

# Using Homogeneous Coordinates to Solve The Problems of Determining The Orientation Parameters of Non-Metric Cameras and The Reconstruction of Space Models

Mohammed El-Shafei Abdel-Latif<sup>1</sup> and Ahmed M. Elsonbaty<sup>2</sup>

<sup>1</sup>Faculty of Engineering, Assiut-EGYPT

<sup>2</sup>Institute of Geometry, TU Vienna, AUSTRIA

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## ABSTRACT:

In this paper a new mathematical model is developed for the determination of the orientation parameters of a non-metric camera using at least five control points, among which four points are coplanar, while the fifth point should lie outside that plane.

In case two images of the same space model, taken at different locations by the same camera, are given, the space model can be directly reconstructed without calculating the parameters.

The methods used here are direct and simple. They need no linearization of equations nor complicated techniques.

## 1. INTRODUCTION

Orientation parameters of a camera play an important role in photogrammetric measurements. The accuracy of these orientation parameters are very important in estimating the accuracy of the photogrammetric measurements. Therefore the main objective of this research work is to determine the orientation parameters of a non-metric camera which allows the use of the non-metric camera in photogrammetric work and enable many engineers and scientists in numerous fields to make full use of the technical and economical advantages of photogrammetry.

In this paper a direct method to determine the orientation parameters of a camera is developed. This method can be used to calibrate a non-metric camera. Here lens distortion and film deformation are neglected because they have no mathematical relationship to the central projective geometry.

For the determination of the orientation parameters, at least five control points in general position are needed. Here we consider the case when four of them lie in one plane; the fifth not.

The work is divided into two main parts:

1. In the first part the mathematical model is developed using homogeneous coordinates in the

plane of the four coplanar points. This leads to a simple direct solution to the problem.

2. The second part deals with the reconstruction of a space model subject to the above conditions using two images without calculating the orientation parameters.

## 2. THE MODEL

As shown in fig.(1), the first model consists of known control points  $A_1, A_2, A_3$  and  $A_4$  lying in one plane called the *object plane*  $\alpha$ . The fifth point  $A_5$  lies outside  $\alpha$ .

For simplicity, the space rectangular coordinate system  $O; X, Y, Z$  is chosen such that the  $X$ - and  $Y$ -axes lie in  $\alpha$ .

The control points have the known coordinates:

$$A_i(X_i, Y_i, Z_i) \quad i := 1, 2, \dots, 5$$

with  $Z_5 \neq 0$  and  $Z_i = 0$  for  $i = 1, 2, 3, 4$ .

Further, an image of the above model, taken upon the plane  $\pi$ , is also given. The coordinates of the image points

$$A'_i(X'_i, Y'_i) \quad i = 1, 2, \dots, 5$$

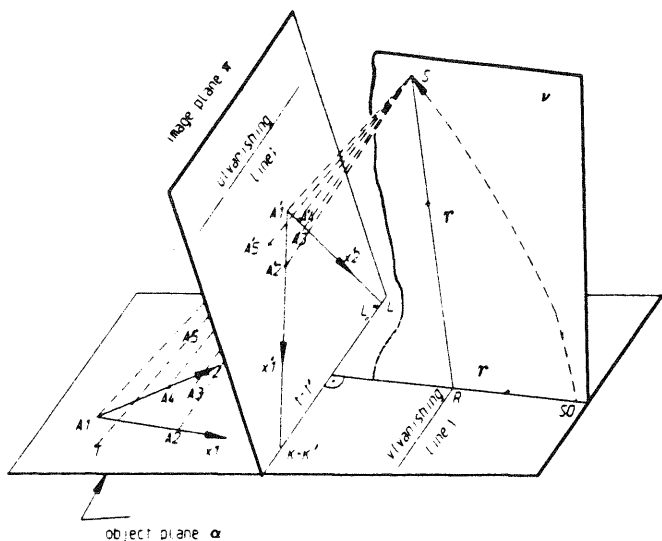


Figure 1: The model

respectively, can be determined relative to an arbitrary coordinate system  $O'; X', Y'$  in  $\pi$ .

