THE SUFFICIENT AREA CONDITION OF AN OBJECT FOR A SPOT- DETECTOR

F. Cheng* and Ph. Hartl
Institute for Navigation, University of Stuttgart, Stuttgart, Germany

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ABSTRACT:

Some objects can not be found or identified on a SPOT-image due to their too small areas. An Object on the ground must be large enough so that it can be detected vertically by a SPOT-detector and recorded as a complete pixel in a SPOT-image. This condition, defined as the sufficient area condition, has been not studied yet. In this paper the condition is theoretically analysed and quantitatively calculated. It may be expressed as follows: 1) the area of an object on the ground must include an ellipse, whose long axis e_1 is in the direction with the greatest ground slope ε and equal to $2R/\cos\varepsilon$, and whose short axis e_2 is horizontal and equal to $2R$; 2) if the ground is smooth (ε =0), the ellipse becomes a circle with the radius R. Here R is the diagonal length of a SPOT-image's pixel: $10\sqrt{2}\,m$ or $20\sqrt{2}\,m$.

1. INTRODUCTION

Some objects can not be found or identified on a SPOT-image due to their too small areas. How large should the area of an object on the ground be, so that it can be detected at least by a SPOT-detector? Many people have met such a question in their works, especially in classifying or identifying objects on satellite images. Some of them hope to use satellite data with smaller pixel size or higher spatial resolution in order to obtain better results (Begni 1988, Dowman et al 1989, Moore et al 1989, Jensen et al 1993, Manavalan et al 1993, Cheng et al 1995, Hartl et al 1995).

There are two types of SPOT-images: vertical and oblique viewing images. Here only the former is discussed. The area of an object on the ground, which makes the object to be vertically detected at least by a SPOT-detector and be recorded as a complete pixel in a SPOT-image, is defined as the sufficient area condition. This paper shows the study of the condition and gives its mathematical expression.

2. THE SUFFICIENT AREA CONDITION ON THE SMOOTH GROUND

2.1 Position change of grids

The figure 1 shows a square grid, which represents the corresponding area of a SPOT-detector on the smooth ground. Here we call it a SPOT-detector-grid (a- side length; d- diagonal length). Suppose that there is a circular object C_e with the radius R on the ground. On the object a Cartesian coordinate system *oxy* is set up, whose origin *o* is at the center of the object and whose *y*-axis points north. Suppose that the center of a SPOT-detector-grid (p) is just overlapped with the object's center and t_1, t_2, $\dots t_8$ are the eight neighbour grids of the grid p (figure 2). In this situation the object can be detected at least by a complete SPOT-detector-grid, if its radius R meets the following condition:

$$R \geq \frac{d}{2}\ . \qquad (1)$$

This is a special situation. But it may be supposed that the grids' position shown in the figure 2 is the original position. The position of the grids on the ground generally changes after a revolving period of the satellite. The figure 3(1) shows such a change. The grids have left the original position in a new revolving period. We may only consider the position change of the grid p instead of all the grids, because the relative position between the different grids is fixed. The position change of the grid p shown in the figure 3(1) can be resolved into a shift and a turn around the origin *o*. i) The grid p shifts first from *o* to p' and then ii) it turns around *o* from p' to p'' (figure 3(2)). In fact any position change of the grids can be resolved as above.

Suppose that the grid p shifts first from the original position (0,0) to an other position (x_p, y_p) (figure 4(1)). The shift amount is equal to the distance between the two points. We define X as the component of this distance on the *x*-axis and Y as its component on the *y*-axis. Then the following equalities are tenable: $X=|x_p|$ and $Y=|y_p|$. In the situation shown in the figure 4(1) the object C_e can be detected by a complete SPOT-detector-grid, if its radius R meets:

$$R = \left[(X+\frac{a}{2})^2 + (Y+\frac{a}{2})^2 \right]^{\frac{1}{2}} \qquad (2)$$

See the figure 4(2). The grid p turns around *o* in the next revolving period of the satellite. Then R should meet:

$$R = \left\{ \left[(X+\frac{a}{2})\ \cos\rho\ +\ (Y+\frac{a}{2})\ \sin\rho \right]^2 + \left[(Y+\frac{a}{2})\ \cos\rho\ -\ (X+\frac{a}{2})\ \sin\rho \right]^2 \right\}^{\frac{1}{2}}$$

*Present address: Company for applied Remote Sensing (GAF), 80636 Munich, Germany

$$=\left[(X+\frac{a}{2})^2+(Y+\frac{a}{2})^2\right]^{\frac{1}{2}}. \qquad (3)$$

It is known from the above calculations that the demanded object radius R has no relationship with a turn of the grids around *o*. Therefore only the grids' shift needs to be considered.

