THE PHOTOGRAMMETRIC SURVEY OF THE COLISEUM IN ROME

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ABSTRACT

The problem of the photogrammetric survey of the Coliseum is examined both for the vertical external walls (with terrestrial photogrammetry, scale 1:50-1:100) and for the planoaltimetric map (with aerial photogrammetry, scale 1:100-1:200). A centimetric accuracy is to be reached.

A 1st order triangulation net around the monument is set up; subsequent secondary nets allow the determination of the controls on the wall, in a bi- and tri-dimensional approach.

A particular kind of aerial triangulation is then set up and utilized to complete the controls on the vertical walls. Geometric and photogrammetric problems, depending on the variable elliptic curvature of the monument, are solved.

Some samples of the computations, and the analytic and analog plotting of a sample of the second external wall are finally presented.

The operations are going on for the remaining walls and for the aerial map.

A strict cooperation with other Departments and Institutes is foreseen for different disciplinary developments, i.e.Architecture, Science of Constructions, Ancient Topography, Computer Graphics, etc.

CHAPTER 1

GENERAL SET-UP

Introduction

1.1. - The photogrammetric survey of the Coliseum introduces really tremendous problems, if we want to get a survey with strict requirements of homogeneity, continuity and high accuracy. In fact, the setup which we plan is the following one:

Goals to be achieved

a. - The survey of the vertical walls in a single "cartographic" system, based on unique geometric surfaces to which the "heights" of the single points are related. This shall be performed by terrestrial photogrammetry.

This approach - which was proposed in its general lines by Ferrara and Giannoni in the paper [1]- is quite similar to terrestrial cartography; each single point is thus defined by three coordinates (s h q), corresponding to the terrestrial coordinates (Φ Ω h), or (E N H).

b. - The survey of the plano-altimetric map of the entire monument, with the means and the techniques of aerial photogrammetry.

c. - The definition of the controls for the conventional terrestrial survey of the remaining architectural elements, where the photogrammetry cannot arrive (internal planimetries, prospects, walls, etc.), and for the survey of thematic features, for istance with termography.

Accuracy

Defined by rmse of 1≈2 cm in the three coordinates.

Scale

1:50 for the external walls; 1:100 for the map.

Costs and times

About 1,000,000 US $; about 3 years.

We hope to get the financial support not only from the University, but also from other Agencies - i.e. the Soprintendenza Archeologica, the Ministero dei Beni Culturali, etc. - and from private firms and enterprises; contacts in this way are in course.

The geometric problem

1.2. - The plan of the Coliseum has an elliptic shape; this is evident from a simple glance to the 1:500 aerophotogrammetric map surveyed in 1981 by the firm AEROFOTOGRAMMETRICA di R. Nistri, by order of the Soprintendenza Archeologica (see Annex 1). We shall see later a check of this assumption.

In the same map appear more different ellipses, presumably concentric and parallel; they refer to different walls and foundations, one external and 5 internal. We shall verify also this hypothesis; by now we limit our examination to the external wall, assuming that it is representable by an elliptic right vertical cylinder: from the above said map we obtain the following approximate values for the semi-axes and for the ellipticity of the basic ellipse:

$$A = 95 \text{ m}; \quad B = 80 \text{ m}; \quad e^2 = 0.2909$$

The canonic equation of this ellipse is:

$$X^2/A^2 + Y^2/B^2 = 1$$

The planimetric location of any point P on the wall may be defined in this canonic reference (XY); a third coordinate Z = altitude must be added, referred to a horizontal zero plane. However, this reference is scarcely suitable for a "cartographic" representation of the wall: we must use something similar to the terrestrial cartographic representation, where not the geocentric (XYZ) coordinates, but the superficial coordinates (Φ Ω) are used - or their plane transformed (EN) - to which the altitude H is added *a posteriori*. We shall use therefore the superficial (sh) coordinates, where **s** is the length of the elliptic arc counted from whatever origin O, and **h** is the above said Z altitude, counted along the vertical from a horizontal origin plane h = 0. To define the spatial location of P we must add a third coordinate q, that is the distance of the point from the surface of a suitable mean elliptic cylinder, which we assume as reference surface for the heights q (its correspondant in the terrestrial cartography is the ellipsoid; the geoid has no meaning here).

To the (shq) reference we give the name of *mean cylindric reference*; the mean cylinder will be defined in para 1.4,c).

1.3. - After the institution of a convenable geometric framing net in an arbitrary reference (xyz), it is possible to measure on the ground the (xyz) coordinates, but certainly not the (shq) coordinates. On the other hand these coordinates are indispensable for the continuity of the cartographic representation;

we must therefore perform the transformation (xyz) => (XYZ) => (shq).

To this purpose we shall take into account the following considerations:

a) - the mean cylinder is a plane applicable surface; the above said transformation is therefore much simpler than the plane representation of the terrestrial ellipsoid. The perfect similitude is obtenable; no linear, areal, angular deformations are to be feared;

b) - the definition of the elliptic arc s is however a problem of a considerable complexity, as it implies the use of elliptic integrals. Serial expansions are therefore necessary, and they must proceed well beyond those issued by the treaties of Geodesy, or of simple Geometry. For instance, the approximate length of the whole ellipse is given by well known formulas, like:

$$L = \pi[3(A + B)/2 - \sqrt{(AB)}]$$

which, with the above values, gives L = 550.7904 m, with a very little error; but such formulas are not valable for an arc of ellipse, and cannot be used for the computation of the arc s, to which we request an accuracy of 1≈2 cm; that is, on L/4, an accuracy of $10^{-4} \approx 10^{-5}$.

In the paras 2.1 to 2.5 we report the treatment of this problem; here the above said accuracy is obtained by a serial expansion up to the term in $e^8 \cos^8 u$ (u = reduced latitude).

We must furthermore point out that there are programs for a quick and precise electronic computation of defined integrals (i.e. the quadrature method of Gauss-Legendre, utilized in the program SOLV of the HP 15 computer [3]). Anyhow, we deem that the computation program which we propose (see 2.6 and Annex 3) is perfectly suitable for the solution of this problem;

c) - for the definition of the heights **q** the consideration is necessary of the elliptic normal N, and then of the "latitude" Φ. The geometry of the ellipse allows to compute Φ from the (XY) coordinates, when e^2 is known, and furthermore the "reduced latitude" u to be used in the above said serial expansion.

The topographic problem

1.4 - The operations for the definition of the (shq) coordinates of single points on the wall may be set up as follows:

a) - establishment of a framing trigonometric net around the Coliseum in a general (xyz) reference, with high accuracy (see paras 3.1, 3.2 and Annex 4);

b) - with the support of such net, determination of a set of points on the wall, including the controls for the photogrammetric survey (see paras 1.9, 3.3, 3.4 and Annex 6). In any case their precision - whatever be the reference and the procedure employed for their determination - must be better than 1 cm rmse in each one of the three coordinates;

c) - determination, by least squares techniques, of a best fitting ellipse on the whole of such points. The computation will first demonstrate the reliability of the elliptic assumption, and will then issue the 5 parameters which fully define the said ellipse in the (xy) reference. This ellipse is assumed as *mean ellipse* of the wall; the elliptic right vertical cylinder based on it is called *mean cylinder*; the "cartographic" representation of the wall will be referred to it;

d) - transformation of the general (xy) coordinates into canonic (XY) coordinates by Helmerts' formulas, and of the (XY) coordinates into superficial (sq) coordinates by the formulas in Chapter 2. For the altitudes we shall simply assume $h = Z = z$.

1.5 - The operations a) and b) have already been carried out utilizing modern devices and procedures. The angular and distance measurements have been performed by the Wild T 2000 "total station"; the 8 points of the 1st order framing net a) have been materialized by solid pillars with fixed auto-centering devices; the controls b) by 12x12 cm square signals, painted with reflecting varnish capable of distantiometric answer, glued or cemented on the wall (see fig. 6).

The computation of the 1st order net has been executed following two procedures, resp. bi- and tri-dimensional; the results are completely indifferent, and the overall accuracy is very good (rms errors of about 1 mm). The same has been done on a sample of the first internal wall for the control points b); due to the differences of their altitudes, the discrepancies between bi- and tri-dimensional results are more evident (up to 12 mm). We deem it preferable the bi-dimensional approach, followed by separate altimetric computation, as it gives better residuals and better rms errors.

In Chapter 3 the whole problem is treated in detail; the computations programs are described, and the output results fully reported.

1.6 - Within some limits, the actual proximity of the mean cylinder to the whole of the points on the wall has not a great importance; in fact, not the absolute heights q are needed, but their values related to a unique reference surface, so that the relative height situation of the points be well defined.

Consequently, we have selected for each ellipse a good number of points on the aerophotogrammetric map (see Annex 2), and graphically measured their coordinates in the general (cadastral) reference (xy). On these points we have performed the best-fitting and transformation operations (Chapter 2), and obtained for the parameters of the ellipses and for the planimetric residuals the values which appear in the computation output Annex 3.

The planimetric residuals are nothing but the heights q of the single points above the mean cylinder; their random distribution and their low values (about 0.5 m, comparable with the observation errors) demonstrate that the plan of the Coliseum is an ellipse.

The photogrammetric problem

1.7 - The great extension of the survey, the necessity of its continuity and the high accuracy which is requested, impose to modify and to adapt the conventional procedures of terrestrial photogrammetry, generally employed for the survey of façades and fronts. In fact, it is opportune to base the absolute orientation of the single models much more on the controls located on the wall than on the coordinates of the taking centers and the angular parameters of each plate, pre-determined or pre-imposed.

There is a second reason for that. The requested plotting accuracy needs short taking distances (≈30 m with Wild P 31 camera); as the wall is considerably higher (up to ≈50 m), it is not possible to take it in a single photogram. The difficulty could be avoided by taking two plates from the same station,resp. with horizontal and inclined axes, but the second plate would have very different characteristics from the first one, and could involve a very different accuracy. We preferred to take two superimposed photograms, both with horizontal axes, the first one from the ground, the second one from a convenable altitude (about 20 m), utilizing an elevator carriage which hoisted up the camera and the operator. Obviously the orientation elements of the second taking cannot be determined *a priori* with a sufficient accuracy: we are in the same situation of the aerial photogrammetry, hence the necessity of utilizing control points.

1.8 - From this situation it derives the opportunity of taking two strips on the wall, resp. from ground level and from an altitude of about 20 m, with a strong transversal overlap (about 30%); and to fully utilize the techniques of aerial triangulation for the control. This is possible, as the whole of the wall points is defined in an unique superficial reference - the mean cylindric reference - in its three coordinates. In the lower strip the absolute orientation elements of the camera could be pre-determined; in the block computation they would certainly strengthen the adjustment. However, we deem this unnecessary; the only actual necessity is the regularity and the continuity of the takings. An accurate study of the camera's locations, particularly for the upper strip, is imperative; the relevant problems have been solved on the ground, in each singular case.

1.9 - The computation of the "aerial" triangulation for the whole wall could be done in an unique block. However, we deemed it opportune to subdivide the computation into 4 blocks, one for each elliptic quadrant, also in consideration af the decreasing accuracy of the s coordinate with the length of the arc. These blocks will be afterwards linked by known techniques, keeping at least one common model between two subsequent blocks.

In this approach each block includes about 30 models, subdivided into 2 strips. The control points are located within them with the usual distribution for the aerial triangulation, locally reinforced in consideration of the high accuracy and the peculiarity of this survey. We have adopted the following distribution:

a) - the initial and final models of each strip are fully controlled with the usual 5 points. We would have therefore in the whole 8 fully controlled models, located at the vertices of the ellipse. This in theory; in the practical application the distribution has been adapted to the actual extension of the walls;

b) - along the upper and lower borders of the wall, 1 point each 2-3 models;

c) - in the transversal overlapping area, 1 point each 4-5 models.

The location of the b) and c) points is not strictly fixed; it is convenable to put them in accessible areas, or in coincidence with natural features.

The application of "aerial" triangulation for one quadrant of the Coliseum is treated in para 2.8.