It is required to find the space position of the image plane  $\pi$  and the camera station  $S$  relative to the space coordinates  $X, Y, Z$ . This will be done by finding the line of intersection  $t$  of  $\pi$  with  $\alpha$  (called trace), as well as its angle of inclination  $\omega$ , as well as the space coordinates of  $S(X_S, Y_S, Z_S)$  will be determined. The principal point  $P'$  and the focal length  $f$  can then be easily calculated.

### 3. DETERMINATION OF THE ORIENTATION PARAMETERS

As said before, exactly five control points are needed for the solution of the problem.

#### 3.1 Homogeneous coordinate systems

In the object plane  $\alpha$ , a homogeneous coordinate system is chosen with origin at  $A_1$ , the  $x_1$ - and  $x_2$ -axes passing through  $A_2$  and  $A_4$  respectively, fig.(2), and the  $x_0$ -axis at infinity. The homogeneous coordinates of a given point are the three ratios  $(x_0 : x_1 : x_2)$ , not all zeros, which can be arbitrarily multiplied by any non-zero factor. From fig.(2), our control points have the following Homogeneous coordinates:

$$A_1(1 : 0 : 0), A_2(1 : b : 0), A_3(1 : c_1 : c_2), A_4(1 : 0 : d),$$

where, since  $x_0$  is taken = 1;  $b, d, c_1$  and  $c_2$  are metric lengths, which can be calculated from the space coordinates, or directly measured.

A similar homogeneous coordinate system is chosen in the image plane  $\pi$  as shown in fig(3). The coordinates

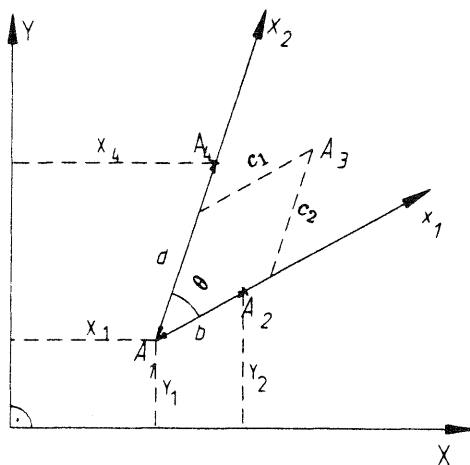


Figure 2: Homogeneous and rectangular coordinates in  $\alpha$

of the image points are:

$$A'_1(1 : 0 : 0), A'_2(1 : b' : 0), A'_3(1 : c'_1 : c'_2), A'_4(1 : 0 : d')$$

and  $A'_5(1 : t'_1 : t'_2)$ .

where, as before,  $b', c'_1, c'_2, d', t'_1$  and  $t'_2$  are metric lengths.

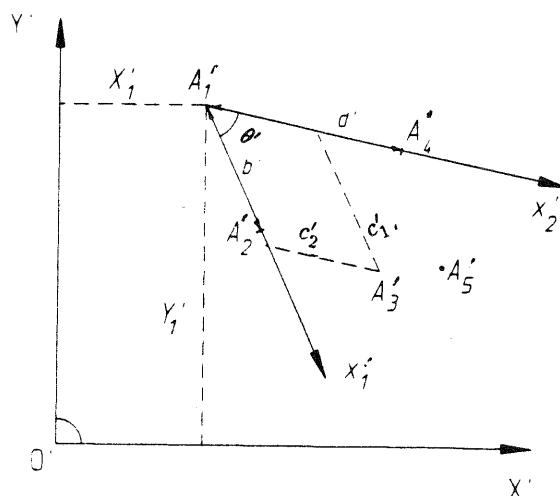


Figure 3: Homogeneous and rectangular coordinates in  $\pi$

#### 3.2 Steps of solution:

We are going to carry out the following steps:

1. The collinear relation between the correspondence

$$A_i \in \alpha \longleftrightarrow A'_i \in \pi \quad i = 1, 2, 3, 4$$

will first be determined.

2. The vanishing line  $v \in \alpha$  and  $u' \in \pi$  are found.
3. A pair of congruent corresponding lines  $t \in \alpha \equiv t' \in \pi$  is determined.  $t$  should be the trace of  $\pi$  on  $\alpha$ .