2.2 Shift extent of grids

The lower bounds of the shift amount X and Y are obviously zero (figure 2). What are the upper bounds? See the figure 2 and the figure 5. The figure 2 shows the original position of the grids. In the figure 5(1) X and Y are smaller than a/2. In the figure 5(2) X and Y are equal to a/2. Contrast the figure 5(1) and 5(2) with the figure 2. It will be known that the demanded object radius R is longest in the figure 5(2) and shortest in the figure 2. In fact R increases with the increase of the shift's distance, if $0 \le X \le a/2$, $0 \le Y \le a/2$. How does R change, if $X \ge a/2$, $Y \ge a/2$?

1) $\frac{a}{2} < |x_p| \le a$, $0 < |y_p| \le \frac{a}{2}$:

The figure 6(1) shows such an example. In this situation the neighbour t_8 of the grid p is a nearest grid to the center of the object C_e. C_e can be detected by a complete SPOT-detector-grid, if its area is large enough to include the grid t_8. In fact it may be held that the grids are formed by the method that a grid appears repeatedly at intervals of 10*m* or 20*m* on the *x*-axis and on the *y*-axis. The grids have the same periodic characteristic on the *x*-axis and on the *y*-axis in other words. Therefore the distance $\overline{ot_8}$ may be treated as the equivalent shift of the grid p. But it is tenable: $0 < |x_{t8}| \le a/2$ and $0 < |y_{t8}| \le a/2$.

2) $0 < |x_p| \le \frac{a}{2}$, $\frac{a}{2} < |y_p| \le a$:

The figure 6(2) shows an example of this situation. It is similar to 1) that the distance $\overline{ot_6}$ may be regarded as the equivalent shift of the grid p. It is also tenable: $0 < |x_{t6}| \le a/2$ and $0 < |y_{t6}| \le a/2$.

3) $\frac{a}{2} < |x_p| \le a$, $\frac{a}{2} < |y_p| \le a$:

See the figure 6(3). Similarly $\overline{ot_7}$ may be regarded now as the equivalent shift of the grid p and it is tenable: $0 < |x_{t7}| \le a/2$ and $0 < |y_{t7}| \le a/2$.

4) $|x_p| > a$, or $|y_p| > a$ or $|x_p| > a$ and $|y_p| > a$:

See the figure 6(4). No matter how long the distance of the shift is, a grid *t'* can be always found in the neighbourhood of the object's center o, which meets $0 < |x_{t'}| \le a/2$ and $0 < |y_{t'}| \le a/2$. $\overline{ot'}$ is the equivalent shift of the grid p.

Therefore the upper bounds of X and Y are equal to a/2. The extent of the grids' shift is expressed as $0 \le X \le a/2$, $0 \le Y \le a/2$. In this extent the demanded object radius R increases with the increase of X and Y. R should reach its maximum, when $X = a/2$ and $Y = a/2$ appear at the same time. The following is its mathematical proving.

2.3 The sufficient area condition

Suppose $Z=R^2$ ($R>0$). Then the formula (2) is turned into the following form:

$$Z=(X+\frac{a}{2})^2+(Y+\frac{a}{2})^2,$$
$$(0 \le X \le \frac{a}{2},\ 0 \le Y \le \frac{a}{2}). \qquad (4)$$

The region D_0 defined by $0 \le X \le a/2$, $0 \le Y \le a/2$ is bounded and closed (figure 7). Therefore Z has a minimum and a maximum on D_0. The former is already known (figure 2). The latter is the sufficient area condition for an object on the smooth ground. Let us first consider that to which direction the grid p shifts so that Z can get a maximal increment. For this purpose the gradient of Z at the point (0,0) is calculated: $\vec{\nabla} Z|_{(0,0)} = (a, a)$.