1.10 - In the usual aerial triangulation the terrestrial curvature must be kept into account, at least for the altimetry. In our "aerial" triangulation the problem is much heavier, due to the strong curvature of the reference surface and to its variability. In fact, between the tangent plane and the elliptic wall we have strong variable differences, both in height and in planimetry. The geometric and photogrammetric solution of this problem is treated in paras 2.8 and 2.9.

The aerophotogrammetric problem

1.11 - We have to build the aerophotogrammetric map of the whole monument, at the scale 1:100 ≈ 1:200, with an accuracy of 1-2 cm in the three coordinates.

This operation presents no particular difficulties, except those deriving from the high requested accuracy. With the classic nadiral cameras f = 152 mm, 23x23 cm, in order to reach such accuracy - particularly in the spot elevations - we must keep a relative flight height of 150-200 m, i.e. a photographic scale of about 1:1000. The use of helicopters is then necessary, also considering that the Coliseum is in the city center, where so low flights are forbidden.

With a height of 200 m the side of the photograms obtained with the said camera is about 300 m; one only strip, with axis coincident with the major axis of the ellipse, is sufficient for the whole coverage of the Coliseum.

However, on request of the Soprintendenza Archeologica, and to include the survey in the context of the so called "Valley of the Coliseum", it is opportune to take a block of 3-4 strips of 5-6 photograms each. With a longitudinal 60% and a lateral 20% overlap we have thus a stereoscopic coverage of about 900x900 m, with a photographic scale of about 1:1300.

The control points of this block must be determined in the (xyz) reference, with centimetric accuracy. Also here the use of aerial triangulation is opportune, where the block's controls will be derived from the 1st order framing net of para 1.4.a). Some other control points in the interior area of the Coliseum must be determined, after the institution of a set of intermediate points on the top of the monument's external wall; these ones must be viewable and collimable both from the internal and external side of the monument, and will be signalized before the taking flight.

CHAPTER 2

THE GEOMETRY OF THE COLISEUM

Definition of the mean ellipse

2.1 - In the following analysis the coordinates of the perimetral points of the monument - desumed from the aerophotogrammetric 1:500 scale map in Annex 2, or determined on the wall by topographic operations - are treated as independent and accidental observation data. In fact, this is only an approximation, as these coordinates derive from an unique survey, which obviously implies systematic effects and complex correlations. However, we presume that the approximation due to the above said assumption is practically acceptable; a more rigorous approach would be very onerous, and would not give appreciable improvements.

We want now to perform the analysis of the observation data, in the hypothesis that the external wall is a right vertical elliptic cylinder. The problem may be put as follows: "*having a statiscally valid series of n observed points on the wall, define the ellipse which best fits on the horizontal projections of such points (mean ellipse); define its equation and numerical parameters; evaluate the accuracy of these determinations*".

2.2 - The well known general equation of a conic section in homogeneous cartesian coordinates is:

$$x\ A\ x^T = 0$$

and in heterogeneous coordinates, with $A_{33} = 1$:

1) $$ax^2 + bxy + cy^2 + dx + ey + 1 = 0$$

where:

$$a = a_{11};\ b = 2\ a_{12};\ c = a_{22};\ d = 2\ a_{13};\ e = 2\ a_{23}$$

With these assumptions the matrix A is:

2) $$A = \begin{Bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & 1 \end{Bmatrix}$$

where the characteristic determinants and the orthogonal invariant are:

$$3)\begin{cases} |A_{13}| = (be - 2cd)/4; \quad |A_{23}| = (2ae - bd)/4; \quad |A_{33}| = (4ac - b^2)/4; \\ |A| = |A_{13}|d/2 - |A_{23}|e/2 + |A_{33}| \\ I = a + c \end{cases}$$

Hence with known formulas we obtain the 5 parameters which define the conic section, i.e.:

the coordinates of the center:

$$4) \qquad x_c = \frac{|A_{13}|}{|A_{33}|}; \qquad y_c = -\frac{|A_{23}|}{|A_{33}|};$$

the lengths of the semi-axes:

$$5) \qquad a = \sqrt{\left|\frac{\gamma}{\alpha}\right|}; \; b = \sqrt{\left|\frac{\gamma}{\beta}\right|}$$

where:

$$\left.\begin{matrix}\alpha \\ \beta\end{matrix}\right\} = \frac{I \pm \sqrt{I^2 - 4|A_{33}|}}{2}; \; \gamma = \frac{|A|}{|A_{33}|};$$

the orientation θ, given by:

$$6) \qquad tg\, 2\theta = b/(a-c)$$

and last, the eccentricity, given by:

$$7) \qquad e^2 = (a^2 - b^2)/a^2$$

2.3 - With the assumption of para 2.1, the proposed problem may be considered as an adjustment of indirect observations, where 1) is the generating equation. The generated system may be written:

$$8) \qquad x_i{}^2 a + x_i y_i b + y_i{}^2 c + x_i d + y_i e + 1 = v_i$$

with $i = 1,n$; $n > 5$. This is already a linear system; however, it is convenable - in order to operate on small figures, and to attenuate the rounding errors - to go on to barycentric coordinates, by assuming:

9) $x_i = x_i - x_m$; $y_i = y_i - y_m$;

where (x_m, y_m) are the coordinates of the barycenter of the observed points.

The system 8) may now be treated with the ordinary l.s. techniques. We will not report the details of this application, which is surely trivial; the program of electronic computation is based on a subroutine MQ, which gives the normalisation of the system, the elements α_{ik} of the reciprocal matrix, the adjusted values of the unknowns, the variance-correlation matrix. The terms of this matrix are:

$$c_{ii} = \alpha_{ii} \; ; \; c_{jk} = \varrho_{jk} = \frac{\alpha_{jk}}{\sqrt{\alpha_{ii} \cdot \alpha_{kk}}}$$

2.4 - We can now compute the parameters of the conic section, by introducing the adjusted unknowns a b c d e in eq. 3; then, by substituting the values hence obtained in eq. 4), 5), 6) we have the adjusted parameters x_c y_c A B θ of the mean ellipse. The A_{33} determinant is positive, as we shall see in the computation; then the conic section is an ellipse.

So these parameters are function of the coefficients a b c d e, which being adjusted quantities are correlated. In this respect it is enough to consider that the generic parameter p is a function of the coefficients:

$$p = f(a\ b\ c\ d\ e)$$

and therefore its mse is given by:

$$\mu_p^2 = \left(\frac{\partial f}{\partial a}\right)^2 \cdot \mu_a^2 + \dots + \left(\frac{\partial f}{\partial e}\right)^2 \mu_e^2 + 2\,\frac{\partial f}{\partial a} \cdot \frac{\partial f}{\partial b} \cdot \mu_a \mu_b \varrho_{ab} + \dots + 2\,\frac{\partial f}{\partial d} \cdot \frac{\partial f}{\partial e} \cdot \mu_d \mu_e \varrho_{de}$$

The second member has 15 terms; happily, being $\rho_{ii} = 1$, and $\rho_{ik} = \rho_{ki}$, it may be written in the following form, which is well suitable to electronic computation:

10) $$\mu_p^2 = \sum_{i=1}^{5} \sum_{k=1}^{5} \frac{\partial f}{\partial i} \cdot \frac{\partial f}{\partial k} \cdot \mu_i \mu_k \varrho_{ik}$$

With this, the difficulty is reduced to the evaluation of the partial derivatives $\partial f/\partial a$, ...,$\partial f/\partial e$, from the equations of para 2.2. Assuming:

$$u = 4\,|A_{13}|\ ; \quad u_1 = 4\,|A_{23}|\ ; \quad v = 4\,|A_{33}|$$

we obtain for the coordinates of the center:

11)
$$\begin{cases} \dfrac{\partial x_c}{\partial a} = -\dfrac{u}{v^2}\,4c\ ; \quad \dfrac{\partial x_c}{\partial b} = \dfrac{ev + 2bu}{v^2} \\[2ex] \dfrac{\partial x_c}{\partial c} = \dfrac{-2dv + 2bu}{v^2}\ ; \quad \dfrac{\partial x_c}{\partial d} = -\dfrac{2c}{v}\ ; \quad \dfrac{\partial x_c}{\partial e} = \dfrac{b}{v} \end{cases}$$

$$\begin{cases} \dfrac{\partial y_c}{\partial a} = \dfrac{2ev - 4cu_1}{v^2}\ ; \quad \dfrac{\partial y_c}{\partial b} = \dfrac{-d_v + 2bu_1}{v^2} \\[2ex] \dfrac{\partial y_c}{\partial c} = -\dfrac{4au_1}{v^2}\ ; \quad \dfrac{\partial y_c}{\partial d} = -\dfrac{b}{v}\ ; \quad \dfrac{\partial y_c}{\partial e} = \dfrac{2a}{v} \end{cases}$$

while for the semiaxes A,B, being:

12)
$$A^2 = \frac{\gamma}{\alpha}\ ; \ B^2 = \frac{\gamma}{\beta}\ ; \ a' = \frac{\gamma'\alpha - \alpha'\gamma}{2a\alpha^2}\ ; \ b' = \frac{\gamma'\beta - \beta'\gamma}{2b\beta^2}$$

it is first convenable to compute the derivatives α' β' γ' respect the variables a b c d e. Assuming:

$$\begin{cases} U = b\,d\,e - c\,d^2 - a\,e^2; \qquad V = 4\,a\,c - b^2 \\ W = \sqrt{(a-c)^2 + b^2} \end{cases}$$

we have the expressions:

$$\begin{cases} \dfrac{\partial \gamma}{\partial a} = -\dfrac{e^2V + 4cU}{V^2}\ ; \quad \dfrac{\partial \gamma}{\partial b} = \dfrac{deV + 2bU}{V^2} \\[2ex] \dfrac{\partial \gamma}{\partial c} = -\dfrac{d^2V + 4aU}{V^2}\ ; \quad \dfrac{\partial \gamma}{\partial d} = \dfrac{be - 2cd}{V}\ ; \quad \dfrac{\partial \gamma}{\partial e} = \dfrac{bd - 2ae}{V} \\[2ex] \left.\begin{matrix}\dfrac{\partial \alpha}{\partial a} \\ \dfrac{\partial \beta}{\partial a}\end{matrix}\right\} = \dfrac{1}{2}\left(1 \pm \dfrac{a-c}{W}\right)\ ; \quad \left.\begin{matrix}\dfrac{\partial \alpha}{\partial b} \\ \dfrac{\partial \beta}{\partial b}\end{matrix}\right\} = \pm\dfrac{b}{2W} \\[2ex] \left.\begin{matrix}\dfrac{\partial \alpha}{\partial c} \\ \dfrac{\partial \beta}{\partial c}\end{matrix}\right\} = \dfrac{1}{2}\left(1 \mp \dfrac{a-c}{W}\right)\ ; \quad \dfrac{\partial \alpha}{\partial d} = \dfrac{\partial \beta}{\partial d} = \dfrac{\partial \alpha}{\partial e} = \dfrac{\partial \beta}{\partial e} = 0 \end{cases}$$

which, when substituted in eq. 12), allow to immediately obtain the partial derivatives of the semiaxes respect the unknowns.

Substituting in eq. 10) the values 11) and 12) we have the rmse of the ellipse's parameters. The computation program, suitably operating on the indexes of the involved quantities, solves the problem with simplicity and elegance; the Annex 3 reports the program and its application to the points measured on the aerophotogrammetric map Annex 2. From these we have the following values for the semiaxes and the excentricities of the external E1 and the first internal E2 walls of the monument:

E1: $A = 95.505$ m; $B = 78.910$ m; $e^2 = 0.317$

E2: $A = 80.681$ m; $B = 63.844$ m; $e^2 = 0.374$

Length of an arc of ellipse

2.5 - At this point we have the coordinates x_c y_c of the center of the mean ellipse, and the orientation $r = xX$ of its major axis in the (xy) reference. It is easy to pass, by a roto-translation, to the (XY) coordinates of any point in the canonic reference of the said ellipse.