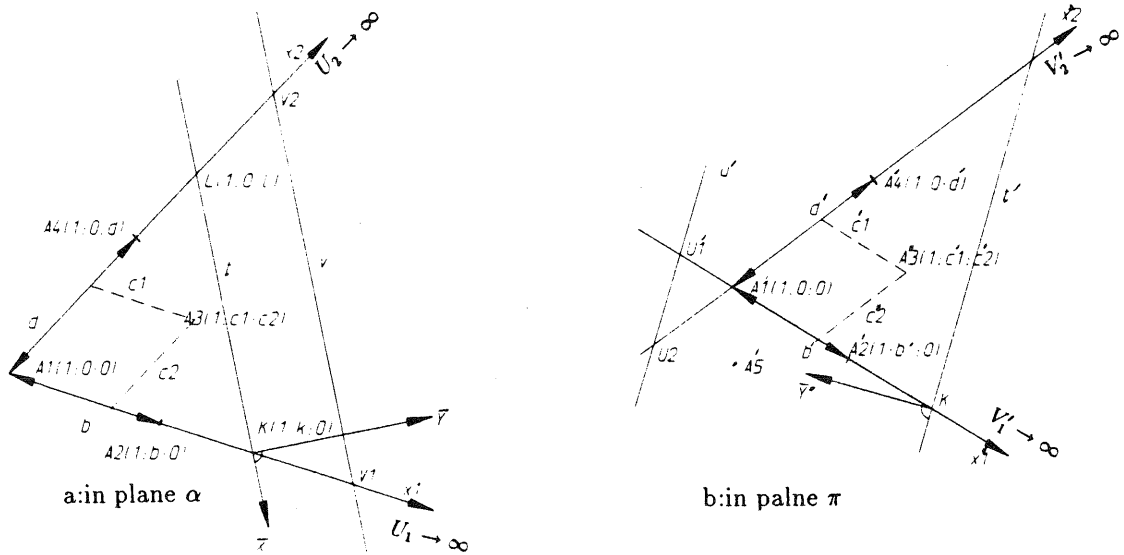


Figure 4: Homogeneous and related coordinates.

4. The plane  $\pi$  is set to coincide with  $\alpha$ , such that the congruent lines  $t$  and  $t'$  identically coincide. In this position, the space coordinates of the image points are determined, as though they belong to  $\alpha$
5. The collineation between the two sets in  $\alpha$  is determined through the perspectivity in the plane  $\alpha$ : axis  $t$  and center  $S_0$ .
6. The true position of the camera station  $S$  in space lies on the circle of rotation of  $S_0$  about the vanishing line  $v$  [1]. Its plane  $\nu$  is perpendicular to  $t$ .
7. Through the collineation in step 5 above, a point  $T \in \alpha$  is found to correspond to  $A'_5$  (still in  $\alpha$ ). Hence the point  $T \in \alpha$  and the control point  $A_5$  in space must be collinear in space with  $A'_5$  in its true position in  $\pi$  in space. The line joining  $A_5$  and  $T$  must therefore pass through  $S$  in the plane  $\nu$ .

### 3.3 Transformation Equations between $\pi$ and $\alpha$

Fig.(4a,b) show the four pairs of corresponding points  $A_i(x_0 : x_1 : x_2) \rightarrow A'_i(x'_0 : x'_1 : x'_2)$  with their homogeneous coordinates as given before. The equations of such linear transformations can be expressed in the form:

$$\left. \begin{aligned} x'_0 &= a_{00}x_0 + a_{01}x_1 + a_{02}x_2 \\ x'_1 &= a_{10}x_0 + a_{11}x_1 + a_{12}x_2 \\ x'_2 &= a_{20}x_0 + a_{21}x_1 + a_{22}x_2 \end{aligned} \right\} (1)$$

where  $a_{ij}$  are constant coefficients.

