbf{x}_2$  corresponding to two different datums, then /16/

 $x_2 = Sx_1$  and  $Qx_2 = SQx_1S^T$ ;  $S = I - C(BC)^{-1}B$ .

The rows of B (7xn) must be independent from each other and from the rows of A. The columns of C (nx7) must span A. For example the G matrix presented above fullfils these conditions.

The use of the S-transformation saves also some computations, because the (BC) matrix to be inverted is only a 7x7 matrix. In our case whole  $Q_{x_1}$  must be formed for the computation of  $Q_{x_2}$ , but fortunately not stored completely.

### 3.4 Installed software

A software package for bundle adjustment, which functions with the principles mentioned in this chapter has been developed in the Helsinki University of Technology and will be installed into a commercial system called Mapvision /11/. The problem of overparametrization has been under heavy simulation, and the presented method seems to work nicely. The S-transformation is currently being installed.

From a practical point of view one can solve both forced and free networks consisting of image and distance observations. Also all the parameters can be treated as free, stochastic or deterministic variables. As an extended model one can choose from four sets presented by Ebner /4/, Brown /2/, Kilpelä /18/ and Juhl /17/. Of course the coordinates of the principal point is unknown for each camera.

## 4. Potentials for online stability control

This chapter gives some ideas how to control the stability of the station against changes in the calibration. Usually we measure single points for determining the form of the object. This is done by space intersection with help of the results from the calibration. Now, if something happens either to the inner or outer orientation of the camera, it can be detected from the standard deviation of the intersected object coordinates. When this happens, it would be nice automatically to detect the reason for it or even to recalibrate the malfunctioning part of the system. This needs a new set of observations.

With digital cameras one can quickly measure some 10 to 50 points for the checking. Anyhow, often the conditions are such that they produce a much weaker geometry than is needed for the total calibration. So, we have to find simpler methods for detecting the error or even for correcting it. One possibility would be to analyze the residuals of the intersection, but as we know from the reliability studies, is not always straightforward to draw conclusions from them . A safer, and computationally

quite a quick method can be found.

We know that the malfunctioning part of the system is the inner or outer orientation of some of the cameras. So, it is simple just to free one of these components after another and to fix all the other ones, and to test if the new parameters differ significantly from the old ones. This can be done based on a statistical test, if we have stored the variance-covariance matrix of the calibration. In case of eight cameras this would mean the computation of eight outer and eight inner orientations.

When the malfunctioning camera is detected, there are two possibilities:

- 1) If a multicamera station is in use, it may be possible to drop the detected camera from the intersection, to make a notice and to continue the operation.
- notice and to continue the operation. 2) The new parameters of the camera can be utilized. This would violate only a little the free network.

# 5. Concluding remarks

This paper has dealt with the geometric calibration of a multicamera station in digital close-range photogrammetry. It has been emphasized that the internal instabilities of the cameras, like the jittering problem arising from the use of the analog video signal can be limited to a reasonable level. Also, a method for tackling the joint problem of overparametrization and zero-order design during the calibration of the station, is presented. The kernel of the method is the utilization of the singular value decomposition. Finally, some ideas for the automated stability control of the operatively running stations are given.

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