SYNTHETIC TECHNIQUE FOR BLUNDER DETECTION IN CLOSE RANGE PHOTOGRAMMETRY

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Abstract

A new concept about gross error was discussed , According to the fuzzy sets theory. Through considering the error subsets and their power set as well as the whole error set , it can be seen that traditional blunder detection can not be fully efficient . Based on fuzzy comprehensive estimation theory, a synthetic technique that combines data snooping , least sum method and some prior information to detetect blunders was proposed . The technique was tested by using close range photogrammetric data and the results were encouraged.

1. Introduction

In the practical methods of blunder location , data snooping and a lot of function of weight with robust properties for blunder detection show effective in a certain situation . But sometimes not any case in these methods can obtain fully satisfactory result at different system of adjustment . Whether the statistical quantity in data snooping or robust weight is the function of residuals , because gross error have been smoothed and distributed over many observations after the least squares treatment , the residuals can only reflect the observations that may be contain blunders in a certain extent, so the methods of blunder detection would be affected by the least squares adjustment . Sometimes the influence can not be overlooked because in some serious situation it might brińg about some pernicious result . In the matter of fact , since the factors that lead to blunders are complicated , the blunders have some fuzzy properties . From the view of fuzzy sets theory , the paper discussed the phonomena of error's fuzzy properties and the interaction of random error set, systematic error set , as well as gross error set , revealed from another point of view the reason why the methods of blunder detection can not be fully efficient, and proposed a synthetic technique for blunder detection .

2. The description of gross error's fuzzy properties

Under normal condition , the causes resulting gross error relate to many aspects , it may be due to the condition of observation , the instruments , the skilled level of observer even his mood or psychologyical quality /8/ or may be the interation result of these respects . Thus the gross errors appear some indefinite characters in statistics and some fuzzy properties .

2.1 The phonomenon of non-normal distribution of random error

When describing and treating random error , traditional theory of error almost suppose that random error is with normal distribution strictly . That is the necessary result of central limit theorem . The assumption can simplified the analysis and solution of random error . But we should pay attention to the presuppositions of central limit theorem :

(1) The numbers of terms of sum must be very large ;

(2) The contribution of every trem to the sum is very small .

Normal distribution is a approximation to the practical distribution while the above presuppositions have been satisfied to a certain extent . In the fact , it is difficult to say any practical case can result in a real normal distribution . Obviously , outside about $\pm 2\sigma$ of normal distribution density function , especially beyond the $\pm 2.5\sigma$, It's hard to believe the normal distribution is correct depiction to the practical situation . This trend is much conspicious when there are some blunders in observations . We know that normal distribution is based on the following inference :

There are equal errors $\pm \varepsilon_i/2$ (i=1,......k) , they play the same role in the sample , then the density function can be given by :

$$f(li)= \frac{1}{\sqrt{2\pi}\,\sigma_\ell} \exp\left\{- \frac{(li-e(li))}{2\,\sigma_{l_i}^2}\right\} \qquad (2.1.1)$$

however, in the practice, it is possible in some special condition , such as exsiting blunder , that L elements in the k elements do not play roles or the roles not equal , if the effective elements are m=k-l, then we can get the following deformed normal distribution density function /1/ :

$$f(li)= \frac{1}{\varepsilon\sqrt{2\pi}} \int_0^K \exp\left[\frac{-2\,\ell^2}{m\,\varepsilon}\right] \frac{f(m)}{\sqrt{m}}dm \qquad (2.1.2)$$

where : $f(m)=(1+\alpha)\,k^{-(\alpha+1)}\,m^{\alpha}$

$\alpha>0$: variable parameter . depend on the observation condition

As matter of fact, the random errors often trend to Eq.(2.1.2) ,the deformed normal distribution density function. So the methods of blunder location that based on the normal distribution of random errors must have some approximate properties, Sometimes it can cause unsatisfactory result .

2.2 The indefinite and discrete properties of gross error

In the observation process , because of the complication of cause that result in blunder, the distribution and the interaction of blunder will present some indefinition and discretion. Suppose there is a blunder ε_g in the data of observation, its symbol can not be determined , the practical blunder may be $+\varepsilon_g$ or $-\varepsilon_g$, we can consider it would be one of them in equal probability .