Substituting the coordinates of an object point in the

R.H.S. of equation(1) and those of the corresponding image point multiplied by an unknown factor in the L.H.S., we get three equations. Totally, we get twelve equations in the nine coefficients and the four unknown factors. Since only the ratios between the factors are of interest, one of them can be arbitrarily chosen. The solution of these equations are simple and yield:

$$\left. \begin{aligned} a_{10} &= a_{20} = a_{21} = a_{12} = 0 \\ a_{00} &= bc_1c_2d(b'd' - c'_1d' - c'_2b') \\ a_{11} &= b'd'c'_1c_2(bd - c_1d - c_2b) \\ a_{22} &= b'd'c'_1c'_2(bd - c_1d - c_2b) \\ a_{01} &= c_2[b'c_1d(c'_2 - d') - bc'_1d'(c_2 - d)] \\ a_{02} &= c_1[b'c'_2d(b - c_1) - bc_2d'(b' - c'_1)] \end{aligned} \right\} (2)$$

Similar formulas are found for the inverse transformation

$$\left. \begin{aligned} x_0 &= a'_{00}x'_0 + a'_{01}x'_1 + a'_{02}x'_2 \\ x_1 &= a'_{11}x'_1 \\ x_2 &= a'_{22}x'_2 \end{aligned} \right\} (3)$$

with coefficients:

$$\left. \begin{aligned} a'_{00} &= \frac{c'_2}{c_2} a_{11}, & a'_{01} &= -\frac{c'_2}{c_2} a_{01}, & a'_{02} &= -\frac{c'_1}{c_1} a_{02}, \\ a'_{11} &= \frac{c'_2}{c_2} a_{00}, & a'_{22} &= \frac{c'_1}{c_1} a_{00} \end{aligned} \right\} (4)$$

### 3.4 The Vanishing lines

The vanishing line  $v$  in  $\alpha$  corresponds to the line at infinity in  $\pi$ . It can be determined using the points at infinity of both  $x'_1$  and  $x'_2$  axes, whose coordinates are  $(0 : 1 : 0)$  and  $(0 : 0 : 1)$  respectively.

Substituting in equation(3), we get the vanishing points:

$$V_1(a'_{01} : a'_{11} : 0), V_2(a'_{02} : 0 : a'_{22}). \quad (5)$$

The vanishing line  $v$  joins both  $V_1$  and  $V_2$ . Similarly, the vanishing line  $u'$  in  $\pi$  is determined through the two vanishing points.

$$U'_1(a_{01} : a_{11} : 0), U'_2(a_{02} : 0 : a_{22}). \quad (6)$$

### 3.5 The Trace $t$

As shown in fig.(1), the trace  $t$  can be considered as belonging to both  $\alpha$  and  $\pi$ . Let  $t'$  denotes it in  $\pi$  when separated from  $\alpha$ .  $t$  and  $t'$  must be congruent beside being corresponding to each other. It is well known that they are parallel to the vanishing lines in space [4]. Let  $K(1 : k : 0)$  and  $L(1 : 0 : l)$  be the points of intersection of  $t$  with  $x_1$  and  $x_2$ , and let their corresponding points be  $K'(1 : k' : 0)$  and  $L'(1 : 0 : l')$ . The metric length  $KL$  and  $K'L'$  must be equal.

Since  $KL$  is parallel to  $v$  fig.(4a), then  $\frac{A_1 L}{A_1 K} = \frac{A_1 V_2}{A_1 V_1}$ , hence from (5) and (4),

$$\frac{A_1 L}{A_1 K} = \frac{l}{k} = \frac{a'_{22}/a'_{01}}{a'_{11}/a'_{01}} = \frac{a_{01}}{a_{02}} \quad (7)$$

Similarly, for the corresponding points  $K'(1 : k' : 0)$  and  $L'(1 : 0 : l')$ , it can be shown that

$$\frac{l'}{k'} = \frac{a'_{01}}{a'_{02}} \quad (8)$$

Substituting the coordinates of  $L$  and  $K$  into (1), we get the coordinates of  $K'$  and  $L'$  as follows:

$$K'(a_{00} + a_{01}k : a_{11}k : 0), L'(a_{00} + a_{02}l : 0 : a_{22}l) \quad (9)$$

