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## ABSTRACT

The method of determining the actual geometric shape of the surface of the cooling tower after a failure was presented in the paper. The method of surrounding tangents was used to establish deformation in horizontal sections. Deformation walues calculated for each horizontal section and for each photogrammetric stand.

## INTRODUCTION

Construction and geometric shape of cooling towers determine strictly defined methods of gedetic and photogrammetric measurements. The fact, that it is not possible to have points of signalization on the tower surface, implies the application of surrounding tengents method as the optimum method for the measurement of the shape of the cooling tower surface.

The inventory measurement for determining the actual shape of the surface or testing deformation on the basis of control measurements is the aim of photogrammetric measurements of the cooling tower. Determining the actual surface shape on the basis of photogrammetric measurements is much more difficult. It is determined most often on the basis of approximation of rotational object radii and deformation in vertical planes [1,3]. This method is efficient when one studies cooling tower axis deflections and displacements using cyclic measurements. However, when the task is to determine the actual shape and to compare it with the theoretical one, the problem should be considerably developed. Different way of solving the problem results from the basic assumptions of the method of surrounding tangents in which the points of tangency to the surface of rotational hyperboloid shape are the subject of measurements.

## METHODOLOGY OF DETERMINATION

The approximate value of a radius, which is determined for individual horizontal sections, is one of the parameters which is calculated on the basis of photogrammetric measurements.

The radius values in all horizontal sections were defined on the basis of the tangent equations and their distance from the circle centre (fig.1), in accordance with the equations:

$$
\begin{aligned}
& y_{i}=m_{i} \cdot x_{i}+y_{0} \\
& x=\left|\frac{m_{i} p-q+y_{0}}{\sqrt{m_{i}^{2}+1}}\right|
\end{aligned}
$$

and after transformations we get the following expressions for each section:

$$
\begin{equation*}
r^{\prime}=p \cdot \sin \varphi_{i}-q \cdot \cos \varphi_{i}+Y_{i} \cdot \cos \varphi_{i}-X_{i} \cdot \sin \varphi_{i} \tag{2}
\end{equation*}
$$



Fig.1. Determination of the horizontal section radii.

Approximating equation, from which the approximate values $p_{0}$. $q_{0}, r_{0}$ are established, are as follows:

$$
\begin{equation*}
v_{j}=p_{0} \cdot \sin \varphi_{j}-q_{0} \cdot \cos \varphi_{j}-X \cdot \sin \varphi_{j}+Y \cdot \cos \varphi_{j}-r_{0} \tag{3}
\end{equation*}
$$

where: $p_{0}, q_{0}$ - approximate coordinates of horizontal section
$x$ centre,
X, $X$ - coordinates of photogrametric stands,
$\alpha$ - azimuth of the direction tangent to cooling tower surface.
$\varphi=90^{\circ}-\alpha$

In the case when high cooling towers are observed from a short distance the values $p_{0}, q_{0}$, and $r_{0}$ are calculated taking dism placement of tangent points.

The calculated values $r_{i}$ and approximate corrections define the degree of cooling tower deformation in horizontal planes. A full picture of surface deformation can be obtained comparing the position of the points of tangency with the theoretical values.

The set of the intersection points of the conical surface marked out by vectors perpendicular to the straight line $\mathrm{p}\left(\overrightarrow{\mathrm{w}_{\mathrm{p}}}\right.$ ) and the vectors on the straight line $\mathrm{p}\left(\mathrm{W}_{\mathrm{s}}\right.$ ) in the pointspif tangency to the surface of the rotational hyperboloid (fig.2) are the geometric locus of the points of tangency


Fig.2. Observed points of tangency to the hyperboloid
This relationship can be put down as the scalar product of two vectors:

$$
\vec{w}_{\mathrm{ps}} \cdot \vec{w}_{\mathrm{s}}=0
$$

where the components of the perpendicular to the hyperboloid surface are:

$$
\vec{w}_{p s}=\left[-z_{x},-z_{y}, 1\right]
$$

Calculated partial derivatives $\left(z_{x}\right)$ and $\left(z_{y}\right)$ are:

$$
z_{x}=\frac{\lambda_{0} x}{z}, z_{y}=\frac{\lambda_{0} y}{z}, \text { gdzie }=\frac{b^{2}}{a^{2}}
$$

The components of the tangent vector to the hyperboloid are:

$$
\vec{w}_{s}=\left[x-x_{0}, y-y_{0}, z-z_{0}\right]
$$

Thus:

$$
\stackrel{w}{p s} \cdot \stackrel{\rightharpoonup}{w}_{s}=\left[\frac{-\lambda x}{z}, \frac{-\lambda y}{z}, 1\right] \cdot\left[x-x_{0}, y-y_{0}, z-z_{0}\right]=0
$$

After transformations we get the equation of the plane com prising the points of tangency:

$$
\begin{equation*}
z=\lambda \frac{y_{0}}{z_{0}} \cdot y+\lambda \frac{x_{0}}{z_{0}} \cdot x-\frac{b^{2}}{z_{0}} \tag{3}
\end{equation*}
$$

The shape of the cooling tower surface, observed from each stand is different according to the stand coordinates ( $x_{0}$, $y_{0}, z_{0}$. In order to define the geometry of cooling tower deformation it is necessary to determine additional components for the planes XY and ZX in the from of the difference betwen the actual and theoretical values.