If there are two independent discret blunders $\pm\varepsilon_{g1}$, and $\pm\varepsilon_{g2}$, then the sum $\Delta\varepsilon$ would be the following possible value :

$$\Delta\varepsilon = \varepsilon_{g1} + \varepsilon_{g2} \qquad \Delta\varepsilon = \varepsilon_{g1} - \varepsilon_{g2}$$
$$\Delta\varepsilon = -\varepsilon_{g1} + \varepsilon_{g2} \qquad \Delta\varepsilon = -\varepsilon_{g1} - \varepsilon_{g2} \qquad (2.2.1)$$

let three blunders be $\pm\varepsilon_{g1}$, $\pm\varepsilon_{g2}$ and $\pm\varepsilon_{g3}$, then the sum $\Delta\varepsilon$ would be the following value :

$$\Delta\varepsilon = \varepsilon_{g1} + \varepsilon_{g2} + \varepsilon_{g3} \qquad \Delta\varepsilon = -\varepsilon_{g1} - \varepsilon_{g2} - \varepsilon_{g3}$$
$$\Delta\varepsilon = \varepsilon_{g1} + \varepsilon_{g2} - \varepsilon_{g3} \qquad \Delta\varepsilon = -\varepsilon_{g1} - \varepsilon_{g2} + \varepsilon_{g3}$$
$$\Delta\varepsilon = \varepsilon_{g1} - \varepsilon_{g2} + \varepsilon_{g3} \qquad \Delta\varepsilon = -\varepsilon_{g1} + \varepsilon_{g2} - \varepsilon_{g3} \qquad (2.2.2)$$
$$\Delta\varepsilon = \varepsilon_{g1} - \varepsilon_{g2} - \varepsilon_{g3} \qquad \Delta\varepsilon = -\varepsilon_{g1} + \varepsilon_{g2} + \varepsilon_{g3}$$

that is to say , the sum of blunder may be in a definite way . Assume the number of blunders is m , the forms of sum would be 2^m . consequently , the indefinite and discrete properties of gross error can lead the whole error to deviate from normal distribution , and make the normal curve appear longer-tailed distribution . Also , the tail of the curve will present indefinite character with the apperance of the number,the quantity of gross error . This must bring about a certain difficult to the method of robust weight function that remove the affection of the tail. Because in such indefinite condition , any robust function hardly idealy approach to the variety of parctical condition .

2.3 Error sets and their extension

Error set means the whole error elements that have some special properties . Let :

X : The true value
li : The value of observation , (i=1,.....n)

Then the true error is :

$$\Delta_i = l_i - X$$

So, the field of discussion of true error set :

$$u = \{\Delta_1, \Delta_2 \ldots\ldots\ldots, \Delta_n\}$$
$$= \{\Delta_i / \Delta_i \text{ true error elements}\} \qquad (2.3.1)$$

Let ε_{ri} be random error elements (i=1,......n) , then the random error set can be given by :

$$A = \{\varepsilon_{r1}, \varepsilon_{r2}, \ldots\ldots\ldots, \varepsilon_{rn}\}$$
$$= \{\varepsilon_{ri} / \varepsilon_{ri} \text{ random error elements}\} \qquad (2.3.2)$$

Let ε_{si} be systematic error elements (i=1,......n), then the systematic error set can be given :

$$B = \{\varepsilon_{s1}, \varepsilon_{s2}, \ldots\ldots\ldots, \varepsilon_{sn}\}$$
$$= \{\varepsilon_{si} / \varepsilon_{si} \text{ systematic error elements}\} \qquad (2.3.3)$$

If there is a certain number of gross errors in the observation , and the possibility that gross error may appear is not small , in such condition , it can be said that gross errors have exceed the bounds of random error . Let ε_{gi} be the gross error elements (i=1,......m) that may present in observation , then the gross error set is given :

$$C=\{\varepsilon_{g1},\ \varepsilon_{g2},\ \ldots\ldots\ldots,\ \varepsilon_{gm}\}$$
$$=\{\varepsilon_{gi}\ /\ \varepsilon_{gi}\ \text{gross error elements}\} \qquad (2.3.4)$$