The direction of $\vec{\nabla} Z|_{(0,0)}$ is $\zeta_X = \zeta_Y = 45^o$. Suppose that the grid p shifts with a very small distance δ in this direction. The new coordinates of the grid p are $(X_1,\ Y_1)$. Certainly it is tenable: $X_1 = Y_1$. $\vec{\nabla} Z$ at the point $(X_1,\ Y_1)$ is calculated as follows: $\vec{\nabla} Z|_{(X_1,Y_1)} = (2X_1+a,\ 2Y_1+a)$. Its direction is also equal to 45^o. The direction of $\vec{\nabla} Z$ does not change, if the grid p shifts again by δ in the direction 45^o. If the above process goes on, the gradient line originating from *O* can be obtained. It is a straight line from the origin *O*(0,0) to the point *K*(a/2, a/2) (figure 7). Therefore Z_{max} appears at the end of this vector $X_{max} = Y_{max} = a/2$ (figure 8):

$$Z_{max} = Z|_{(\frac{a}{2},\frac{a}{2})} = 2a^2 = d^2. \qquad (5)$$

Obviously the maximal radius R_{max} is equal to d. The sufficient area condition S_b is calculated as follows (figure 5(2)):

$$S_b = \pi\ R_{max}^2 = \pi\ d^2.$$

(6)

For a panchromatic SPOT-image there are

$d = 10\sqrt{2}$ (*m*) and $S_b = 200\pi$ (m^2).

For a multispectral SPOT-image there are

$d = 20\sqrt{2}$ (*m*) and $S_b = 800\pi$ (m^2).

An object on the smooth ground can be detected at least by a panchromatic (multispectral) SPOT-detector-grid, if its area on the ground can include a circle with the radius $10\sqrt{2}\,m$ ($20\sqrt{2}\,m$). This radius is equal to the diagonal length of a panchromatic (multispectral) SPOT-image pixel.

3. THE SUFFICIENT AREA CONDITION ON A SLOP

Suppose: a) the ground E_ε has a slope, whose maximum ε lies in the direction D_h; b) there is an object on E_ε (figure 9(1)). What is the sufficient area condition in this situation? It is known from geometry that the area of the object on the ground should include an ellipse (B_ε), whose long axis e_1 is in the direction D_h and whose short axis e_2 is horizontal (figure 9(2)):

$$e_1 = \frac{2R_{\max}}{\cos\varepsilon}, \quad e_2 = 2R_{\max}, \qquad (7)$$

so that the object can be detected at least by a complete SPOT-detector-grid. The area of the ellipse $S_{b\varepsilon}$ is easily calculated:

$$S_{b\varepsilon} = \frac{\pi \; R_{\max}{}^2}{\cos\varepsilon} = \frac{\pi \; d^2}{\cos\varepsilon} \qquad (8)$$

The formula (8) is identical with the formula (6), if ε =0. Therefore the formula (8) has universality.

4. CONCLUSIONS

The sufficient area condition guarantees that an object on the ground can be detected at least by a panchromatic (multispectral) SPOT-detector-grid and recorded as a complete pixel in a SPOT-image.

This condition is that the object's area on the ground must include an ellipse, whose long axis e_1 is in the direction with the greatest ground slope ε and equal to $2R/\cos\varepsilon$, and whose short axis e_2 is horizontal and equal to $2R$. If the ground is smooth (ε =0), then this ellipse becomes a circle with the radius R ($R = 10\sqrt{2}\,m$ for a panchromatic SPOT-image; or $20\sqrt{2}\,m$ for a multispectral SPOT-image).

The above conclusions may be applied to other satellite images, if they are similar to SPOT-images in the aspects of imagery principle and pixel form.

Here it should be particularly pointed out that an object, which can be surely identified or which can become at least a identifiable pixel on a SPOT-image, should meet not only the sufficient area condition, but also an other condition: there should be a considerable difference in spectral responsivity between the object and the other objects around it. The difference condition should be studied in the future.