We must now obtain, from the XY coordinates, the superficial coordinate s = length of the arc of ellipse between the point and a selected origin O (fig. 1). As it is well known, the length of an arc of ellipse cannot be computed in finite terms, as it derives from an elliptic integral. Serial expansions are needed,

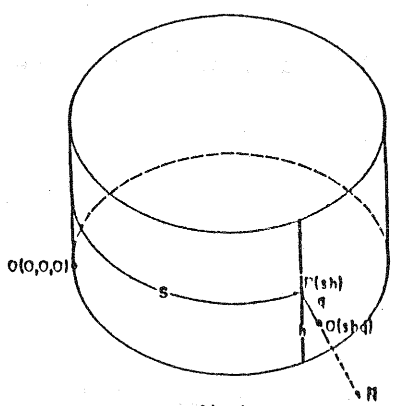

Fig. 1

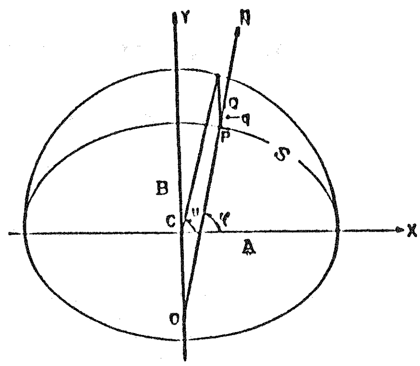

Fig. 2

with a considerable number of terms, which generally are not given by the ordinary treaties of Geodesy or Geometry; in fact, we want a high approximation in presence of a strong excentricity.

2.6 - When the parameters A, e^2 of the ellipse are known, there are many ways to set up the above said expansions. We have chosen the following one, based on the consideration of the "reduced latitude" u. When the coordinates $X_P Y_P$ of any point of the ellipse are known, we have from known formulas:

13) $$\operatorname{tg}\varphi = \frac{Y_P}{X_P(1-e^2)} = \sim \frac{Y_Q}{X_Q(1-e^2)}$$

14) $$\operatorname{tg} u = \sqrt{1-e^2}\cdot \operatorname{tg}\varphi$$

15) $$s = a(1-e^2)\int_0^{\varphi}(1-e^2\operatorname{sen}^2\varphi)^{-3/2}d\varphi = a\int_0^{U}(1-e^2\cos^2 u)^{1/2}du$$

By developing the integrand into binomial series we have:

16) $$(1-e^2\cos^2 u)^{1/2} = 1-\frac{1}{2}e^2\cos^2 u-\frac{1}{8}e^4\cos^4 u-\frac{1}{16}e^6\cos^6 u-\frac{10}{256}e^8\cos^8 u-\ldots$$

and being:

17) $$\begin{cases} \cos^2 u = \frac{1}{2}+\frac{1}{2}\cos 2u\ ;\ \cos^4 u = \frac{3}{8}+\frac{1}{2}\cos 2u+\frac{1}{8}\cos 4u \\ \cos^6 u = \frac{5}{16}+\frac{15}{32}\cos 2u+\frac{3}{16}\cos 4u+\frac{1}{32}\cos 6u \\ \cos^8 u = \frac{35}{128}+\frac{7}{16}\cos 2u+\frac{7}{32}\cos 4u+\frac{1}{16}\cos 6u+\frac{1}{128}\cos 8u \end{cases}$$

we get, substituting the 17) into the 16) and then in the 15):

18) $$s = a\cdot(I_0+I_2+I_4+I_6+I_8+\ldots) = a\sum_{k=0}^{2n} I_k$$

where:

19) $$\begin{cases} I_0 = u\ ;\ I_2 = -\frac{1}{2}e^2\left(\frac{1}{2}u+\frac{1}{4}\operatorname{sen}2u\right) \\ I_4 = -\frac{1}{8}e^4\left(\frac{3}{8}u+\frac{1}{4}\operatorname{sen}2u+\frac{1}{32}\operatorname{sen}4u\right) \\ I_6 = -\frac{1}{16}e^6\left(\frac{5}{16}u+\frac{15}{64}\operatorname{sen}2u+\frac{3}{64}\operatorname{sen}4u+\frac{1}{192}\operatorname{sen}6u\right) \\ I_8 = -\frac{10}{256}e^8\left(\frac{35}{128}u+\frac{7}{32}\operatorname{sen}2u+\frac{7}{128}\operatorname{sen}4u+\frac{1}{96}\operatorname{sen}6u+\frac{1}{1024}\operatorname{sen}8u\right) \\ I_{10} = \sim -\frac{7}{256}e^{10}\left(\frac{1}{2}u+\ldots\right) \end{cases}$$

With the approximate values of the parameters:

A = 95 m; B = 80 m; e^2 = 0.2909

we have for a quadrant of ellipse ($u = \pi/2 = 1.570796$):

$$
20)\quad \begin{cases}
I_0 = \dfrac{\pi}{2} = 1{,}570796; \qquad I_2 = -\dfrac{e^2}{4}\dfrac{\pi}{2} = -0{,}1142361 \\
I_4 = -\dfrac{e^4}{8}\dfrac{3}{8}\dfrac{\pi}{2} = -0{,}0062309; \qquad I_6 = -\dfrac{e^6}{16}\dfrac{5}{16}\dfrac{\pi}{2} = -0{,}0007552 \\
I_8 = -\dfrac{10}{256}e^8\dfrac{35}{128}\dfrac{\pi}{2} = -0{,}0001201 \\
I_{10} = -\dfrac{7}{256}e^{10}\cdot\dfrac{1}{2}\dfrac{\pi}{2} = -0{,}0000447
\end{cases}
$$

and therefore, with the 18):

$$s_{\pi/2} = 95\ 1.4494092 = 137{,}694 \text{ m}$$

$$s_{2\pi} = 550.775 \text{ m}$$

From the last one of the 20) it results that the contribution of I_{10} is about $5*10^{-5}$, and is therefore negligible; the expansion 18) may be limited to the term I_8.

If we use the first formula in the 15), which gives s as a function of Φ - following Jordan [2], whose formulas are used in [1] - we have a much less convergent expansion. This depends on the fact that Jordan starts with a 1st kind elliptic integral with a -3/2 power of the radicand; while, with the simple transformation Φ -> u, we have a 2nd kind elliptic integral with the 1/2 power of the radicand.

2.7 - If the point Q which we consider does not belong to the mean ellipse, but lies not too far from it, we can consider coincident the elliptic normals through Q and P . But as soon as the distance of Q from the mean ellipse reaches some extent - say 1 m - we have to take into consideration the fact that the two normals do not coincide.

Therefore to compute the superficial height q it is opportune to compute first the latitude Φ of the point Q resp. the mean ellipse (fig. 3). We shall do that by Rinner's formula [3]:

21) $$\mathrm{tg}\,\Phi = C \cdot Y_Q / X_Q$$

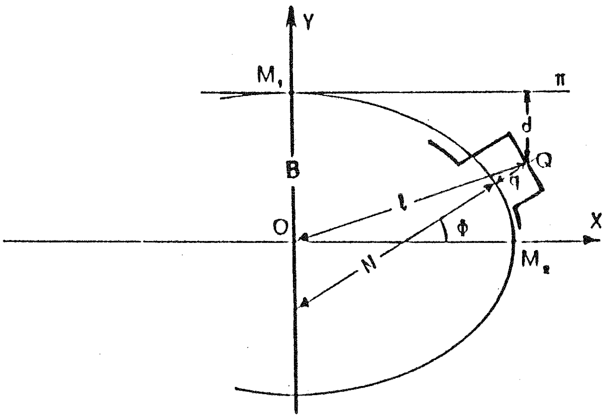

Fig. 3

where:

22) $C = 1 + e'^2/(1 + qV/B)$; $V = \sqrt{(1+e'^2\cos^2\Phi)}$; $e'^2 = (A^2 - B^2)/B^2$

It is convenable to operate by successive approximations. Assuming:

23) $$\ell = \overline{OQ} = \sqrt{(X^2_Q + Y^2_Q)}$$

we have:

24) $$qV/b = (\ell/B - V)V - B/2\ell \cdot (e'^2 - \eta^2) \cdot \eta^2/V$$

with:

$$\eta^2 = e'^2 \cdot \cos^2\Phi$$

As first approximation we take $q_1 = 0$, and then:

$C_1 = 1 + e'^2$ and from 21): $\mathrm{tg}\,\Phi_1 = C_1 \cdot X_Q / Y_Q$

With this value we compute, by 23) and 24), the 1st approximate value $q_1 V_1 / B$, and then in 2nd approximation:

25) $$C_2 = 1 + e'^2/(1 + q_1 V_1 / B)$$

from which $\mathrm{tg}\,\Phi_2$, and so on.

Three to four iterations are generally sufficient to give the final value Φ, from which the final value of the coordinate q :

26) $$q = X/\cos\Phi - N = X/\cos\Phi - A/W \; ; \quad W = \sqrt{(1-e^2\sin^2\Phi)}$$

The computation program easily solves this problem; we shall use this solution also when the point which we consider is a measured point on the wall or on the map, in order to obtain the adjustment residuals resp. the mean ellipse.

2.8 - We want to set up the control points for the whole monument by "aerial" triangulation. If we consider a single quadrant of the monument, and start the "aerial" triangulation from the vertex M_1 of the ellipse (fig.3), it is well known that all the models are reported to the absolute orientation of the first one, that is to the normal ("vertical") N_1 and to the tangent plane π. For any point Q we get therefore the "strip" coordinates (X h d), where h = z = Z, and the height d has generally a negative value, i.e. it is a depth.

To refer Q to the mean cylinder we need instead the superficial coordinates (s **h** q). Being $Y_Q = B - d_Q$, the above formulas 18) and 26) allow the transformation:

$$(x\ d) \Rightarrow (X\ Y) \Rightarrow (s\ q);$$

by this way we have for each control point Q, in any model, the superficial coordinates in the mean elliptic reference.

To carry on the "aerial" triangulation on the wall of each single quadrant we shall operate by independent models. We shall therefore:

a) - perform the relative orientation of the first model of the strip, in the neighbourhood of the vertex M_1, and go on with conventional techniques up to the last model,in the neighbourhood of the vertex M_2. In each model we shall observe the pass-points and the existing "ground" control points, if any, already determined on the wall in the general (s **h** q) reference;

b) - by a first chaining we have the (X h d) "strip" coordinates of all the observed Q points, and then - by the above said procedure - their superficial (s **h** q) coordinates;

c) - by using these coordinates we shall finally perform a conventional adjustment, by any procedure, on the existing "ground" control points.

2.9 - Now let us consider a single model, already oriented on its controls Q; its orientation on the mean elliptic surface is correct, but the plotting of its points is affected with the errors due to the variable curvature of the reference surface. We have to keep this into account, and correct these errors.

The problem is rather complex, due to the variability of the elliptic curvature; it should be solved for each single point by correcting the plotting procedure, or - in an analytical plotter - the plotting program.

But the source programs are not available. Then we have devised an expedient solution, based on the consideration that in one single plate we may correct the total variation of the x distance between the principal point and the fiducial marks, due to the elliptic curvature,by imposing it as a false shrinkage of the film. We propose the following approach:

We suppose to know the camera data d_r = x distance between the fiducial marks , and F = focal length, whence the field angle:

$$\alpha = \text{arctg}\ d_r/2F$$

and also to know the coordinates x_p y_p of the taking station P (fig 4). We shall first compute the canonic coordinates X_A Y_A, X_B Y_B of the intersections A B between the straight lines r_A r_B and the mean ellipse, knowing α and the"latitude" Φ from the 21). The equation of the straight line r_A through P, with angular coefficient $m_A = tg(\Phi - \alpha)$, and the canonic equation of the mean

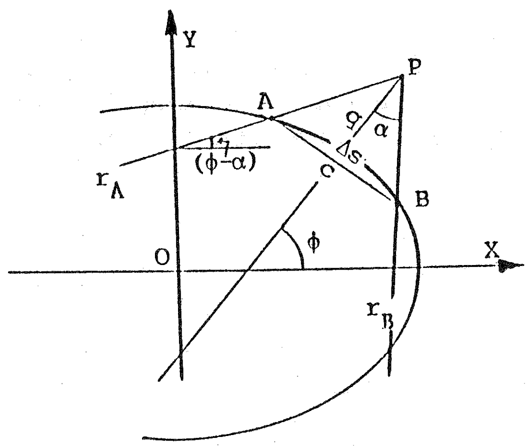

Fig. 4

ellipse give the system:

28)
$$\begin{cases} Y = m_A X + p_A \\ X^2/A^2 + Y^2/B^2 = 1 \end{cases}$$

where $p_A = Y_P - m_A X_P$. Its solutions are:

29)
$$\begin{cases} X_A = 1/(\ell + m_A^2)\cdot(-m_A p_A \pm \sqrt{(b^2(\ell+m_A^2) - \ell p_A^2)} \\ Y_A = m_A X_A + p_A \end{cases}$$

where $\ell = 1 - e^2$.