The theoretical projection of the set of tangency points on the plane XY can be determined solving the equations:

$$
\begin{align*}
& z=\lambda \frac{x_{0}}{z_{0}} \cdot x+\lambda \frac{y_{0}}{z_{0}} \cdot y-\frac{b^{2}}{z_{0}} \\
& z^{2}=\lambda\left(x^{2}+y^{2}\right)-b^{2} \tag{4}
\end{align*}
$$

Accepting the position of the camera in the plane $Z Y$ and the designation:

$$
\lambda \frac{y_{0}}{z_{o}}=k \quad, \quad \frac{b^{2}}{z_{o}}=n
$$

we obtain:

$$
\frac{x^{2}}{\frac{\left(k^{2} b^{2}-b^{2}-n^{2} \lambda\right)}{\lambda\left(k^{2}-\lambda\right)}-\frac{\left(y-\frac{k \cdot n}{k^{2}-\lambda}\right)^{2}}{\frac{\left(k^{2} b^{2}-b^{2} \lambda-n^{2} \lambda\right)}{\left(k^{2}-\lambda\right)^{2}}}=1.210 .}
$$

Finally the equation representing the projection of the geometric locus of the points of tangency from a given position on the plane XV is of the following form:

$$
\begin{equation*}
\frac{x^{2}}{\frac{\left(y_{0}^{2} b^{2}-z_{o}^{2} a^{2}-a^{2} b^{2}\right) a^{2}}{b^{2} y_{o}^{2}-a^{2} z_{o}^{2}}}-\frac{\left(y-\frac{a^{2} b^{2} y_{0}}{b^{2} y_{o}^{2}-a^{2} z_{o}^{2}}\right)^{2}}{\frac{z_{o}^{2} a^{4}\left(y_{o}^{2} b^{2}-z_{o}^{2} a^{2}-a^{2} b^{2}\right)}{\left(b^{2} y_{o}^{2}-a^{2} z_{o}^{2}\right)^{2}}}=1 \tag{5}
\end{equation*}
$$

at the assumption: $z_{0} * 0$ and $x^{2}+y^{2}>a^{2}$ For $\mathrm{J}_{0} \rightarrow \infty$ the equation (5) assumes the form:

$$
\mathrm{y}=\mathrm{o}
$$

Fig. 3 present the shape of the hyperbola defined by the equation (5). It is conditioned by the distance of the camera from the cooling tower ( $y$ ) and the heigh of the camera axis in relation to the interesection point of the hyperbola asymptotes.

Solving the same problem in the projectionoon the $Z X$ plane we also make use of the equations (4). We get:

$$
z^{2}=\left(x^{2}+\frac{1}{k^{2}} z^{2}-\frac{2}{k^{2}} z n+\frac{n^{2}}{k^{2}}\right)-b^{2}
$$

and then:

$$
\frac{\left(z-\frac{n \lambda}{k^{2}-\lambda}\right)^{2}}{\frac{k^{2}\left(\lambda n^{2}-\lambda b^{2}-b^{2} k^{2}\right)}{\left(k^{2}-\lambda\right)^{2}}}-\frac{x^{2}}{\frac{\lambda n^{2}-\lambda b^{2}-b^{2} k^{2}}{\lambda\left(k^{2}-\lambda\right)}}=1
$$

and after transformation:

$$
\begin{equation*}
\frac{\left[z-\frac{a^{2} b^{2} z_{0}}{\left(y_{0}^{2} b^{2}-a^{2} z_{0}\right.}\right]^{2}}{\frac{y_{0}^{2} b^{4}\left(a^{2} b^{2}+a^{2} z_{0}^{2}-b^{2} y_{0}^{2}\right)}{\left(y_{0}^{2} b^{2}-a^{2} z_{0}^{2}\right)^{2}}-\frac{x^{2}}{\frac{a^{2}\left(a^{2} b^{2}+a^{2} z_{0}^{2}-b^{2} y_{0}^{2}\right)}{\left(b^{2} y_{0}^{2}-a^{2} z_{0}^{2}\right)}}=1.12} \tag{6}
\end{equation*}
$$

When $y \rightarrow \infty$ the equation (6) takes the form:

$$
\frac{x^{2}}{a^{2}}-\frac{z^{2}}{b^{2}}=1
$$



Fig.3. Projection of the points of tangency on the horizontal plane
The equation (6) is particulary significant from the point of view of the measurement. It presents a theoretical shape of the constructed cooling towers, which are observed from a given stand in the projection on the vertical plane, perpendicular to the line passing through the cooling tower axis and the stand.

## DETERMINATION RESULTS

Photogrammetric photos were taken by UMK-10/1318 camera, manufactured by Carl Zeiss - Jena, from 11 stands arranged in such a way as to obtain uniformly distributed points of tangency on the cooling tower surface. The geodetic net measurement was supplemented with the measurement of photopoints for the orientation of photogrammetric models in the horizontal and vertical planes.

The measurement of the background coordinates were performed according to the technology given in $[1,3]$.

$0 \longdiv { 1 0 2 0 3 0 4 0 \mathrm { cm } }$

Fig. 4. Actual and approximate shape of the cooling tower surface in the horizontal section.

Fig. 4 presents the actual shape of surface in the horizontal sections and the approximate values of the section radii. The degree of deformation in thehorizontal plane is defined by the difference between the planned values of the radii and the determined values obtained from different stands. For instance for the lowest section the maximum difference between the planned value of the radius and the real one is 6 cm at the approximation accuracy $\pm 2 \mathrm{~cm}$ and the radius determination accuracy $\pm 4 \mathrm{~mm}$ 。

The deformation values in the $2 X$ plane were determined from each stand in relation to the theoretical shape observed from each stand. The example of diffence between the theoretical shape of surface from typical stand and shape of meridian section presented fig. 5 .


Fig. 5. Distortion meridian section from the camera stand.

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