THE APPLICATION OF THE PANORAMIC PHOTOGRAPH BY FISH EYE LENS IN CLOSE RANGE PHOTOGRAMMETRY

Li Guangwen Li Weiqian
Department of Surveying, Tongji University
Siping Road, Shanghai
China
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ABSTRACT

This paper states the principles and experiments of close range photogrammetry by an ordinary camera with an additional fish eyes lens.

The subject matter described in this paper covers the geometric relation and the mathematical transformations between the panoramic and planimetric photograph, the variety on the panoramic photographic scale, the analytic calculation on the panoramic photographic formed by fish eyes, the theory by means of horizontal and vertical line segments as controls and trials process and so on.

1. INTRODUCTION

The fish eye lens, which was designed for scientific research and once called as all-weather lens, was originally used to use at astronomical photography and observation. Nowadays, the fish eye lens is widely used by professional photographer in order to achieve specific effects. It is considered that the lines taken by the fish eye lens are too complex to use for measuring. We sounded out to perform a full of imaginative application in the end of 1985. We used it to take photo in studying geometric scienic space, in direct sense scienic space and architecture on a large scale so as to use for the design of planning in city. Therefore, we proceded with theoretical investigations and repeated tests, thus having made encouraging progress.

The principal advantage about the fish eyes panoramic photography is that it can take huge object at very close range, even less than 1 ~ 2^m and they also have the advantages of being large area of coverage, high resolution and of large image capacity. This paper is mainly concerned with such problems as the basic formula, scaling variety, line controlling theory and analytic process about fish eyes panoramic photograph. According to the proposed theory, we have taken through the tests and verification of the 3-D control network and performed systematically experiments. The results obtained from the tests show that the theory put forward in this paper and the program drawn up is correct.

2. THE GEOMETRIC RELATION OF FISH EYES PANORAMIC PHOTOGRAPH

The photographs used in photogrammetry usually are central projection of the taken object on the horizontal level (the panoramic photograph or called as cylindrical photograph is the central projection on the cylindrical surface).

But the photograph taken by fish eye lens is the central projection on the spherical surface. So the photo scales at each parts of the fish eye panoramic photograph are different from each other.

2-1 The Relational Formula between the Fish Eyes Panoramic Photograph and Phanimetric Photograph

The fish eye panoramic photograph is the central projection on the spherical surface, taking the objective focal length f as radius (Fig. 1). Planimetric photo is then the central projection on the level. Assume that the fish eye panoramic photo and horizontal photo have the communal projection center o', oo' = f, o and o' is the origin point of coordinates on the photoes P and P' respectively (the central point on the fish eye lens photo). Fig. 1 shows that the planimetric photo P is perpendicular to axial

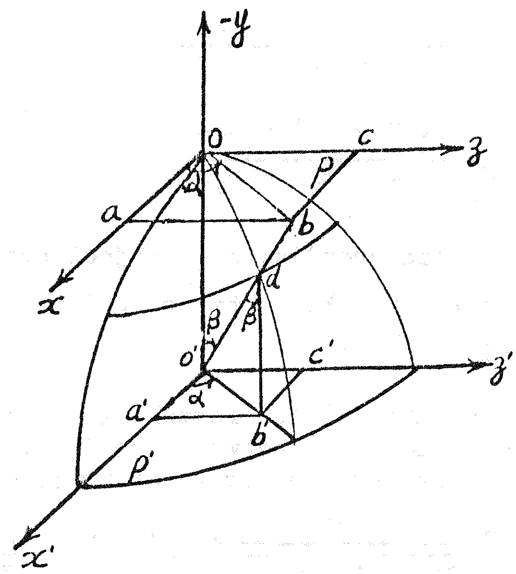


Fig. 1 The projection relation of the fish eye panoramic photograph

y and tangent to spherical, the tangential level is parallel to fish eyes panoramic photo P'. A unified projective relationship can, therefore, be built, and can proceed with coordinate transformation, so that the photo coordinates of the fish eye panoramic could transform into corresponding planimetric photo coordinates. Then according to the relationship of the central projection, the fundamental formula of photogrammetry used in fish eye panoramic photo can be adopted in the calculation of adjustment.

From Fig. 1 we can know, oa = cb = x; oc = ab = z. In the light of geometric principle, we know, $\angle a'o'b' = \angle aob = \alpha$, $x = ob \cos\alpha$; $z = ob \sin\alpha$; $ob = f tg\beta$.