Hence

$$k' = \frac{a_{11}k}{a_{00} + a_{01}k} \quad (10)$$

and

$$l' = \frac{a_{22}l}{a_{00} + a_{02}l} \quad (11)$$

Since  $\overline{LK}^2 = \overline{L'K'}^2$ , then:

$$l^2 + k^2 - 2lk \cos \theta = l'^2 + k'^2 - 2l'k' \cos \theta' \quad (12)$$

where  $\theta$  and  $\theta'$  are the angles subtended by the axes pairs, which can be measured directly or determined from the well-known cosine formula:

$$\begin{aligned} \cos \theta &= \frac{A_1 A_2^2 + A_1 A_4^2 - A_2 A_4^2}{2A_1 A_2 A_4} \\ \cos \theta' &= \frac{A'_1 A'_2^2 + A'_1 A'_4^2 - A'_2 A'_4^2}{2A'_1 A'_2 A'_4} \end{aligned} \quad (13)$$

Dividing (12) by  $k^2$  and substituting from (7),(8) and (10), we get after several reductions:

$$\begin{aligned} (a_{00} + a_{01}k)^2 &= \\ &= \frac{a_{11}^2 a_{02}^2}{a_{02}^2} \cdot \frac{a_{01}^2 - 2a_{01}a_{02} \cos \theta' + a_{02}^2}{a_{01}^2 - 2a_{01}a_{02} \cos \theta + a_{02}^2} = \delta^2 \end{aligned} \quad (14)$$

It can be shown that the R.H.S. of (14) is a positive quantity and hence it is set to  $\delta^2$ .

Solving equation(14), yields two values for  $k$ . The practical value of  $k$  is chosen, such that, for a positive photo,  $t$  has a position similar to that shown in fig.(1) and(4), in which:

$|A_1 K| < |A_1 V_1|$ , or

$$k < \left| \frac{a'_{11}}{a'_{01}} \right| = \left| \frac{a_{00}}{a_{01}} \right| \quad (15)$$

Since from(14)  $k = \frac{-a_{00} \pm \delta}{a_{01}}$ ,

hence

$$k = \begin{cases} \frac{-a_{00} + \delta}{a_{01}} & \text{for } a_{00} > 0 \\ \frac{-a_{00} - \delta}{a_{01}} & \text{for } a_{00} < 0 \end{cases} \quad (16)$$

Determining  $k$ , we can calculate the value of  $l, k'$  and  $l'$  from equation(7),(10) and (11) respectively. The homogeneous coordinates of  $K, L, K'$  and  $L'$  are therefore known.

### 3.6 Setting $\pi$ to coincide with $\alpha$

Let  $\pi$  be put upon  $\alpha$  such that  $K'$  and  $L'$  coincide with the corresponding points  $K$  and  $L$  respectively. In this position the two planes are centrally collinear. The collineation axis is  $t$  and the collineation center is  $S_0$  as shown in fig.(5). Mathematically, this is a

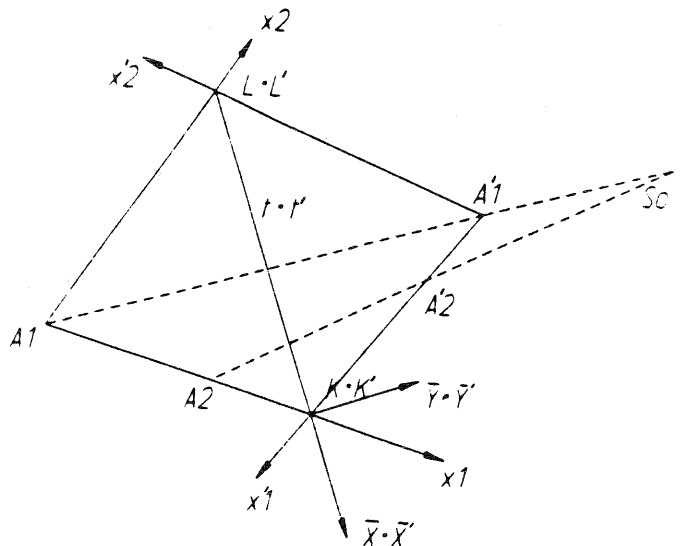


Figure 5: Coincidence of  $\pi$  with  $\alpha$

achieved by referring both planes to a common cartesian system of coordinates.