In the same way, assuming $m_B = tg(\Phi + \alpha)$, we have the coordinates of B:

$$X_B = 1/(\ell + m_B^2)\cdot(-m_B p_B \pm \sqrt{(b^2(\ell+m_B^2) - \ell p_B^2)}$$

30)

$$Y_B = m_B X_B + p_B$$

and then the length of the chord AB:

31) $$c = \overline{AB} = \sqrt{[(X_A - X_B)^2 + (Y_A - Y_B)^2]}$$

We shall now obtain the length s_{AB} of the arc $\widehat{AB}$ as the difference between the arcs s_A, s_B, given by the 18) with the 21); and finally:

32) $$\Delta s = s_B - s_A; \quad \Delta = \overline{AB}/\widehat{AB} = c/\Delta s$$

which solve the problem. The computation subroutines easily give - after a suitable analysis on the 29) and 30) to select the correct solution - and the $\Delta\%$, which applied to the x coordinates of the fiducial marks convenably lengthens their distance. In Annex 3 the results are reported for the Wild P31 camera employed for the takings, whose calibration data are (fig. 5):

f = 99.64 mm

fid. point	x	y
5	0	-27.501
6	-57.498	-0.001
7	0	57.503
8	57.494	-0.001

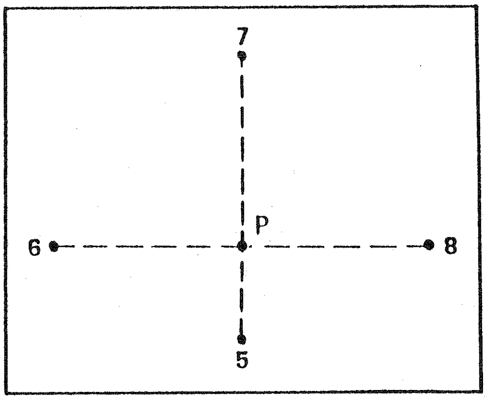

Fig. 5

hence:

$$d_r = 57.498 + 57.494 = 114.992$$

$$tg\ \alpha = 114.992/(2*99.64) = 0.577037; \quad \alpha = 29^\circ.987 = \approx 30^\circ$$

Obviously we have the maximum $\Delta\%$ value in the neighbourhood of the vertex M_2, where is the maximum curvature; its value is, for the ellipse E_2:

$$\Delta\%\ max = \approx 1.67\%$$

to which corresponds a lengthening of about 0.85 mm in the x_6 and x_8 above.

CHAPTER 3

THE FRAMING AND CONTROL NETS

The framing net

3.1 - Following the ordinary setup of the architectural photogrammetry, we have first planned and established a basic framing net all around the Coliseum, with vertices materialized by pillars which assure the stability and the repeatability of the observations. The design of this net is not fully based on mathematical requirements: a compromise was necessary with the necessity of minimizing the perturbations of the ambient caused by durable constructions such as the observation pillars, placed amidst an area of a great monumental and touristic interest. The result of this compromise is reported in the Annex 1, where is the planimetric sketch of the net, with its 8 vertices; the low number of connexions among non adjacent vertices is due to intervisibility problems.

The pillars are built in reinforced concrete, with external dimensions 0.35 x 0.35 x 0.90 m; they are protected from thermal effects by a cohibent shell covered with brick masonry, which makes them similar to the surrounding ruins. On their top an inoxid steel plate is cemented, with 120° cuts for the forced centering of the measuring devices. The distances among the observed vertices are of the order of 100 m; only one side is over 250 m (side 101-105 = 273.05 m). The maximum slope of the zenithal directions is below 5 grades.

The instruments which we employed are an electronic Wild T 2000/S theodolite, with GRE registrator; a Wild DI 5 distance meter; a set of rods and reflectors of the same firm.

3.2 - This octagonal net must be considered as a primordial net, dedicated to frame the whole survey of the monument: like a first order net in terrestrial triangulation. Its purpose is not only to permit the setting up of local detail nets with shorter sides, but also to block up within fixed limits the error propagation, and to assure the homogeneity and uniformity of the results in any part of the survey.

Obviously this primordial net must be determined with all the accuracy obtainable from the available devices and procedures. A riguorous block adjustment is necessary, if we want to consider its points as fixed and error-free in the derivation of detail nets and single points. The octagonal net of the Coliseum well fits these requirements, as we shall see later on.

Its block computation has been performed in two different approachs:

- the first one is bi-dimensional for the planimetry, immediately followed by the altimetric computation (Bencini's program [5], modified by Birardi);
- the second one is tri-dimensional, with the program by Ferrara - Giannoni [6], see 3.5 and following.

The results obtained in these approachs are completely indifferent; the accuracy that both have reached is very good, as the rmse of the adjusted coordinates are less than 1 mm in planimetry, and 2 mm in height (see Annex 3), notwithstanding the feeble configuration of the net, as said in 3.1.

The control nets

3.3 - The above said octagonal net does not have a point distribution and density sufficient to give a good determination of control points on the external walls by multiple intersection. We must therefore provide local densification nets, not necessarily connected in an unique block adjustment with the primordial net. On the contrary, it seems convenable to provide separate nets, of a limited local extent (one elliptic quadrant at the most), such as not to suffer from the lack of homogeneity due to the different physical moments in which the measurements were necessarily carried out.

In fact, these measurements must be executed together with those for the determination of the controls on the wall, which certainly cannot be signalized and observed in one-two working days; moreover there are strong height differences among the control and detail points, some of which must be situated on the top of the external wall in order to derive the net relevant to the

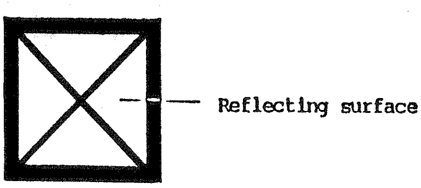

Fig. 6 - Signal for the control points

interior area of the Coliseum; furthermore, if we want to determine the detail and control points in an unique block - as it is advisable - we have to keep into account the different collimation conditions to the high and low points, and the considerable errors due to this situation, particularly in the heights[1]; last, the stations for the local nets are generally executed on tripod - seldom on the few pre-existing parapets and prop walls - with no claims for stability and repeatability, that is with very different requirements than the octagonal net. In one word, a block adjustment of the observations for the primordial, detail and control nets is not advisable.

3.4 - However, to probe the question - mainly for practical and operational purposes - we have considered a sample limited to a local detail and control net, which includes 1 point of the octagonal net, 3 ground detail points and 4 control points on the wall. Here we have done the observations which result in the sketch Annex 5; the adjustment has been done following the two procedures proposed in 3.2, and its results are resumed in Annex 6. We can see here slightly bigger differences than those in Annex 3, as they reach 12 mm in planimetry and 9 mm in altimetry; but anyhow we can say that the differences between bi- and tri-dimensional adjustment are practically negligible. We must point out that the mean length of the sides is here very small, so that the detrimental effects of the athmospheric refraction are negligible; in the ordinary terrestrial triangulation the tri-dimensional adjustment gives worse results.

In other tests we have checked the block adjustment of all the above said (8+3) points versus the adjustment of the sole 8 points of the primordial net, obtaining indifferent results (see Annex 3).

As an operative conclusion, we shall adopt the computation procedure which results from Annex 5, i.e.:

- local block adjustment supported by few points of the primordial net, considered as fixed and error-free;
- bi- and tri-dimensional computing approach, for a reciprocal checking of the results.

The tri-dimensional approach

3.5 - In each station we have first performed the station adjustment, and computed the mean values of the unit weight; we have then checked, by the Bartlett's test, the hypothesis that all the measurements belong to the same

[1] The signalization of the controls involves heavy problems relevant to their directional collimation, their distantiometric answer, their individuation on the photograms. After several attempts we decided to use plastic laminate signals, 12x12 cm square, with reflecting surface (see fig. 6); they give a good distantiometric answer in a wide azimuthal and zenithal field.

They have been cemented to the wall with silicon glue; to do this operation in the higher areas an elevator carriage was used, kindly conceded by the municipal ACEA agency.

corresponding class. Assuming that the correlations are null, the diagonal weight matrix which we have introduced in the computation is of this kind:

$$P = \begin{bmatrix} I & 0 & 0 \\ 0 & \dfrac{m^2_{dir}}{m^2_{dist}} * I & 0 \\ 0 & 0 & \dfrac{m^2_{dir}}{m^2_{zen}} * I \end{bmatrix}$$

with I = unit matrix, dimensioned on the number of the measured elements.

3.6 - In a tri-dimensional model, suitable for local nets, the observation equations concern the azimuthal and zenithal directions, and the slope distances. They may be written:

$$V_{ij} = -dV_0 + \frac{Y^0_j - Y^0_i}{d^2_{ij}} dx_i - \frac{X^0_j - X^0_i}{d^2_{ij}} dy_i - \frac{Y^0_j - Y^0_i}{d^2_{ij}} dx_j + \frac{X^0_j - X^0_i}{d^2_{ij}} dy_j + $$

$$+ \ell^0_{ij} - (\ell_m + V^0)$$

$$V_{ij} = -\frac{X^0_j - X^0_i}{D_{ij}} dx_i - \frac{Y^0_j - Y^0_i}{D_{ij}} dy_i - \frac{Z^0_j - Z^0_i}{D_{ij}} dz_i + \frac{X^0_j - X^0_i}{D_{ij}} dx_j + $$

$$+ \frac{Y^0_j - Y^0_i}{D_{ij}} dy_j + \frac{Z^0_j - Z^0_i}{D_{ij}} dzj + D^0_{ij} - D_m$$

$$V_{ij} = -\frac{X^0_j - X^0_i}{d^2_{ij}} \frac{Z^0_j - Z^0_i}{D_{ij}} dx_i - \frac{Y^0_j - Y^0_i}{d^2_{ij}} \frac{Z^0_j - Z^0_i}{D_{ij}} dy_i + \frac{X^0_j - X^0_i}{d^2_{ij}} \cdot$$

$$\cdot \frac{Z^0_j - Z^0_i}{D_{ij}} dx_j + \frac{Y^0_j - Y^0_i}{d^2_{ij}} \frac{Z^0_j - Z^0_i}{D_{ij}} dy_j + \frac{d_{ij}}{D^2_{ij}} dz_i \frac{d_{ij}}{D^2_{ij}} dz_j + \zeta^0 - \zeta_m$$

where x_o y_o z_o are the approximate coordinates of the vertices, d_{ij} and D_{ij} the approximate horizonthal and slope distances. The measured terms are indicated with the index $_m$.

The net has four degrees of freedom, i.e. three translations along the coordinate axes, and one rotation around the z axis. As constraints we have assumed a fixed point and the N coordinate of a second point; the results of this adjustment are reported in the Annex 4.

Afterwards, we have performed a free adjustment of the net, in order to evaluate its internal accuracy. To this purpose we have forced the measured net on the preceding one, by a Helmert's transform. The singularity of the normal matrix, extended to the whole of the net's points, has been removed by imposing:

$$x^T x \equiv \text{minimum}$$

We have assumed as unknowns the sole coordinates, excluding orientation or instrumental constants. For this we did a partial minimization of the matrix' trace, i.e. we have edged the original N matrix of the normal system, for the sole part relevant to the coordinates, with the matrix $[G_x \quad 0]^T$ of the eigen vectors corresponding to the four eigen values = 0.