The coordinate's relationships in fish eyes panoramic photo P' to planimetric photo P can be given:

$$x = f \frac{x'}{\sqrt{f^2 - x'^2 - z'^2}}$$

$$z = f \frac{z'}{\sqrt{f^2 - x'^2 - z'^2}} \tag{1}$$

2-2 The Relationship between Fish Eyes Panoramic Photo and Object Space Coordinates

From photogrammetry we know that the photo coordinates x, z measured from single ground photo can be used to calculate the object coordinates by using the following formula.

$$x - x_s = (y - y_s) \frac{a_1 x + a_2 f + a_3 z}{b_1 x + b_2 f + b_3 z}$$

$$z - z_s = (y - y_s) \frac{c_1 x + c_2 f + c_3 z}{b_1 x + b_2 f + b_3 z} \tag{2}$$

where: x, y, z — photogrammatric coordinate in object space;

x_s, y_s, z_s — coordinates at photo station point;

a_i, b_i, c_i — direction cosine.

Substitute the x, z in formula (1) for formula (2) and let $x_o = z_o = o$, therefore, the coordinates relationships between object point and its corresponding image point on the fish eye panoramic photo can be achieved as follows:

$$x - x_s = (y - y_s) \frac{a_1 f \frac{x'}{\sqrt{f^2 - x'^2 - z'^2}} + a_2 f + a_3 f \frac{z'}{\sqrt{f^2 - x'^2 - z'^2}}}{b_1 f \frac{x'}{\sqrt{f^2 - x'^2 - z'^2}} + b_2 f + b_3 f \frac{z'}{\sqrt{f^2 - x'^2 - z'^2}}}$$

$$z - z_s = (y - y_s) \frac{c_1 f \frac{x'}{\sqrt{f^2 - x'^2 - z'^2}} + c_2 f + c_3 f \frac{z'}{\sqrt{f^2 - x'^2 - z'^2}}}{b_1 f \frac{x'}{\sqrt{f^2 - x'^2 - z'^2}} + b_2 f + b_3 4 \frac{z'}{\sqrt{f^2 - x'^2 - z'^2}}} \tag{3}$$

If the element of exterior orientation on the fish eye panoramic photo is a small angle, then the above formula can be written as:

$$x - x_s = (y - y_s) \frac{f \frac{x'}{\sqrt{f^2 - x'^2 - z'^2}} + f\varphi - kf \frac{z'}{\sqrt{f^2 - x'^2 - z'^2}}}{- f \frac{x'}{\sqrt{f^2 - x'^2 - z'^2}} + f - \omega f \frac{z'}{\sqrt{f^2 - x'^2 - z'^2}}}$$

$$z - z_S = (y - y_S)\frac{kf\dfrac{x'}{\sqrt{f^2 - x'^2 - z'^2}} + \omega f + f\dfrac{z'}{\sqrt{f^2 - x'^2 - z'^2}}}{-\varphi f\dfrac{x'}{\sqrt{f^2 - x'^2 - z'^2}} + f - \omega f\dfrac{z'}{\sqrt{f^2 - x'^2 - z'^2}}} \tag{4}$$

When $\varphi = \omega = k = o$, then the above formula has following forms:

$$\begin{aligned} x - x_S &= (y - y_S)\frac{x'}{\sqrt{f^2 - x'^2 - z'^2}} \\ z - z_S &= (y - y_S)\frac{z'}{\sqrt{f^2 - x'^2 - z'^2}} \end{aligned} \tag{5}$$

2-3 The Scale Variation of the Fish Eye Panoramic Photo

Since the fish eye panoramic photo is spherical central projection, its scale is changeable. The law of its variation is that it varies inversely from central point to boundary as coordinate does. From formula (1) we can see, if there is a sqared network on architectural vertical surface (e.g. windows at identical length and so on), its imaging on the fish eye panoramic photo is shown in Fig. 2.