In the planes  $\alpha$  and  $\pi$ ,  $K$  and  $K'$  are chosen to be the origins of the new systems, while the axes  $\overline{X}$  and  $\overline{X}'$  are chosen along  $LK$  and  $L'K'$  respectively. The  $\overline{Y}$  and  $\overline{Y}'$  axes are perpendicular to them as shown in fig.(4a,b). The two systems will coincide with each other, when both planes coincide as shown in fig.(5). The above procedure is performed through several coordinate transformation:

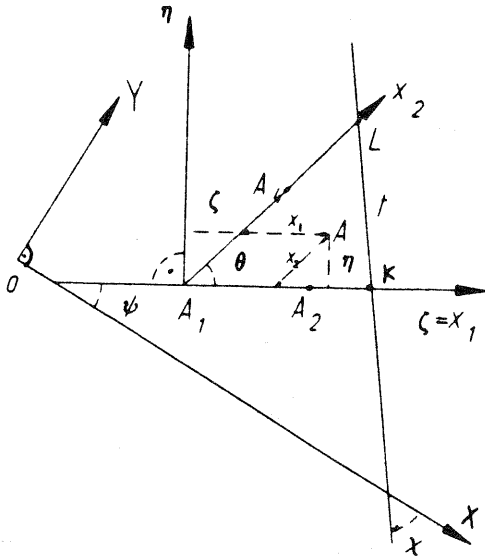


Figure 6: Relationship between Homogeneous and rectangular coordinates

1. For any finite points, let  $x_0$  &  $x'_0$  be taken = 1, then the transformation from the homogeneous coordinate  $(1 : x_1 : x_2)$  to rectangular  $\zeta, \eta$  (fig.6) is as follows:

$$\begin{aligned} \eta &= x_2 \sin \theta \\ \zeta &= x_1 + \eta \cot \theta \end{aligned} \quad (17a)$$

and conversely

$$\begin{aligned} x_2 &= \eta / \sin \theta \\ x_1 &= \zeta - \eta \cot \theta \end{aligned} \quad (17b)$$

2. From fig.(6) we get the relation between the space coordinate  $(X, Y)$  and  $(\zeta, \eta)$  as follows:

$$\begin{aligned} Y &= \eta \cos \psi + \zeta \sin \psi + Y_1 \\ X &= -\eta \sin \psi + \zeta \cos \psi + X_1 \end{aligned} \quad (18a)$$

and conversely

$$\begin{aligned} \zeta &= (Y - Y_1) \sin \psi + (X - X_1) \cos \psi \\ \eta &= (Y - Y_1) \cos \psi - (X - X_1) \sin \psi \end{aligned} \quad (18b)$$

where  $\psi = \sphericalangle(\overrightarrow{X}, \overrightarrow{A_1 A_2}) = \arctan((Y_2 - Y_1)/(X_2 - X_1))$ .

3. From equation(17) (18), the space coordinates of  $K(1 : k : 0)$  and  $L(1 : 0 : l)$  are found as follows:

$$\begin{aligned} X_K &= X_1 + k \cos \psi \\ Y_K &= Y_1 + k \sin \psi \end{aligned} \quad (19a)$$

and

$$\begin{aligned} X_L &= X_1 + l \cos(\psi + \theta) \\ Y_L &= Y_1 + l \sin(\psi + \theta) \end{aligned} \quad (19b)$$

4. Let  $\chi = \sphericalangle(\overrightarrow{X}, \overrightarrow{LK})$ , then

$$\chi = \arctan((Y_K - Y_L)/(X_K - X_L)) \quad (20)$$

5. Finally we get the relation between  $(X, Y)$  and  $(\overline{X}, \overline{Y})$ :

$$\begin{aligned} X &= X_K + \overline{X} \cos \chi - \overline{Y} \sin \chi \\ Y &= Y_K + \overline{X} \sin \chi + \overline{Y} \cos \chi \end{aligned} \quad (21a)$$

and

$$\begin{aligned} \overline{X} &= (X - X_K) \cos \chi + (Y - Y_K) \sin \chi \\ \overline{Y} &= -(X - X_K) \sin \chi + (Y - Y_K) \cos \chi \end{aligned} \quad (21b)$$