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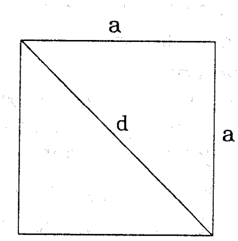

Figure 1. A SPOT-detector-grid
(a - side length; d - diagonal length)

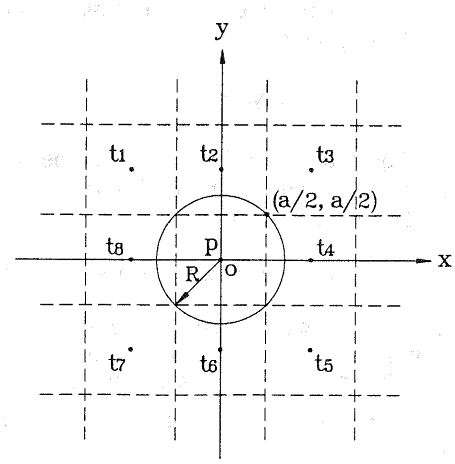

Figure 2. The original position of the grids
(o - object's center; t_1, t_2 ...- eight neighbours of p)

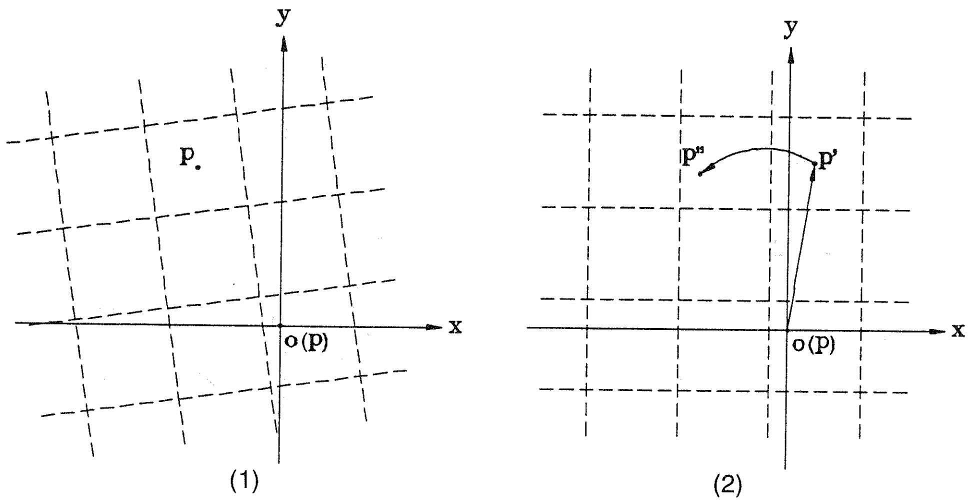

Figure 3. (1) The position change of the grid p: from o to p;
(2) Its resolving: the shift from o to p' and the turn around o from p' to p.

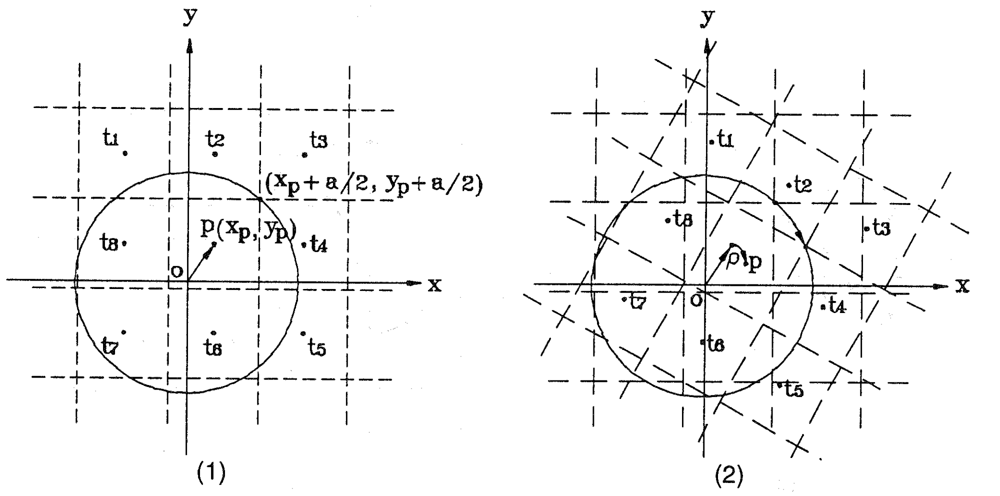

Figure 4. The position change of the grid p: (1) shifting from *o* to p; (2) turning by ρ around *o*.