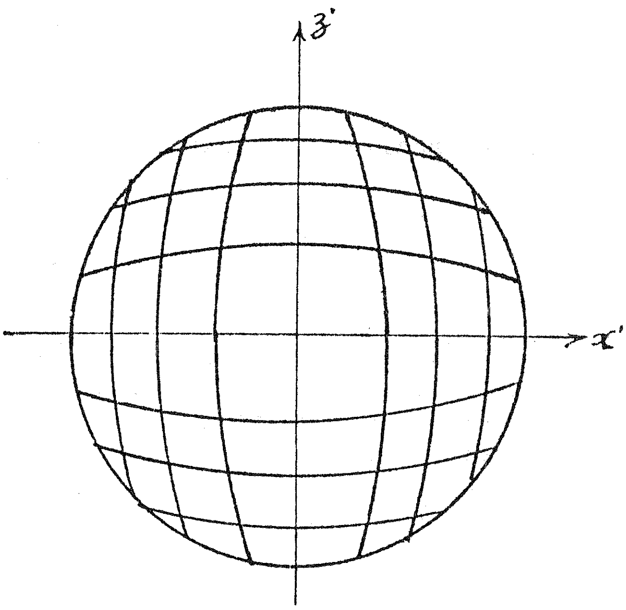


Fig. 2 Geometric distortion of the fish eye photograph on the grid plane

From formula (1), we can get its anti function:

$$\begin{aligned} x' &= f\frac{x}{\sqrt{f^2 + x^2 + z^2}} \\ z' &= f\frac{z}{\sqrt{f^2 + x^2 + z^2}} \end{aligned} \tag{6}$$

From formula (6) we can easily see that the scale variation on fish eye panoramic photo is symmetric distribution and is the function of the coordinates (x_i, z_i).

The mathematical formula of strict scale on the fish eye panoramic photo is given:

$$\frac{1}{m} = \frac{y^{*2}}{(y - y_s)\sqrt{E' \sin^2 r - G' \cos^2 r + F' \sin r}} \tag{7}$$

where E', F', G' are Gauss' cofficients, their values can be determined by the following formula:

$$\begin{aligned} E' &= \left(\frac{\partial x^*}{\partial x'} y^* - x^* \frac{\partial y^*}{\partial x'}\right)^2 + \left(\frac{\partial z^*}{\partial x'} y^* - z^* \frac{\partial y^*}{\partial x'}\right)^2 \\ F' &= \left(\frac{\partial x^*}{\partial x'} y^* - x^* \frac{\partial y^*}{\partial x'}\right)\left(\frac{\partial z^*}{\partial z'} y^* - x^* \frac{\partial y^*}{\partial z'}\right) \\ &\quad + \left(\frac{\partial z^*}{\partial x'} y^* - z^* \frac{\partial y^*}{\partial x'}\right)\left(\frac{\partial z^*}{\partial z'} y^* - z^* \frac{\partial y^*}{\partial z'}\right) \\ G' &= \left(\frac{\partial x^*}{\partial z'} y^* - x^* \frac{\partial y^*}{\partial z'}\right)^2 + \left(\frac{\partial z^*}{\partial z'} y^* - z^* \frac{\partial y^*}{\partial z'}\right)^2 \end{aligned} \tag{8}$$

Formula (7) shows that the scale of any point on fish eye panoramic photo depends not only on preliminary value $f, \varphi, \omega, k, y, y_s, x'$ and z' but also on the direction angle r in which its scale has been determined. If $r = o$, the general formula on scale in parallel with the direction of axis z on the fish eye photo is obtained:

$$\frac{1}{m} = \frac{y^{*2}}{(y - y_s)\sqrt{G'}} \tag{9}$$

If $r = 90^\circ$, the general formula on scale in parallel with the direction of axis z of the photo then is given:

$$\frac{1}{m} = \frac{y^{*2}}{(y - y_s)\sqrt{E'}} \tag{10}$$

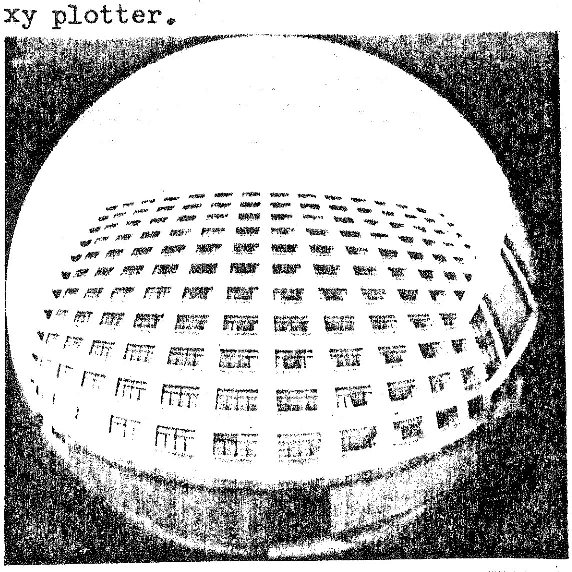
From expression G' and E' in the formula (9) and (10) we know that axis x' is transformed into axis z' and the rest items are all the same, that is, the variant regularity on scale of the fish eye panoramic photo is symmetric from each other and is changeable continuously with different point location. It is a function (x_i, z_i). From this, we can reach the conclusion that whether the primary optic axis is perpendicular to object plane or oblique to object space plane, the scale of the fish eye photo is a variation.

3. EXPERIMENT

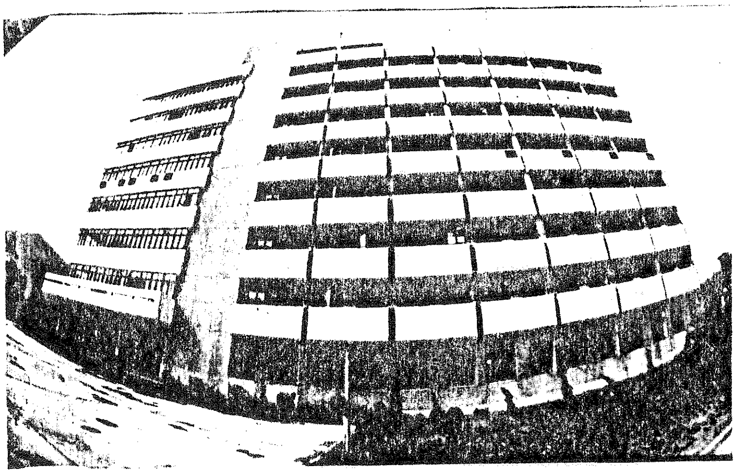
In order to test and verify that the theory about using fish eye panoramic photo to perform close range photogrammetry is correct, and that its application is possible, we use home-made camera model sea-gull DF-135 with an added fish eye lens of focal length 7.5^{mm} or 16^{mm}. We took photoes on a great deal of tall buildings in the Tongji University's campus round Jan. 1986, such as, foreign students building, foreign language department building, electrical department's building and publishing house's building and so on. We measured their elevation view. In order to test the precision of 3D-object space coordinate determined by the fish eye photo, we built an indoor 3D test site with more than 80 marked points made by metal to perform accuracy test. We used a theodohte model 010 to carry out forward intersection and polygon altimetric survey so as to determine the 3D coordinates of 37 featured points on the electrical building in comparison with the accuracy of space coordinates determined by using fish eye panoramic photo. The results of accuracy test are encouraging. The surveying elevation view and 3D accuracy test are presented respectively as follows.

3.1 The Analytic Process of Single Photograph

The analytic process of single photograph is mainly by using the single photo taken by fish eye lens (shown in Fig. 3), to measure the photo coordinates x', z' of all the feature points (door, window, intersecting point of exterior outline etc.) on the building's elevation view by means of handwheel P , q of the stereocomparator. Their measured accuracy is required to be up to ± 0.01mm. We can then proceed with calculation using programmed Fortran and Basic language at microcomputer IBM-XT and draw elevation view with xy plotter.



(a) The foreign students building in Tongji University taken by the fish eye lens of focal length 7.5mm



(b) Electrical department's building taken by the fish eye lens of focal length 16mm

Fig. 3

They are shown in Fig. 3 to Fig. 5. The fundamental principle is to use formula (6) to measure straight line (line segment) L on plane xz and object space coordinates of the ends and to determine the values.

$$L^2 = (x_2 - x_1)^2 + (z_2 - z_1)^2 \tag{11}$$

The equation of the correction for each line segment can be formed as follows:

$$A\delta y + B\delta\varphi + C\delta\omega + \ell = V \tag{12}$$

Written into matric form: $\bar{A}\delta + L = V$ (13)

The equation of the correction is under the condition of $[vv] = \min.$, it can be solved by relaxation method.