The N matrix, decomposed in sub-matrices, is:

$$\begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix}$$

with N_{11} matrix relevant to the coordinates. The edged matrix is:

$$\left[\begin{array}{cc|c} N_{11} & N_{12} & G_x \\ N_{21} & N_{22} & 0 \\ \hline G_x^T & 0 & 0 \end{array}\right]$$

It is inversible, and its inverse, decomposed into the correspondant sub-matrices, is:

$$\left[\begin{array}{cc|c} Q_{11} & Q_{12} & U_{11} \\ Q_{21} & Q_{22} & U_{22} \\ \hline U^T_{11} & U^T_{22} & 0 \end{array}\right]$$

The internal accuracy of the net may be computed by the formula:

$$\sigma_p = \sigma_0 \sqrt{(tr(Q_{jj})/n)}$$

where $s_0 = \pm\sqrt{(V^T PV/r)}$, and r = number of the degrees of freedom of the net = $n_{eq} - n_{unkn} + 4$.

The rmse of the point obtained by this free adjustment of the net resulted 0.308 mm.

CHAPTER 4

THE PHOTOGRAMMETRIC SETUP

The photogrammetric taking

4.1 - The realization of the photogrammetric taking is the most important operation in the general photogrammetric setup; in fact, it involves the definition of all the parameters which qualify the survey itself. In the case of our monument, whose shape and dimensions imply not easy geometric and photogrammetric problems, there is the danger of errors hardly recoverable afterwards: it is enough to consider the cataloguing and documenting to be carried out on line with the taking operations. We want to issue an "historic" document, with any possible information on the present state of the monument; therefore it is necessary not only to consider its geometric and topographic aspects, but also the utilisation of the picture as an historic and artistic document. The planning of the takings has undergone several variations, in order to fit the solution of the different problems, arisen in the first experiments. The input data of this planning, such as they result in this first approach, are resumed as follows:

a) - image definition and quality
b) - check of the projection deformations
c) - obtainable accuracy.

4.2 - The image definition of the smallest observable object - which, in terms of visual sharpness, may be evaluated in 3÷4 mm for our taking distances - requires a photographic scale S_m between 1:270 and 1:360; we shall assume 1:300. In order to have good lighting conditions of the wall we have scheduled a time table for the takings, so that deep shadows and hard contrasts be avoided also with bright sunshine. In the first quadrant we have seen that, in September, midday is the best time for the taking.

4.3 - We have examined several projection procedures of the curve elliptic surface on the photographic and "cartographic" plane, in order to obtain a polivalent solution suitable for archaeology, restoration, static analysis, etc.

As we have seen above, the (**shq**) reference assures the continuity of the representation of the whole wall. The analytical projection of any detail on it may follow two ways:

a) - *a priori* transformations of the plate coordinates (x_p, y_p), by a suitable law which takes into account the differences of length between arc and chord. This is the way which we have followed (see 2.9);

b) - *a posteriori* transformation of the model coordinates (x_m, y_m, z_m). In this case we have two or more archives, but there is the possibility of going back to the model coordinates in order to obtain new projections. The direct drawing from the model coordinates can be done only for limited extensions; for an arc of ≈ 20 m the deformation may reach ≈ 15 cm.

4.4 - The accuracy of the final coordinates, which are function of the number and location of the control points, may be evaluated as follows [7]:

$$M_x = M_y = 1.5\ s_0 * \Sigma = 1.5 * 0.02 * 360 = 10.8 \text{ mm}$$

$$M_z = 2.1\ s_0 * \Sigma = 2.1 * 0.02 * 360 = 15.1 \text{ mm}$$

where s_0 is the rmse of the y-parallax, and $\Sigma = D/f$ the scale factor. This in the hypothesis that i) - only accidental errors are present; ii) - a correct LS adjustment is carried out; iii) - we have normal takings, i.e. with axis normal to the front of the survey.

We plan to check *a posteriori* the above accuracy by tests performed on some points determined with high precision ground triangulation.

4.5 - For the preparation of a project of taking, which can give more than one plotting possibility, two approachs may be considered:

a) - as in aerial photogrammetry: takings with constant interval equal to the computed base;longitudinal overlap= 60%; controls obtained by "aerial triangulation". The base and overlap must be computed in such a way that the plotting of any single model presents deformations within fixed limits. If the deformation must not exceed 3 cm, the maximum length of the arc s in the model should not exceed 14.5 m for the external E1 ellipse, and 12.0 m for internal E2 ellipse:

$$s_{max,E1} < 14.5 \text{ m} ; \qquad s_{max,E2} < 12.0 \text{ m}$$

b) - as in terrestrial photogrammetry: takings from predetermined stations, oriented with good accuracy on the normal to the model exceeds 13 m, a projective coordinate transformation is necessary, based on the computed arc/chord ratio.

In this approach a poliedric rapresentation of the monument is obtained. This is not suitable for aerial triangulation, but is the only way with analog plotters. Although analytic plotting is foreseen, we deemed it opportune to carry out the taking and plotting of a sample in both approachs, in order to get the greatest flexibility for the future users.

4.6 - We resume here the essential data used for the taking of a sample with the P31 camera:

1. - taking distance: D=30 m, corrisponding to a:
2. - photographic scale: S_m = 1:300;
3. - overlap: ≈ 67%;
4. - base: b = ≈ 11 m;
5. - ground side of the photogram: L = ≈ 33 m;
6. - stereoscopic arc: L_{st} = ≈ 22 m.

4.7 - The operations were carried out as follows:

- we executed the taking on the SW side, relevant to about one half of the whole first internal ellipse E2 (see the sample Annex 8). Two strips were taken, a "low" one on tripod, and a "high" one on elevator carriage. 14 stations were executed in each strip, at ≈ 1.6 m and resp. ≈ 17.0 m heights. In no case the Ω inclination exceeded 5 grades;
- negative films b/w and colour diapositives were taken, format 4"x5", in order to have photograms suitable both for precise plotting and for photo-interpretation;
- the photograms were recorded following the rules of the Italian Istituto Centrale per il Catalogo e la Documentazione. Each record contains all the data to be used in the plotting or photo-interpretation operations, i.e.:
  - a. - general information and geographic data;
  - b. - general information on the place, time and execution modalities;
  - c. - orientation data:
    - c_1 - internal orientation;
    - c_2 - external orientation, preliminar dimensioning, general Φ and Ω angles;
    - c_3 - external orientation, control points, coordinates and monographies;
  - d. - coordinates of the taking stations;
  - e. - graphic sketch of the control location;
  - f. - card with the stereoscopic pictures;
  - g. - graphic planimetric sketch of the taking stations;
  - h. - graphic altimetric sketch of the taking stations;

The sample plotting

4.8. - Two sample plottings were carried out, in order to evaluate the actual problems relevant to levels definition, plotting times, and quantity of the points to be plotted for the numerical recording. To this purpose we effected an analytical plotting at the Kern DSR 11/H and resp. an analogical plotting at the Galileo Simplex IIc instrument.

In the analytical plotting we kept into account the information requirements fixed in the general planning. We set up a particular "information system" in a data base, including areas of geometric, architectural, construction-technical, historical, ambiental, etc. interest. This presumes that the plotting was correctly digitized in the planned levels since its first execution, obviously in order to perform an easy editing and classification at the graphic workstation.

The first definition of the codifying levels is reported in the Table I; it is a first approach of the problem, which is more deeply investigated in the paper "Data Acquisition and Standard Metafile..." by R. Carlucci and A. Paoluzzi, to be presented here. The sample in the Table I is limited to one "high" model; the various levels are reported in different colours.

The analogical plotting at the Simplex IIc instrument (with encoders) was carried out to evaluate times and possibilities of the analog versus the numerical approach, particularly for interpretation purposes. The sample, which is reported in the Table II, was plotted by two students for their diplome thesis; it includes the whole wall extended to four arcades, starting from the W vertex of the ellipse. The drawing includes the contours, on a separate sheet; it cannot issue a "technical" classification, as in the analytical approach, but gives a more "artistic" representation, due to the hand drawing which follows the plotting.

Planning the future

4.9. - In the next three years we hope to complete the survey of the E1 and E2 walls, the internal prospects and the aerophotogrammetric map at the scale 1:100.

Contemporarily, the architectural and thematic works (static, thermographic, archaeological, etc.) will be carried out, together with the ground survey of the internal parts.

We hope to present the results - with God's help - at the next 1992 quadriennal Congress of our ISPRS.