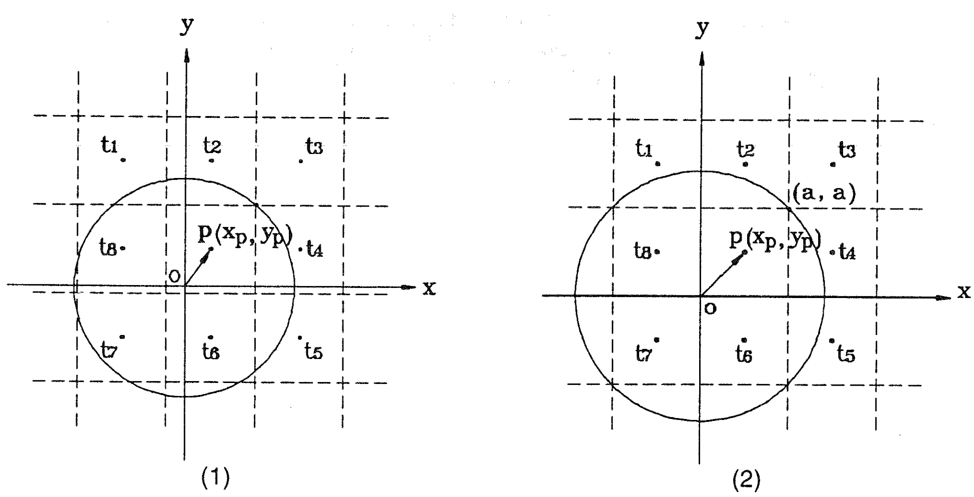

Figure 5. Change of R: (1) $0 < |x_p| < a/2$ and $0 < |y_p| < a/2$; (2) $|x_p| = a/2$ and $|y_p| = a/2$.