3.2 The Analytic Process of Stereograph of the Fish Eye Photograph

The analytic process of the fish eye stereograph is mainly stereopair of the split photography taken by fish eye lens with the focal length 16^{mm}. By using stereomeasurement or single photo measurement at stereocomparator, the measurement accuracy on the comparator coordinates x, y, p, q or x , y , x_2, y_2 of marked points measured is up to $\pm 0.02^{mm}$ (p, q is $\pm 0.01^{mm}$). Then, the object space coordinates of all marked points are run on microcomputer IBM-XT with programmed Fortran language, i.e. program FDLT (Fish Eye Direct Linear Transformation) and FDDLT (Fish Eye Distortion Direct Linear Transformation).

3.2.1 FDLT formula

The photo coordinates formula of the fish eye photo are considered on the basis of direct linear transformation (DLT) formula. The formula of the direct linear transformation of the fish eye photo FDLT, which the results have been derived, is as follows:

$$\begin{aligned} x' + \Delta x_f + \frac{l_1 x + l_2 y + l_3 z + l_4}{l_9 x + l_{10} y + l_{11} z + 1} &= 0 \\ z' + \Delta z_f + \frac{l_5 x + l_6 y + l_7 z + l_8}{l_9 x + l_{10} y + l_{11} z + 1} &= 0 \end{aligned} \tag{14}$$

where:

$$\begin{aligned} \Delta x_f &= x' \frac{f}{\sqrt{f^2 - x'^2 - z'^2}} - x' \\ \Delta z_f &= z' \frac{f}{\sqrt{f^2 - x'^2 - z'^2}} - z' \end{aligned} \tag{15}$$

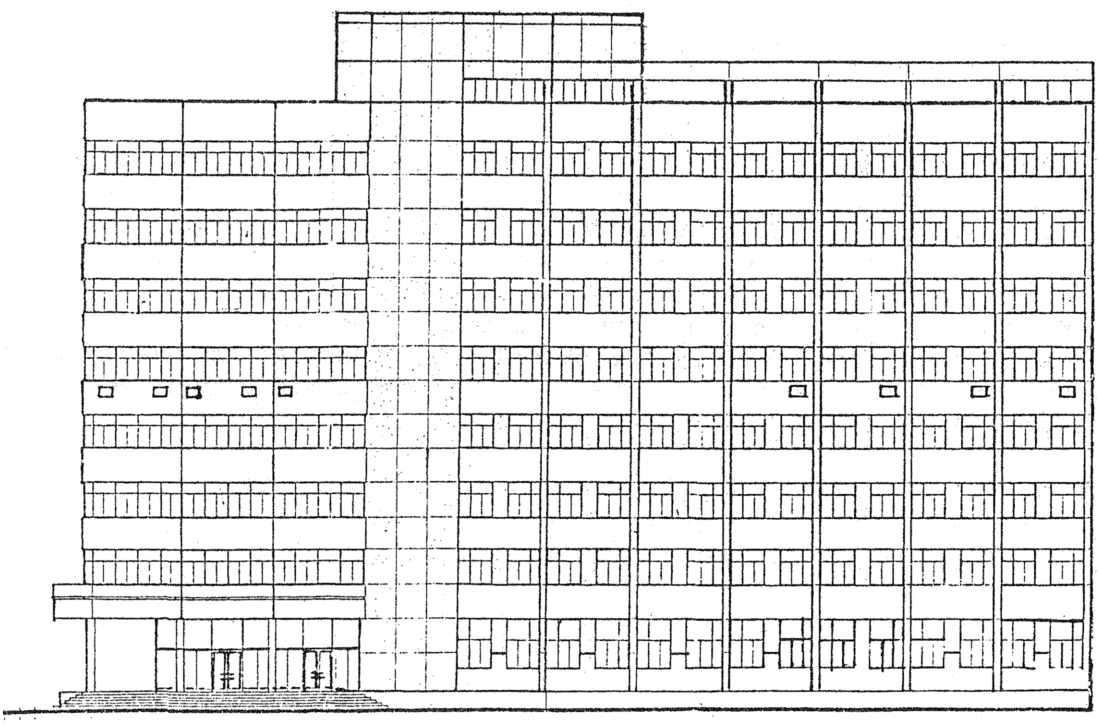
Substitute (15) for (14):

$$\begin{aligned} x' \frac{f}{\sqrt{f^2 + x'^2 - z'^2}} + \frac{l_1 x + l_2 y + l_3 z + l_4}{l_9 x + l_{10} y + l_{11} z + 1} &= 0 \\ z' \frac{f}{\sqrt{f^2 - x'^2 - z'^2}} + \frac{l_5 x + l_6 y + l_7 z + l_8}{l_9 x + l_{10} y + l_{11} z + 1} &= 0 \end{aligned} \tag{16}$$