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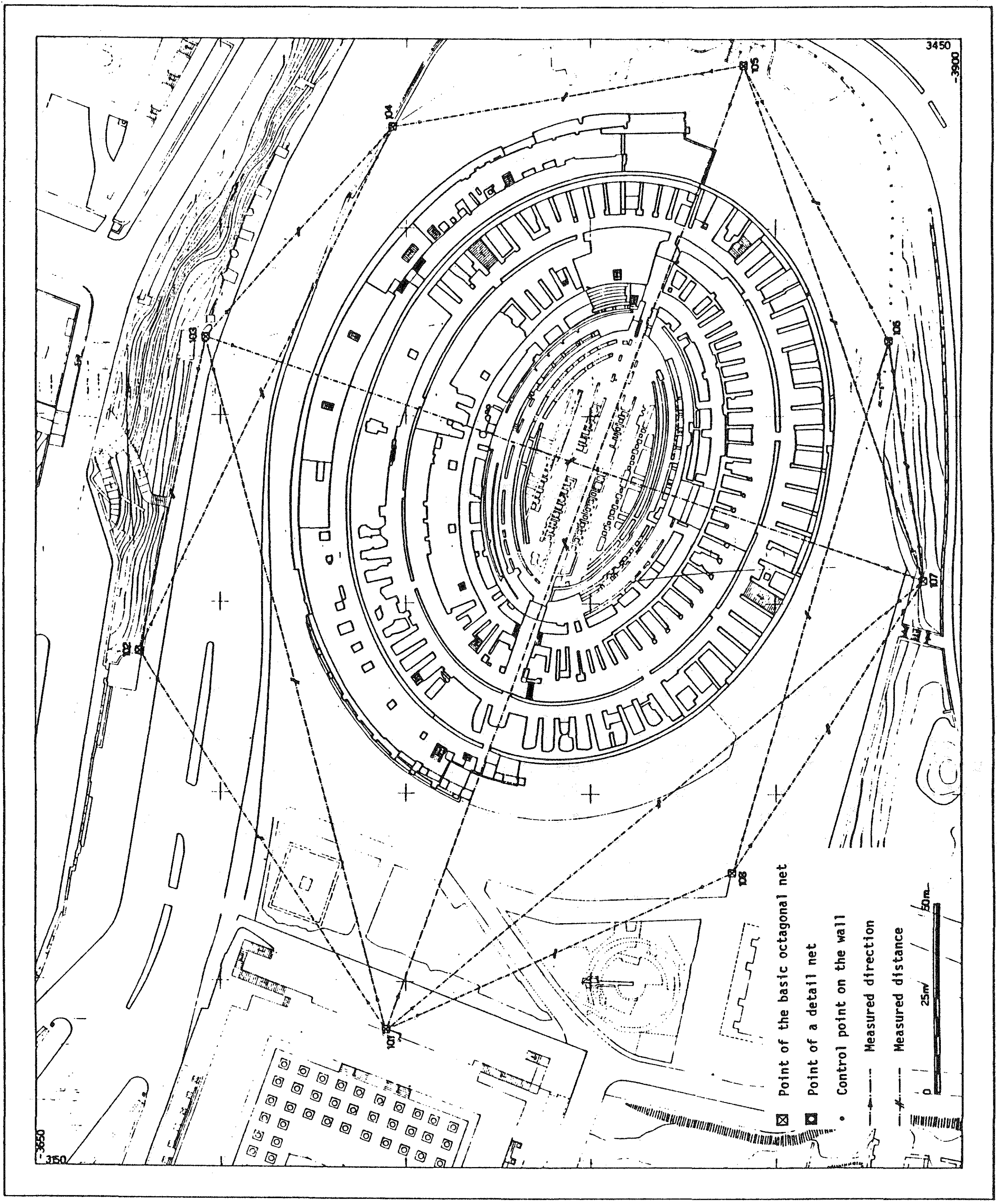
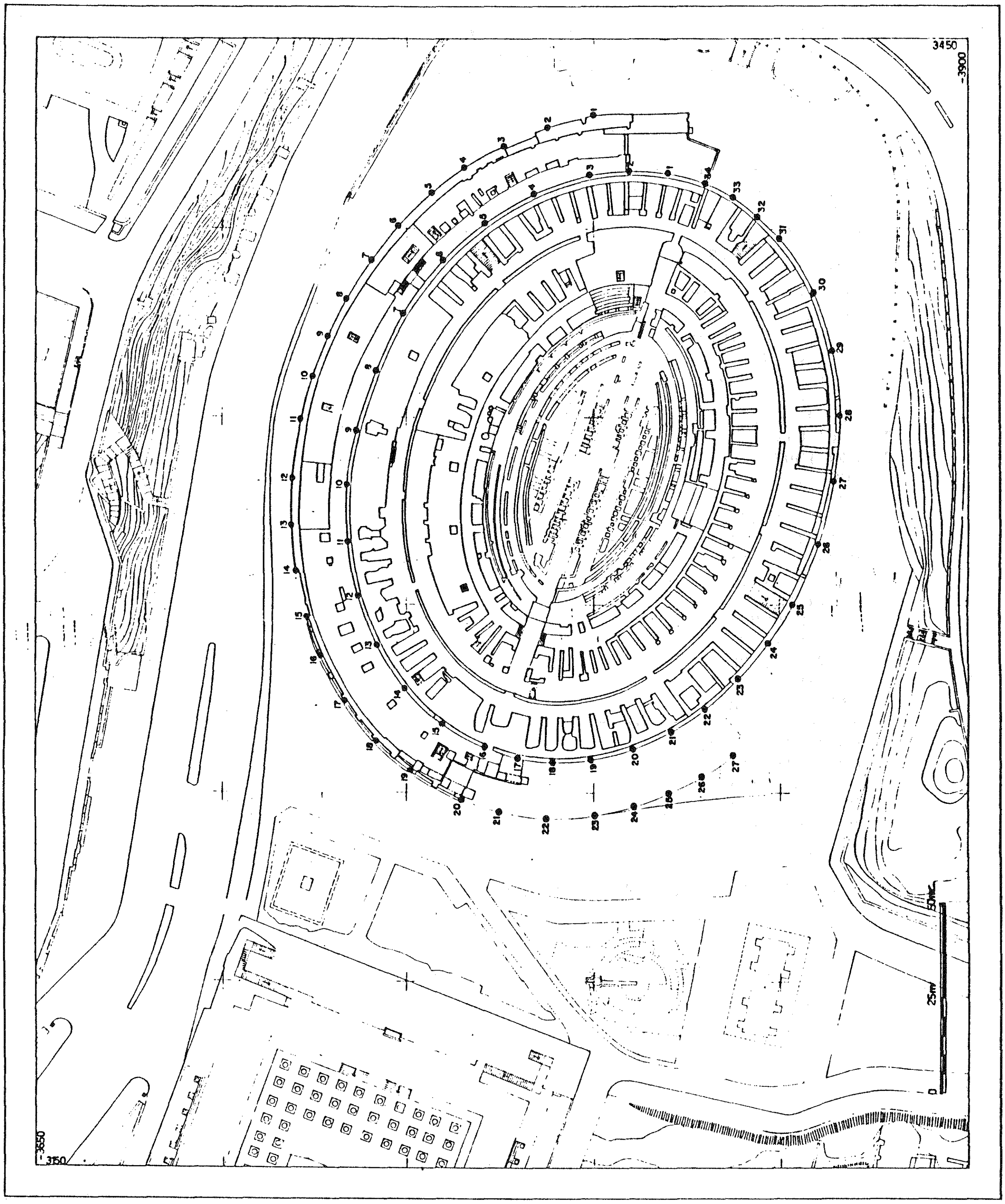
ANNEX 1

Plant of the Coliseum, with the primordial net



ANNEX 2

Plant of the Coliseum, with points graphically measured



ANNEX 3

Samples of the computation program, and computation outputs

```
C       ****************************************
C       *  CALCOLI RELATIVI ALL'ELLISSE MEDIA *
C       ****************************************
        ........

C       ************************
C       * PROGRAMMA PRINCIPALE *
C       ************************
        OPEN(5,FILE='ELLE',STATUS='OLD')
        OPEN(6,FILE='COL',STATUS='NEW')
        PI=3.141592953
        A1G=0.017453293
        REWIND 5
      3 READ(5,*)NELL,NPN
        IF(NELL.EQ.0)GO TO 999
        IF(NELL.GT.100)GOTO 90
        IF(NELL.GT.10)GO TO 80
        DO 71 I=1,NPN
        TT(I)=0.
        DO 72 K=1,5
     72 CO(I,K)=0.
     71 CONTINUE
        SOM=0.
        SOM1=0.
        DO 1 I=1,NPN
        READ (5,*)NO(I),X(I),Y(I)
        SOM=SOM+X(I)
        SOM1=SOM1+Y(I)
      1 CONTINUE
        XM=SOM/NPN
        YM=SOM1/NPN
        DO 73 I=1,NPN
        X(I)=X(I)-XM
        Y(I)=Y(I)-YM
        IF(NO(I).GT.100)GO TO 73
        CO(I,1)=X(I)**2
        CO(I,2)=X(I)*Y(I)
        CO(I,3)=Y(I)**2
        CO(I,4)=X(I)
        CO(I,5)=Y(I)
        TT(I)=1.
     73 CONTINUE
        CALL MQ(NPN,5,CO,TT,AQ,RES,DT,RQQ)

        ..........
C
C       CALCOLO DEGLI SCARTI E DELLA PRECISIONE
C
        U=A13*4.
        V=A33*4.
        DR(1)=-U*4.*C/V**2
```

```
      DR(2)=(E*V+2.*B*U)/V**2
      DR(3)=(-2.*D*V+2.*B*U)/V**2
      DR(4)=-2.*C/V
      DR(5)=B/V
      N=1
      GO TO 40
   32 U=A23*4.
      DR(1)=(2.*E*V-U*4.*C)/V**2
      DR(2)=(-D*V+2.*B*U)/V**2
      DR(3)=-U*4.*A/V**2
      DR(4)=-B/V
      DR(5)=2.*A/V
      N=2
      GO TO 40
   33 U=B*D*E-C*D**2-A*E**2
      V=4.*A*C-B**2
      W=SQRT((A-C)**2+B**2)
      GA=1/(2.*AG*ALFA**2)
      GB=1/(2.*BG*BETA**2)
      DS(1)=(-V*E**2-U*4.*C)/V**2
      DS(2)=(V*D*E+U*2.*B)/V**2
      DS(3)=(-V*D**2-U*4.*A)/V**2
      DS(4)=(B*E-2.*C*D)/V
      DS(5)=(B*D-2.*A*E)/V
      DT(1)=(1.+(A-C)/W)/2.
      DT(2)=B/(2.*W)
      DT(3)=(1.-(A-C)/W)/2.
      DT(4)=0.
      DT(5)=0.
      DO 34 J=1,5
   34 DR(J)=GA*(ALFA*DS(J)-GAMMA*DT(J))
      N=3
      GO TO 40
   35 DT(1)=(1.-(A-C)/W)/2.
      DT(2)=-DT(2)
      DT(3)=(1.+(A-C)/W)/2.
      DO 36 J=1,5
   36 DR(J)=GB*(BETA*DS(J)-GAMMA*DT(J))
      N=4
   40 SOM=0.
      DO 41 J=1,5
      DO 42 K=1,5
   42 SOM=SOM+DR(J)*DR(K)*EM(J)*EM(K)*RO(J,K)
   41 CONTINUE
      EQM=SQRT(SOM)
      GO TO(43,44,45,46),N
   43 EXC=EQM
      GO TO 32
   44 EYC=EQM
      GO TO 33
   45 EAG=EQM
```

```
      GO TO 35
   46 EBG=EQM
      EAGG=EAG
      IF(BG.LT.AGG) GO TO 47
      EAG=EBG
      EBG=EAGG
   47 WRITE (6,858)EXC,EYC,EAG,EBG
      WRITE (6,852)
   82 DO 14I=1,NPN
      CALL QUOTES(NO(I),XG(I),YG(I),FI1,RO1,QOT,S)

      ............

C      *********************
C      *                   *
C      * SUBROUTINE QUOTES *
C      *                   *
C      *********************
C
      SUBROUTINE QUOTES(NO,CSI,ETA,FI1,RO1,QOT,S)
      COMMON AG,BG,E2,EL
      CS=ABS(CSI)
      ET=ABS(ETA)
      E12=E2/(1.-E2)
      ACCA=0.
      IND=1
    3 C1=1.+E12/(1.+ACCA)
      FI1=ATAN(C1*ET/CS)
      CF1=COS(FI1)
      TF1=TAN(FI1)
      ETA2=E12*CF1**2
      VU=SQRT(1.+ETA2)
      ELLOP=SQRT(CS**2+ET**2)
      ACCA=(ELLOP/BG-VU)*VU-.5*BG*(E12-ETA2)*ETA2/(ELLOP*VU)
      IND=IND+1
      IF(IND.GT.4)GO TO 4
      GO TO 3
    4 FI=FI1
      W=SQRT(1-E2*SIN(FI)**2)
      RO1=AG*(1.-E2)/W**3
      QOT=CS/COS(FI)-AG/W
      PI=3.141592953
      IF(CSI.LT.0..AND.ETA.GT.0.) FI=PI-FI
      IF(CSI.LT.0..AND.ETA.LT.0.) FI=PI+FI
      IF(CSI.GT.0..AND.ETA.LT.0.) FI=2*PI-FI
      XQ=AG*COS(FI)/W
      YQ=AG*(1.-E2)*SIN(FI)/W
    5 IF(ABS(XQ).GE.0.000001)GOTO 1
      U=PI/2
      GOTO 2
    1 U=ATAN(ABS(YQ/(XQ*SQRT(1-E2))))
```

```
      IF(XQ.LT.0..AND.YQ.GT..0) U=PI-U
      IF(XQ.LT.0..AND.YQ.LT..0) U=PI+U
      IF(XQ.GT.0..AND.YQ.LT..0) U=2*PI-U
    2 TI0=U
      TI2=-.5*E2*(U/2.+SIN(2.*U)/4.)
      TI4=-(E2**2/8.)*(3.*U/8.+SIN(2.*U)/4.+SIN(4.*U)/32.)
      TI6=-(E2**3/16.)*(5.*U/16.+15.*SIN(2.*U)/64.+3.*SIN(4.*U
     1)/64.+SIN(6.*U)/192.)
      TI8=-(10.*E2**4/256.)*(35.*U/128.+7.*SIN(2.*U)/32.+7.*
     1SIN(4.*U)/1128.+SIN(6.*U)/96.+SIN(8.*U)/1024.)
      S=AG*(TI0+TI2+TI4+TI6+TI8)
      RETURN
      END
C     **********************
C     *                    *
C     *   SUBROUTINE INTSZ *
C     *                    *
C     **********************
C     CALCOLA I PUNTI DI INTERSEZIONE (XA,YA),(XB,YB)
      SUBROUTINE INTSZ(XP,YP,EMME,XINT,YINT)
      COMMON AG,BG,E2,EL
      XX=ABS(XP)
      YY=ABS(YP)
      PINT=YY-EMME*XX
      RDC=SQRT(BG**2*(EL+EMME**2)-EL*PINT**2)
      PAR=-EMME*PINT+RDC
      XP1=PAR/(EL+EMME**2)
      YP1=EMME*XP1+PINT
      PAR=-EMME*PINT-RDC
      XP2=PAR/(EL+EMME**2)
      YP2=EMME*XP2+PINT
      DP1=SQRT((XP1-XX)**2+(YP1-YY)**2)
      DP2=SQRT((XP2-XX)**2+(YP2-YY)**2)
      IF(DP1.GT.DP2)GOTO 1
      XINT=XP1
      YINT=YP1
      GOTO 2
    1 XINT=XP2
      YINT=YP2
    2 RETURN
      END
```