In the above equations : $\varepsilon_{ri}\in A$, $\varepsilon_{si}\in B$, $\varepsilon_{gi}\in C$, and $u\supset A$, $u\supset B$, $u\supset C$, generally we have :

$$\Delta=\begin{cases}\varepsilon_{ri}+\varepsilon_{si}+\varepsilon_{gi} & (\varepsilon_{gi}\neq 0)\\ \varepsilon_{ri}+\varepsilon_{si} & \end{cases} \qquad (2.3.5)$$

therefore , the field of discussion :

$$u=\{\ A,\ \ B,\ \ C\ \} \qquad (2.3.6)$$

The sets of above are based on the traditional set theory . In fact , according to fuzzy set theory and the error's fuzzy properties , the demarcation line of the random error set A, systematic error set B and gross error set C is indefinite so that it is difficult to distinguish their bounds . Therefore, when consider error sets , we should transgress the limits of ordinary set theory and extend to the fuzzy sets . So the A, B and C sets should be $\underset{\sim}{A}$, $\underset{\sim}{B}$, and $\underset{\sim}{C}$. Where the $\underset{\sim}{A}$, $\underset{\sim}{B}$, and $\underset{\sim}{C}$ should be comprehended as the fuzzy sets of random error, systematic error and gross error , the so-called fuzzy set means that the set's content is definite but the bounds may be indefinite or having some fuzzy properties. Thus, we can get the extension of error sets ,the field of discussion :

$$u=\{\ \underset{\sim}{A},\ \ \underset{\sim}{B},\ \ \underset{\sim}{C}\ \} \qquad (2.3.7)$$

From (2.3.7) , we can get the power set of error :

$$p(u)=\{\varnothing\ ,\underset{\sim}{A},\ \underset{\sim}{B},\ \underset{\sim}{C},\ (\underset{\sim}{A},\underset{\sim}{B}),\ (\underset{\sim}{A},\underset{\sim}{C}),\ (\underset{\sim}{B},\underset{\sim}{C}),\ (\underset{\sim}{A},\underset{\sim}{B},\underset{\sim}{C})\} \qquad (2.3.8)$$

We see from the (2.3.8) that the error of observation can appear in different forms . Obvously , discussing the terms in the power set of error, the interaction among different errors is more distinct :

(1) The empty set Φ : there is no error in observations. it is impossible in practice .
(2) Only the set $\underset{\sim}{A}$: only having random error in observation, no systematic error and gross error , it's an ideal situation and hardly possible in practice .
(3) Only the set $\underset{\sim}{B}$: there are no random error and gross error but systematic error in observation , such case almost can not be present either .
(4) Only the set $\underset{\sim}{C}$: there are no random error and systematic error but gross error in observations , no possible in reality .
(5) The set $\underset{\sim}{A}$ and set $\underset{\sim}{B}$: there are random

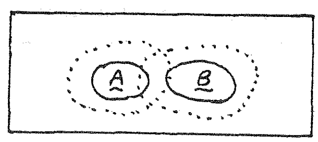


Fig.(2.3.1)

error and systematic error in the observations at the same time and influence each other .(Fig.(2.3.1)),usually the error of observation often appear in this form .

(6) Set $\underset{\sim}{A}$ and set $\underset{\sim}{C}$: there are random error and gross error in observation, no systematic error, it may be an assumption case

(7) Set $\underset{\sim}{B}$ and set $\underset{\sim}{C}$:there are systematic error and gross error but random error in observation,it can be an assumption condition .

(8) Set $\underset{\sim}{A}$, set $\underset{\sim}{B}$ and set $\underset{\sim}{C}$: three kinds of errors exist in observation. It make the error form of observation appear some indefinite and fuzzy properties . Here , if $\underset{\sim}{C}=\Phi$, then the situation exuviates to the case of (5) .