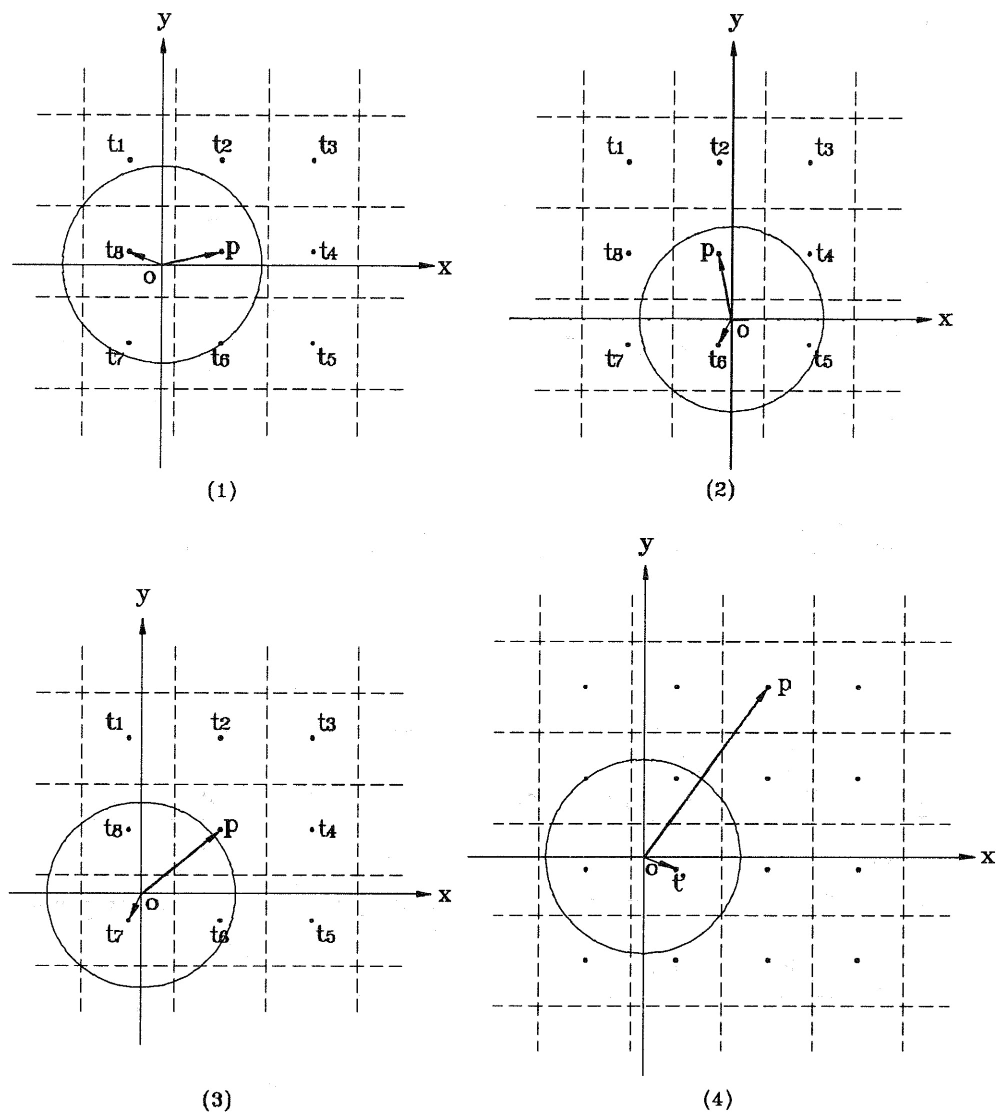

Figure 6. Actual shifts (———→) of the grid p and their equivalent shifts(—→):
(1) $a/2 < |x_p| \leq a$, $0 \leq |y_p| \leq a/2$; (2) $0 \leq |x_p| \leq a/2$, $a/2 < |y_p| \leq a$;
(3) $a/2 < |x_p| \leq a$, $a/2 < |y_p| \leq a$; (4) $|x_p| > a$, $|y_p| > a$;

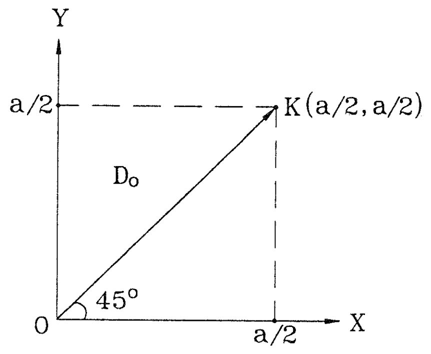

Figure 7. The region D_0 and the gradient line OK

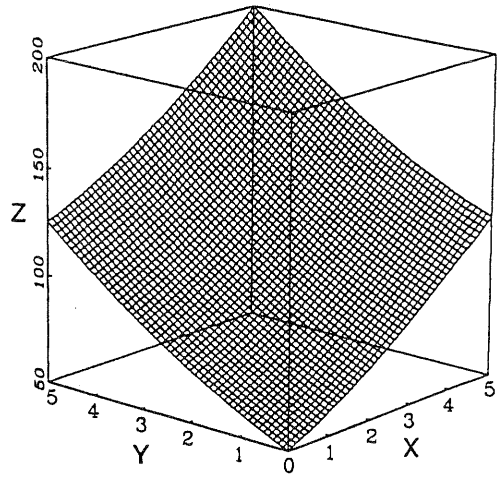

Figure 8. The geometrical form of Z

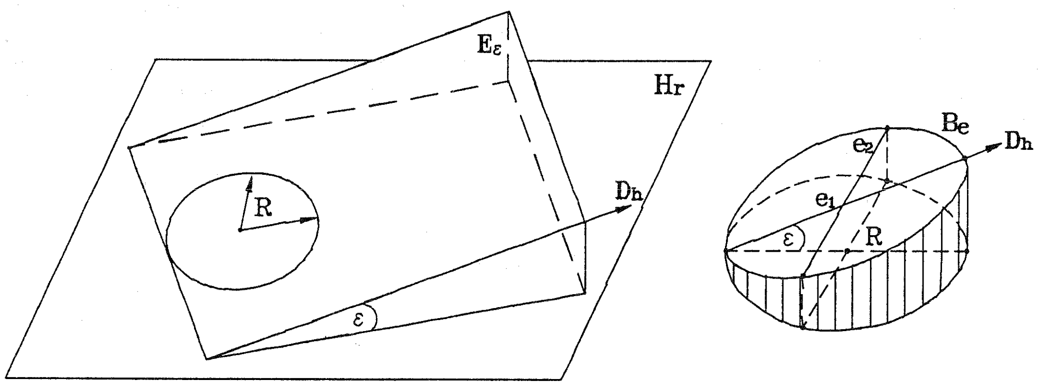

Figure 9. The sufficient area condition on a slope: (1) the ground E_ε with the greatest ground slope ε in the direction D_h and its reference horizontal plane H_r (R on H_r); (2) the ellipse B_e on E_ε corresponding to the circle with the radius R on H_r (e_1 - long axis and e_2 - short axis).