CALCOLO DELL'ELLISSE MEDIA NO. 2

NPN= 34

A= -.0001644 B= -.0000588 C= -.0002349 D= .0006017 E= .0006477
EQM= .0000004 .0000008 .0000006 .0000242 .0000311

455.5825000
XC=3336.519 YC=-3800.080 AG=80.681 BG=63.844 E2= .374 TH= -195552.
EQM= .075 .067 .092 .079

NP	X	Y	XG	YG	FI	RO	Q	S
1	3414.400	-3820.000	80.007	7.822	.15495	51.203	-.065	7.866
2	3415.100	-3810.000	77.256	17.462	.34715	53.986	-.327	17.943
3	3414.100	-3799.500	72.737	26.992	.53596	58.923	-.327	28.568
4	3409.000	-3783.900	62.624	39.919	.79502	69.366	-.244	45.097
5	3401.200	-3770.900	50.860	49.481	.99904	80.071	-.075	60.315
6	3391.200	-3759.600	37.607	56.695	1.17668	89.838	.190	75.414
7	3377.300	-3749.100	20.960	61.828	1.36141	98.136	.172	92.827
8	3362.100	-3741.900	4.216	63.416	1.52927	101.800	-.341	109.672
9	3345.900	-3736.500	-12.855	62.970	1.44370	100.508	-.058	126.800
10	3331.700	-3734.000	-27.056	60.479	1.29717	95.636	.318	141.207
11	3316.300	-3734.200	-41.465	55.042	1.12939	87.307	.242	156.581
12	3301.600	-3737.000	-54.330	47.398	.94768	77.243	.154	171.531
13	3288.900	-3742.200	-64.497	38.180	.75765	67.609	-.126	185.269
14	3277.300	-3749.700	-72.846	27.175	.53769	58.979	-.139	199.136
15	3267.900	-3759.400	-78.376	14.852	.29403	52.997	-.087	212.705
16	3261.700	-3771.000	-80.250	1.833	.03657	50.559	-.397	225.958
17	3258.800	-3779.600	-80.045	-7.240	.14358	51.106	-.114	235.090
18	3257.800	-3789.300	-77.678	-16.700	.33113	53.670	-.183	244.886
19	3258.400	-3799.700	-73.569	-26.273	.51818	58.362	.031	255.331
20	3261.200	-3810.300	-67.324	-35.283	.69660	64.912	.061	266.299
21	3265.900	-3820.800	-59.326	-43.552	.86381	72.791	.209	277.791
22	3271.900	-3829.800	-50.617	-49.968	1.00486	80.394	.205	288.587
23	3280.000	-3838.900	-39.900	-55.762	1.14896	88.366	.242	300.750
24	3289.300	-3847.100	-28.362	-60.300	1.28354	95.045	.507	313.107
25	3299.700	-3853.100	-16.539	-62.396	1.40640	99.560	-.091	325.080
26	3315.800	-3860.000	.948	-63.394	1.56145	101.949	-.445	342.662
27	3332.800	-3864.200	18.362	-61.547	1.38677	98.975	-.611	360.287
28	3350.100	-3866.000	35.239	-57.342	1.20360	91.229	-.088	377.761
29	3367.300	-3864.000	50.727	-49.599	1.00123	80.192	-.048	395.122
30	3382.900	-3858.900	63.654	-39.486	.78002	68.652	.176	411.551
31	3396.900	-3850.000	73.782	-26.347	.51748	58.340	.252	428.128
32	3402.600	-3844.300	77.197	-19.045	.37487	54.571	.177	436.165
33	3407.900	-3837.600	79.896	-10.940	.21467	51.834	.399	444.669
34	3411.600	-3830.200	80.852	-2.722	.05361	50.602	.244	452.902

VQM DELLA Q = .28

MATRICE VARIANZA-CORRELAZIONE

.00000	.21574	-.30704	-.02439	.05645
.21574	.00000	.16650	.04752	-.01540
-.30704	.16650	.00000	-.00066	-.09660
-.02439	.04752	-.00066	.00001	.16065
.05645	-.01540	-.09660	.16065	.00002

PUNTO DI PRESA NO. 101

NP	X	Y	XG	YG	FI	RO	Q	S
1	3366.000	-3711.000	-2.651	93.794	1.55070	101.921	29.977	115.952

COORDINATE PUNTI INTERSEZ. A,B

-6.682	63.624	10.765	63.273

CORDA,ARCO,DIFFERENZA %

17.451	17.472	.123

COORDINATE CORRETTE REPERES

.000	-27.501	.000	-27.501
-57.498	-.001	-57.569	-.001
.000	57.503	.000	57.503
57.494	-.001	57.565	-.001

PUNTO DI PRESA NO. 106

NP	X	Y	XG	YG	FI	RO	Q	S
6	3229.000	-3774.000	-109.969	-12.134	.15049	51.164	30.205	235.444

COORDINATE PUNTI INTERSEZ. A,B

78.008	16.295	80.663	-1.321

CORDA,ARCO,DIFFERENZA %

17.814	17.881	.371

COORDINATE CORRETTE REPERES

.000	-27.501	.000	-27.501
-57.498	-.001	-57.711	-.001
.000	57.503	.000	57.503
57.494	-.001	57.707	-.001

FINE

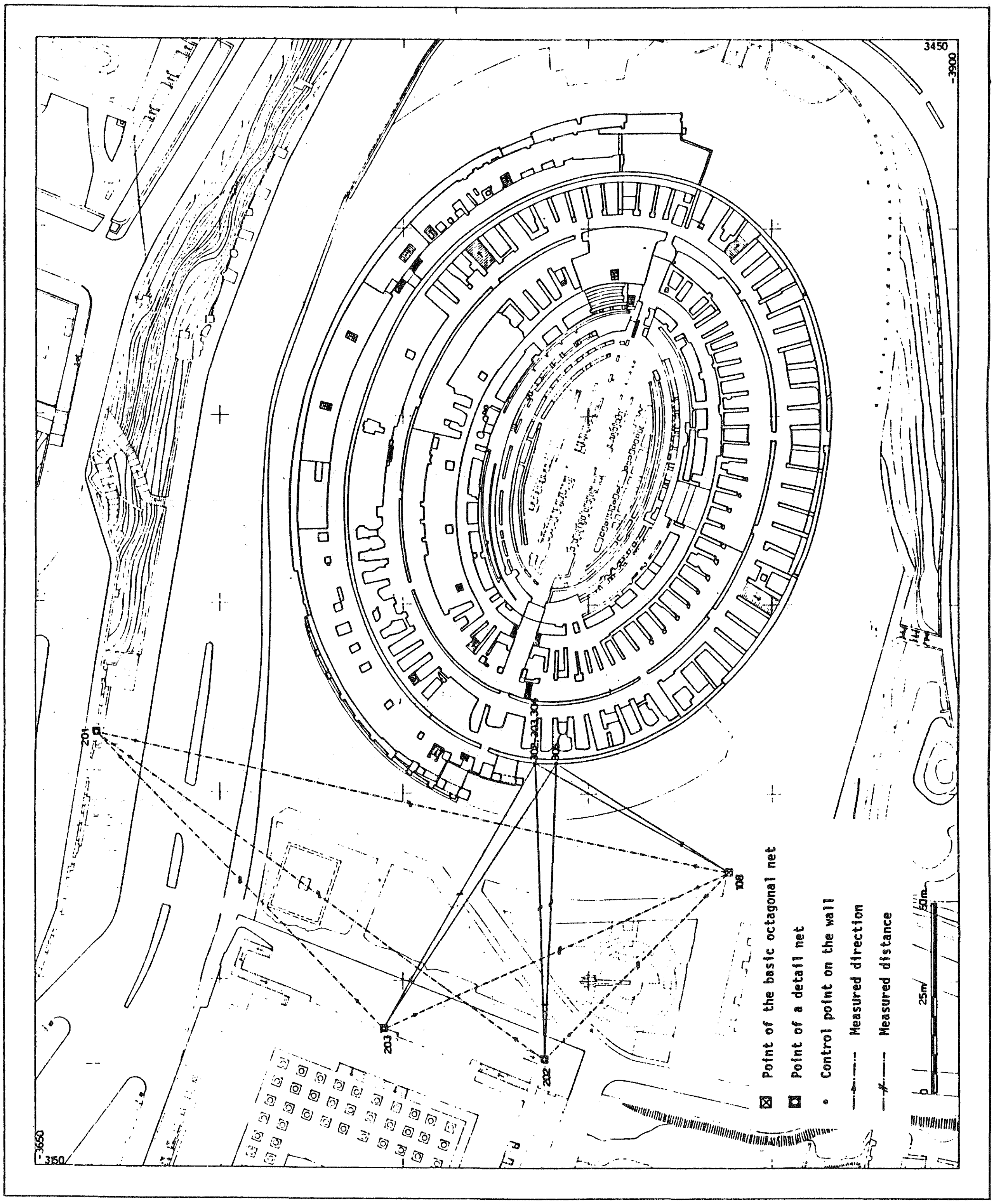
ANNEX 4

Results obtained in the adjustment of the octagonal primordial net, with a. - bi-dimensional + height approach (Bencini- Birardi's program); b. - tri-dimensional approach (Ferrara- Gianonni's program); c. - tri-dimensional approach on 8+3 points (id.id).

Point	Approach	Coordinates (m) N	E	H	R.m.s.e. (mm) N	E	H
	a.	-3744.841	3187.501	32.041	0	0	2
101	b.	.839	.500	.039	-	2	0
	c.	.839	.500	.040	-	1	0
	a.	-3678.251	3286.525	25.232	1	1	2
102	b.	.250	.526	.231	2	2	1
	c.	.250	.526	.231	1	1	0
	a.	-3696.259	3371.820	25.283	1	1	2
103	b.	.258	.822	.283	2	2	1
	c.	.259	.822	.285	1	1	0
	a.	-3746.573	3427.516	26.853	1	1	2
104	b.	.573	.518	.851	2	2	1
	c.	.574	.517	.852	1	1	0
	a.	-3841.993	3442.971	25.070	1	0	2
105	b.	.994	.973	.068	2	1	0
	c.	.994	.972	.069	2	1	0
	a.	-3880.317	3370.606	28.201	1	0	1
106	b.	.317	.607	.202	2	1	0
	c.	.317	.607	.203	2	1	0
	a.	-3890.537	3306.043	28.639	0	0	1
107	b.	.538	.043	.641	1	1	0
	c.	.537	.043	.642	1	1	0
	a.	-3838.780	3228.341	22.950	-	-	-
108	b.	.780	.341	.950	-	-	-
	c.	.780	.341	.950	-	-	-
201	c.	-3666.574	3266.283	32.0481	1	1	0
202	c.	-3788.266	3179.710	26.991	1	1	0
203	c.	-3744.837	3187,501	32.403	1	2	0