3. The synthetic technique for blunder detection

3.1 proposal of the problem

In the technique of blunder detection, usually the statistical test variable /3/ /4/ in data snooping is given by :

$$wi=\frac{f^T P V}{\sigma_0 (f^T P Q_{vv} P f^T)^{\frac{1}{2}}} \qquad (3.1.1)$$

when the observations(l1,l2,......, ln) are independent so the weight matrix is a diagonal matrix , Eq. (3.1.1) become :

$$wi=\frac{P_i v_i}{\sigma_0 (P_i q_{ii} P_i)^{\frac{1}{2}}} \qquad (3.1.2)$$

In the practice , it is convention to represent the σ_0 by the $\hat{\sigma}_0$ of estimation. Thus we can get the statistical test variable with t distribution of freedom degree (n-u-1) :

$$t = \frac{|v_i|}{\hat{\sigma}_0 \sqrt{q_{ii}}} \sim t(n-u-1) \qquad (3.1.3)$$

here $= \left(V^T P V - \frac{P_i V_i^2}{\gamma_i}\right) / n-u-1$

If there is only a blunder , data snooping can obtain an ideal outcome . In this situation , the observation to the largest statistical test variable exceeding the boundary value may be the blunder observation . But when there are a certain number of blunders in observations , the largest one of statistical test variable is uncertain to the observation that may contain blunder . At this time , if we consider the observation to the largest test variable as blunder observation, it is possibe that we may commit the type I error . From the view of statistics, usually.the type I error is worse than type II error. So the problem is which observation contains blunder corresponding to the test variable that exceeds the rejected value ? that is how to make a strategic decision to locate gross error ?

Another technique for blunder detection is robust estimation

/3/ . usually , it may be done iteratively, by successive application of method of weighted least squares . Because the weight is the function of residuals in the foreging adjustment , that is $P=f(v)$. It is difficult to avoid the influence of least squares . In the methods with robust properties for blunder detection , least sum method is effective , it can yield nearly optimal outcome and can avoid the influence of least squares. The residuals of this method can reflect correctly most of the gross errors in observation . Concerning the algorithm , because of the developing of the revised simplex algorithm /7/ of linear programming, least sum method in adjustment system for blunder detection become easy and can be use in practice . But the optimal solution of linear programming is not necessary unique . When there are a certain number of blunders in the system , though most of the residuals can reflect the gross errors of observation , it is not certain that all residuals can do so . In such situation , according to the residuals, we may doubt that some correct observation may contain blunder , this may bring about some uncorrcet location of blunder . How to make a dicission correctly from all residuals to locate gross errors with least sum method is a interesting problem . If we can combine least sum method which is not affected by least squares with data snooping , and some prior information to locate blunders , we can certain obtain an aptimal approach to the blunders .

3.2 The principle of fuzzy comprehensive estimation

(1) fuzzy transformation

Let U, V be finite sets :

$$U=\{u_1, u_2, \ldots\ldots, u_m\}$$
$$V=\{v_1, v_2, \ldots\ldots, v_n\}$$

and let $\underset{\sim}{R}$ be a fuzzy transformation from U to V :

$$\underset{\sim}{R}=\begin{bmatrix} r_{11} & r_{12} \ldots\ldots\ldots & r_{1m} \\ r_{21} & & \\ \vdots & & \\ r_{n1} & \ldots\ldots\ldots & r_{nm} \end{bmatrix}$$

According to the compond calculation of fuzzy matrix, $\underset{\sim}{R}$ determines a transforamtion : let $\underset{\sim}{A}$ be a fuzzy subset of V , we can get another fuzzy subset $\underset{\sim}{B}$ of U :

$$\underset{\sim}{B}= \underset{\sim}{A}\cdot\underset{\sim}{R} \qquad (3.2.1)$$

Eq. (3.2.1) is so-called fuzzy transformation . We can use it to make a strategic decission of a complicated system .