Table 1

Solution	Points number N	3D checking indoor field												Electric building of field survey				
		photographic base $B=1^m$						photographic base $B=2^m$						LG* $y=18.4^m$, base $B=12^m$				
		LG* y m	m_x mm	m_y mm	m_z mm	m_t mm	$\frac{m_t}{y}$	LG* y m	m_x mm	m_y mm	m_z mm	m_t mm	$\frac{m_t}{y}$	m_x mm	m_y mm	m_z mm	m_t mm	$\frac{m_t}{y}$
FDLT	51	2	2.31	2.38	2.83	2.50	$\frac{1}{800}$	3	2.11	2.07	2.31	2.16	$\frac{1}{1388}$	31.48	18.19	43.94	28.20	$\frac{1}{653}$
	69	3	2.21	2.26	2.48	2.31	$\frac{1}{1300}$	3.5	2.07	2.09	2.19	2.11	$\frac{1}{1420}$					
FDDLT	51	2	2.09	2.33	2.71	2.37	$\frac{1}{845}$	3	1.98	1.93	1.78	1.90	$\frac{1}{1578}$	12.10	7.75	12.20	10.68	$\frac{1}{1700}$
	69	3	1.97	2.03	2.05	2.01	$\frac{1}{1492}$	3.5	1.76	1.88	1.69	1.77	$\frac{1}{1977}$					

Remark: LG* — longitudinal distance;
m_t — the mean error of points locations.



Original Scale: 1:300

Fig. 4 Electric department building (northern elevation view)

3.2.2 FDDLT formula

FDLT formula only considers the coordinates relational expression between fish eye photo and general photo, whereas it ignores all errors such as symmetric and asymmetric objective distortion of the fish eye lens. Now, here, we have considered the all correction about the objective distortion etc. The formula on the fish eye distortion DLT, of which the results have been derivated is as follows:

$$x' + \Delta x_f + \Delta x_n + \frac{l_1 x + l_2 y + l_3 z + l_4}{l_9 x + l_{10} y + l_{11} z + 1} = 0$$
$$z' + \Delta z_f + \Delta z_n + \frac{l_5 x + l_6 y + l_7 z + l_8}{l_9 x + l_{10} y + l_{11} z + 1} = 0 \qquad (17)$$

where:

$$\Delta x_f = x - x' = x' \left(\frac{f}{\sqrt{f^2 - x'^2 - z'^2}} - 1 \right)$$
$$\Delta z_f = z - z' = z' \left(\frac{f}{\sqrt{f^2 - x'^2 - z'^2}} - 1 \right) \qquad (18)$$

$$\Delta x_n = (x - x_o)(k_1 r^2 + k_2 r^4 + k_3 r^6) + P_1 [r^2 + 2(x - x_o)^2] + 2P_2 (x - x_o)(z - z_o)$$
$$\Delta z_n = (z - z_o)(k_1 r^2 + k_2 r^4 + k_3 r^6) + 2p_1 (x - x_o)(z - z_o) + p_2 [r^2 + 2(z - z_o)^2] \qquad (19)$$