<u>ANNEX 5</u>

Plant of the Coliseum, with a detail net and control points



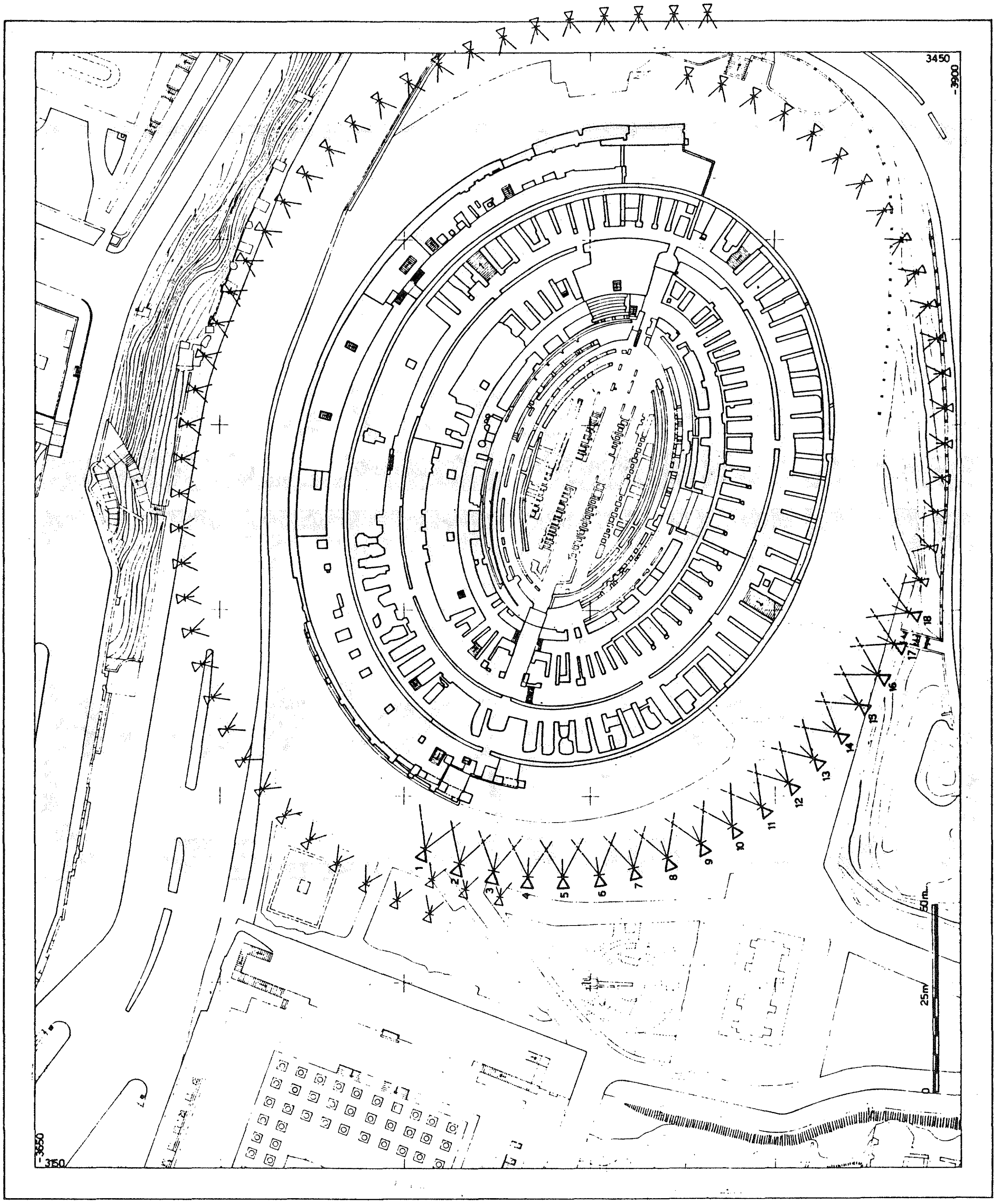
ANNEX 6

Results obtained in the adjustment of a sample of the detail and control nets (1 point of the octagonal net, 3 detail points, 4 control points) with: **a.** - bi-dimensional + height approach (Bencini-Birardi's program); **b.** - tri-dimensional approach (Ferrara-Giannoni's program).

Point	Approach	Coordinates (m) N	E	H	R.m.s.e. (mm) N	E	H
108	a.	-3838.780	3228.341	22.950	-	-	-
	b.	.780	.341	.950	-	-	-
201	a.	-	-	-	-	-	-
	b.	-3666.574	3266.283	32.481	1	1	0
202	a.	-	-	-	-	-	-
	b.	-3788.266	3179.710	26.991	1	1	0
203	a.	-	-	-	-	-	-
	b.	-3744.837	3187.501	32.403	1	2	0
302	a.	-3785.290	3258.279	52.872	3	2	1
	b.	.289	.284	.863	1	1	3
303	a.	-3785.658	3258.050	43.119	3	2	1
	b.	.659	.062	.110	1	1	2
304	a.	-3785.235	3258.156	37.343	3	2	1
	b.	.237	.165	.338	1	1	2
305	a.	-3791.079	3258.046	26.224	3	3	1
	b.	.091	.051	.221	1	1	2

ANNEX 7

Sketch of the taking stations



ANNEX 8

Stereoscopic takings of the sample area

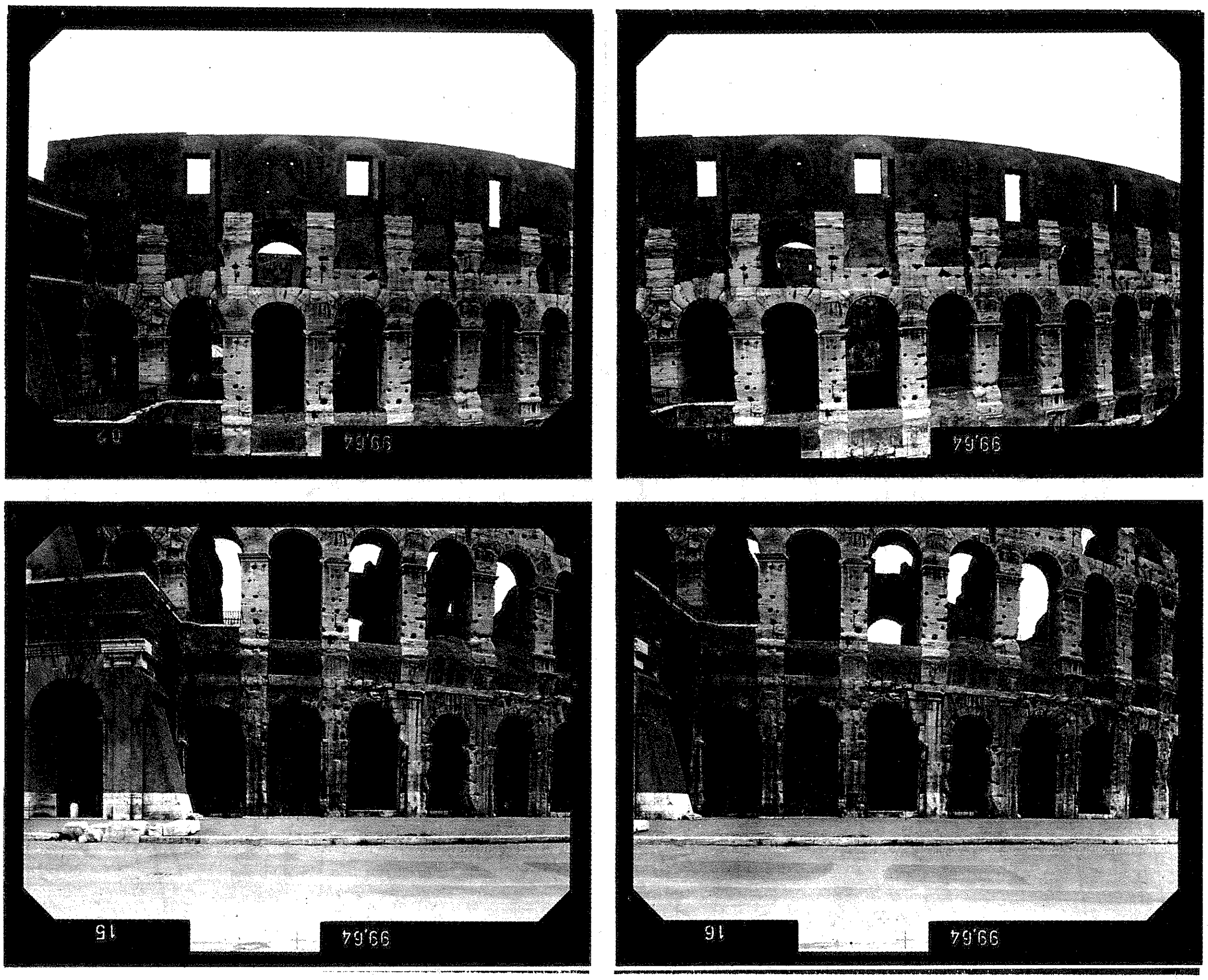


TABLE 1

Analytical plotting

RILIEVO FOTOGRAMMETRICO DEL COLOSSEO

CAMPIONE DI RESTITUZIONE NUMERICA ALLA SCALA 1:50

DI 3 ARCATE - STRISCIATA ALTA - ELLISSE E 2

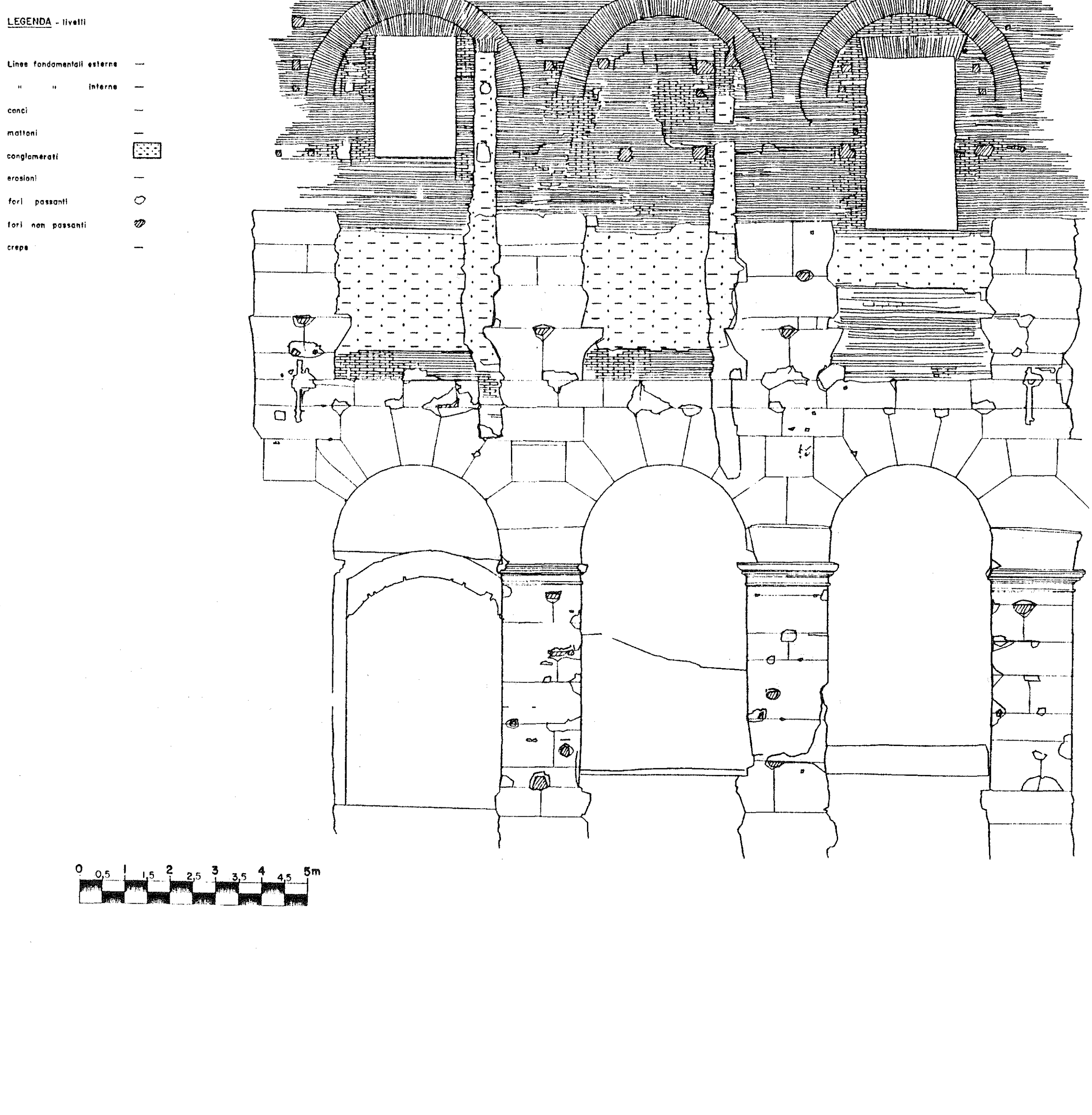


TABLE 2

Analogycal plotting