(2) Fuzzy comprehensive estimation :

Let U be the comment set :

$$U=\{u_1, u_2 \ldots\ldots\ldots\ldots u_m\} \qquad (3.2.2)$$

the number of grade is m . let V be factor set :

$$V=\{v_1, v_2 \ldots\ldots\ldots\ldots v_n\} \qquad (3.2.3)$$

the number of factor is n , if the comment R to the factor i is :

$$\underset{\sim}{R}_i=\{r_{i1}, r_{i2} \ldots\ldots\ldots\ldots r_{im}\}$$

Then the whole comment matrix of n factors is :

$$\underset{\sim}{R}=\begin{bmatrix} r_{11} & r_{12} \ldots\ldots\ldots\ldots & r_{1m} \\ r_{21} & & \\ \vdots & & \\ r_{n1} & \ldots\ldots\ldots\ldots & r_{nm} \end{bmatrix}$$

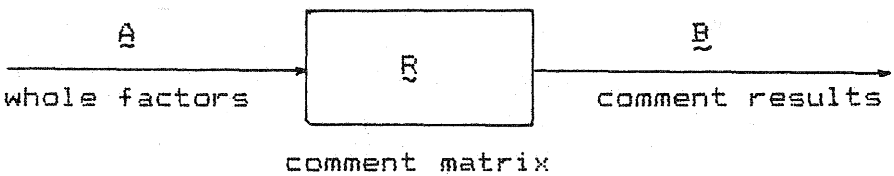
The fuzzy subset $\underset{\sim}{A}$ to V can be given according to the opinion of expert or the experience :

$$\underset{\sim}{A}=(a_1, a_2, \ldots\ldots\ldots\ldots a_n)$$

So the model of comprehensive estimation :

$$\underset{\sim}{B}= \underset{\sim}{A}*\underset{\sim}{R} \qquad (3.2.3)$$

it can be illustrated by the block diagram :



In Eq. (3.2.3) ,let :

$$\underset{\sim}{A}*\underset{\sim}{R} = \underset{\sim}{A}\cdot\underset{\sim}{R}$$

that is : $\underset{\sim}{B}=(b_1, b_2, \ldots\ldots\ldots b_m)=\underset{\sim}{A}\cdot\underset{\sim}{R}$

where $b_j=\bigvee_{i=1}^{n}(a_j \wedge r_{ij})=\max\{\min(a_1,r_{1j}),\min(a_2,r_{2j}),\ldots\ldots\ldots\ldots\min(a_n,r_{nj})\}$ (3.2.4)

Eq. (3.2.4) , are the Zadeh operator /6/ /9/ /10/ .

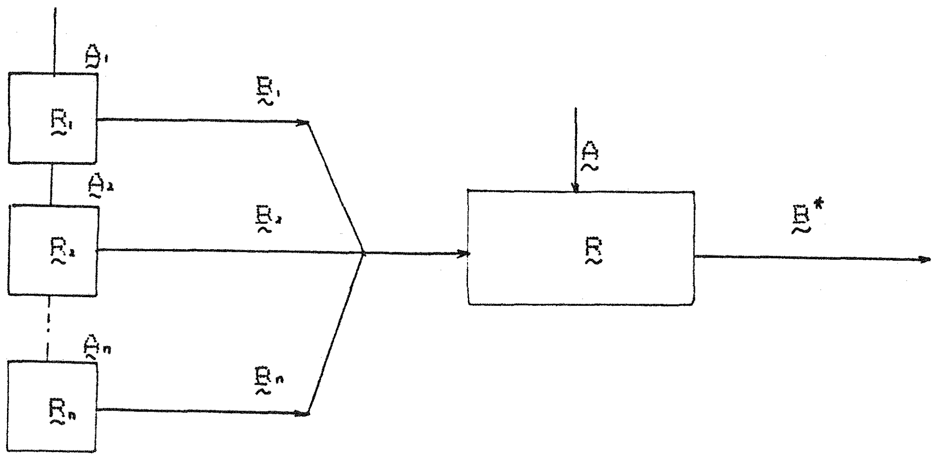
Similary ,futher let R be :

$$\underset{\sim}{R}=\begin{bmatrix} \underset{\sim}{B}_1 \\ \underset{\sim}{B}_2 \\ \vdots \\ \underset{\sim}{B}_n \end{bmatrix} = (b_{ij})_{n\times m}$$

we can get the second stage estimation :

$$\underset{\sim}{B}^*= \underset{\sim}{A}*\underset{\sim}{R} = \underset{\sim}{A} * \begin{bmatrix} \underset{\sim}{A}_1 * \underset{\sim}{R}_1 \\ \underset{\sim}{A}_2 * \underset{\sim}{R}_2 \\ \vdots \\ \underset{\sim}{A}_n * \underset{\sim}{R}_n \end{bmatrix} \qquad (3.2.5)$$

descripted by the following block diagram :