$$r^2 = (x - x_o)^2 + (z - z_o)^2 \qquad (20)$$

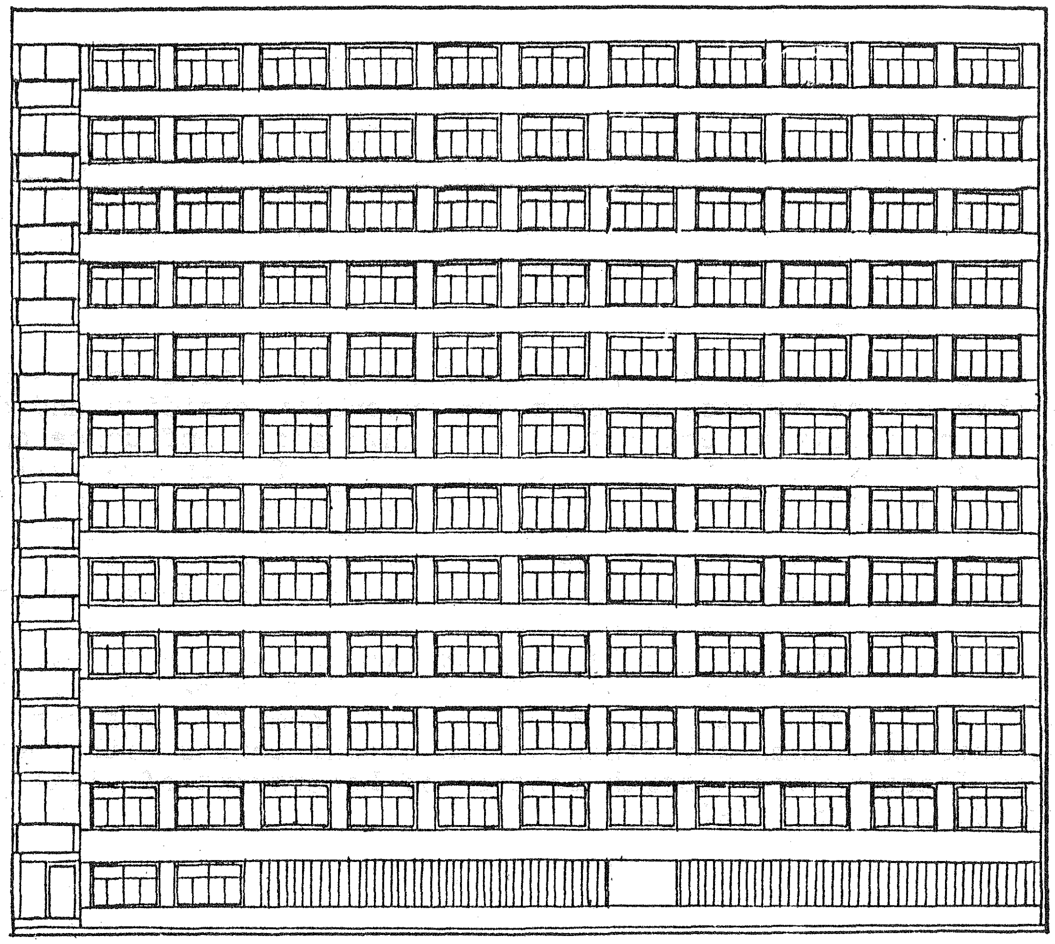
where: x', z' are the comparator coordinate of the fish eye photo;
x, z are the comparator coordinate on general photo which is transformed from comparator coordinate on the fish eye photo;
x_o, z_o are principal point coordinate at the comparator coordinate system;
$l_1 \sim l_{11}$, k_1, k_2, k_3, p_1, p_2 are 16 undetermined coefficients; x, y, z are objective space coordinate. The statistic accuracy of the coordinates derived by the way of the geodesy and the undetermined points solved by FDLT and FDDLT formulas are shown in Table 1.

From Table 1, we can see that the degree of accuracy of the solution by FDDLT is higher than that of the FDLT. It means that when using the photo taken by fish eye lens, besides symmetric linear distortion, in existence there is considerable asymmetric distortion. Therefore, when using fish eye lens to proceed with close range photogrammetry, it would be best to use FDDLT solution.

4. CONCLUSION

The experiments and methods presented in this paper possess very practical or immediate significance. Because the fish eye lens photography has the advantages of large area of coverage, high resolution, light-weight and of a low price. It can take huge object at very short distance, and carry out image process by using analytic method, and it is very applicable and suitable in close rang photogrammetry. It especially has a unique style in design of scenic area and enhances scientific ties between intuition space and geometric space. Through many tests the correctness of mathematical model derived and the reliability of the program worked out has been proved.

The controlling theory on line segment, algorithm and single photo process method of the fish eye panoramic photo introduced in this paper are also especially suitable for surveying in architectural elevation view. The reason is that the outlines of the building are horizontal or vertical and it is very convenient to measure the line segment on the building substituting the controlling line segment method for control point, we thus reduce field work requirements and raise working efficiency of the stereo photogrammetry is carried out by fish eye lens, it would better use the FDDLT solution, as it can correct effectively the scale variation of the fish eye photo and distortion.



Original Scale 1:200

Fig. 5 Southern elevation view of the foreign student building in Tongji University (western part)

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