3.3 The model of synthetic technique for blunder detection

Changing the problem that how to make a strategic decission for blunder detection and elimination in data snooping , least sum method and some prior information into the comprehensive estimation problem , we can get the mathematical model :

The set of factors :

$$V=\{ v_1 , v_2 , v_3 \} \tag{3.3.1}$$

where v_1 : the least sum method detecting vector .
v_2 : data snooping detecting vector .
v_3 : some prior detecting information vector

The set of comment :

$$U=\{ u_1 , u_2 \} \tag{3.3.2}$$

where u_1: contain gross error
u_2: no gross error

From the experience , let the fuzzy subset $\underset{\sim}{A}$ to V be :

$$\underset{\sim}{A}=\{ 0.5 , 0.45 , 0.05 \} \tag{3.3.3}$$

General comment matrix :

$$\underset{\sim}{R}=\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}=\begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \\ r_{31} & r_{32} \end{bmatrix} \tag{3.3.4}$$

Because containing gross error with no gross error is exclusion , if let least sum method's subjection function for detecting blunder be $r_{11}=u_{\underset{\sim}{A}}(x)$,then $r_{12}=\bar{r}_{11}=u_{\underset{\sim}{A}\neg}(x)=1-u_{\underset{\sim}{A}}(x)$, similarly , the rest functions are :

$r_{21}=u_{\underset{\sim}{A}}(d)$, $r_{22}=1-u_{\underset{\sim}{A}}(d)$
$r_{31}=u_{\underset{\sim}{A}}(p)$, $r_{32}=1-u_{\underset{\sim}{A}}(p)$

Therefore the general comment matrix :

$$\underset{\sim}{R}=\begin{bmatrix} u_{\underset{\sim}{A}}(x) , & 1-u_{\underset{\sim}{A}}(x) \\ u_{\underset{\sim}{A}}(d) , & 1-u_{\underset{\sim}{A}}(d) \\ u_{\underset{\sim}{A}}(p) , & 1-u_{\underset{\sim}{A}}(p) \end{bmatrix} \tag{3.3.5}$$

Where Eq. (3.3.5) :

$$u_{\underset{\sim}{A}}(x)=\begin{cases}|v_i|/\max\{|v_i|\} & \max\{|v_i|\}>3\sigma_0\\ |v_i|/3 & \max\{|v_i|\}<3\sigma_0\end{cases} \qquad (3.3.6)$$

$$u_{\underset{\sim}{A}}(d)=\begin{cases}w_i/\max\{w_i\} & \max\{w_i\}>3.3\\ w_i/3.3 & \max\{w_i\}<3.3\end{cases} \qquad (3.3.7)$$

$u_{\underset{\sim}{A}}(p)$: prior information detecting vector . If it is not be given , $u(p)=0.5$

Where Eq. (3.3.6) ,(3.3.7) :

vi : the residuals of least sum method
wi : the statistic test variables

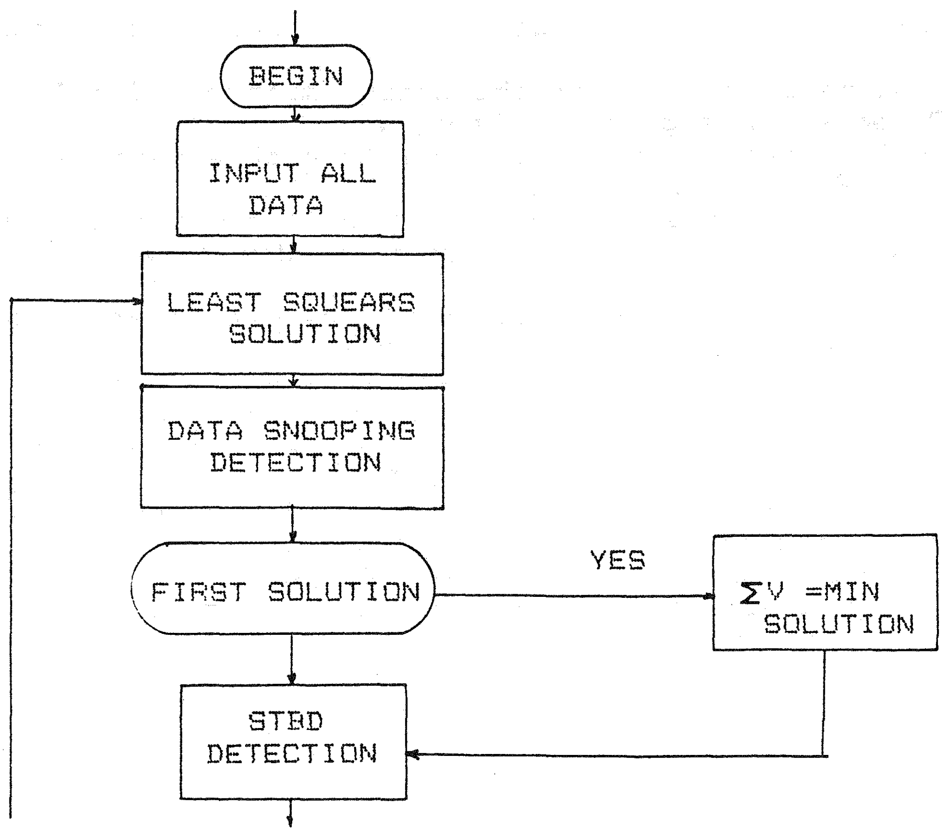
Consequently , the mathematical model of STBD :

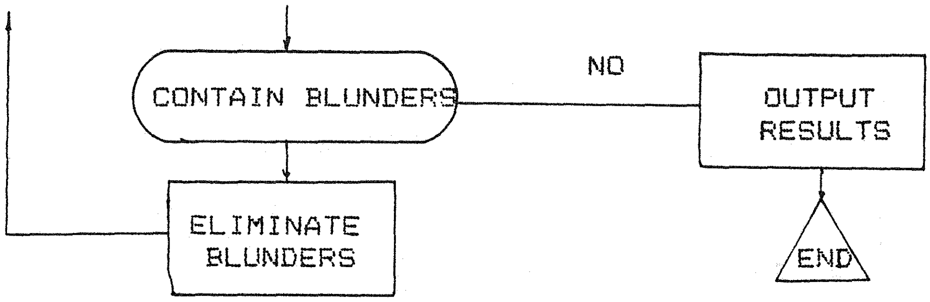
$$\underset{\sim}{B}=\underset{\sim}{A}\cdot\underset{\sim}{R}$$

$$=(\ 0.5,\ 0.45,\ 0.05\ )\cdot\begin{bmatrix}u_{\underset{\sim}{A}}(x) & 1-u_{\underset{\sim}{A}}(x)\\ u_{\underset{\sim}{A}}(d) & 1-u_{\underset{\sim}{A}}(d)\\ u_{\underset{\sim}{A}}(p) & 1-u_{\underset{\sim}{A}}(p)\end{bmatrix} \qquad (3.3.8)$$

for every observation ,we can get Bi , (i=1,2.........,n). Let F be check value , it can be given according to the practical condition.Similar to the second stage estimationn,the observations corrosopond to the containing gross error factors larger than the check value in Bi can be considered as blunder .

The following is the flow chart of caleculation :





4. Conclusion

According to the theory of STBD , we completed a bundle adjustment program of automatic detecting and eliminating blunders in close range photogrammetry . A series of experiments was performed with the data which are extracted from test field. In these experiments, different number of gross errors ,types and their combinations were considered. Compared with single method , such as robust weight function , data snooping and least sum method, the outcomes shown that STBD was more effective in the respects of correct detection of blunder and the solution of stability. Even many of the small blunders lesser than the low detectable boundary /4/ /5/ could be found and located when the check value F was given rigorously . If we can make full use of the symmetric peculiarity when solve the linear programming problem with revised simplex algorithm , the synthetic technique for blunder detection must has great potentiatities in practice .

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