6. Similar formulas can be deduced for the image plane, except for  $\overline{Y}'$  having opposite direction(see fig.(4b)), hence:

$$\begin{aligned} \eta' &= X'_2 \sin \theta' \\ \zeta' &= X'_1 + \eta' \cot \theta' \end{aligned} \quad (22)$$

$$\begin{aligned} X'_{K'} &= X'_1 + k' \cos \psi' \\ Y'_{K'} &= Y'_1 + k' \sin \psi' \end{aligned} \quad (23)$$

$$\begin{aligned} X'_{L'} &= X'_1 + l' \cos(\psi' + \theta') \\ Y'_{L'} &= Y'_1 + l' \sin(\psi' + \theta') \end{aligned} \quad (24)$$

where

$$\psi' = \sphericalangle(\overrightarrow{X'}, \overrightarrow{A'_1 A'_2}) = \arctan\left(\frac{Y'_2 - Y'_1}{(X'_2 - X'_1)}\right),$$

$$\chi' = \sphericalangle(\overrightarrow{X'}, \overrightarrow{L' K'}) = \arctan\left(\frac{Y'_{K'} - Y'_{L'}}{(X'_{K'} - X'_{L'})}\right)$$

$$\begin{aligned} \overline{X}' &= (X' - X'_{K'}) \cos \chi' + (Y' - Y'_{K'}) \sin \chi' \\ \overline{Y}' &= (X' - X'_{K'}) \sin \chi' - (Y' - Y'_{K'}) \cos \chi' \end{aligned} \quad (25)$$

Using (21b) and (25), the coordinates of each point  $A_i$  and its image point  $A'_i$  relative to the common system  $(\overline{X} = \overline{X}', \overline{Y} = \overline{Y}')$  can be calculated.

7. The common point of intersection between the lines  $A_i A'_i$  ( $i = 1 \dots 4$ ) can now be easily found. It is the center of perspective collineation  $S_0(\overline{X}_{S_0}, \overline{Y}_{S_0})$ ; fig(7).

8. The space Coordinate of  $S_0(X_{S_0}, Y_{S_0})$  are calculated from (21a).

9. From (5) :  $V_1(1 : \frac{a'_{11}}{a'_{01}} : 0)$ , hence its coordinates in space  $X_{V_1}, Y_{V_1}$  can be found from (17a) and (18a). The equation of the line  $v$  through  $V_1$  parallel to  $t$  will be:

$$(Y - Y_{V_1}) = (X - X_{V_1}) \tan \chi. \quad (26a)$$

Similarly the equation of  $t$  will be :

$$(Y - Y_K) = (X - X_K) \tan \chi. \quad (26b)$$

10. The distance  $r = \overline{S_0 v} = |\overline{Y}_{S_0} - \overline{Y}_{V_1}|$  can now be found from (26a) to be:

$$r = |(Y_{S_0} - Y_{V_1}) \cos \chi - (X_{S_0} - X_{V_1}) \sin \chi|. \quad (27)$$

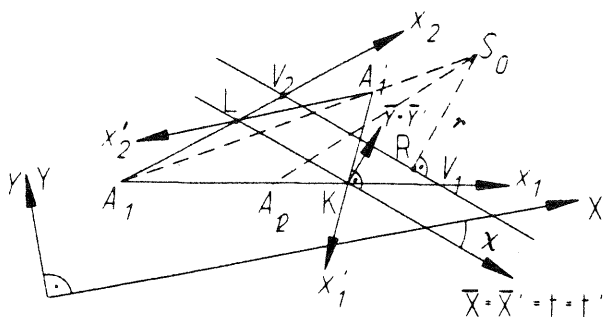


Figure 7: Space coordinates of  $S_0$

### 3.7 The Camera Station $S$

1. The equation of the perpendicular  $n$  dropped from  $S_0$  onto  $t$  is:

$$\bar{X} = \bar{X}_{S_0}$$

or in space

$$(X - X_K) \cos \chi + (Y - Y_K) \sin \chi = \bar{X}_{S_0} \quad (28)$$

Equation(28) represents also a plane  $\nu$  perpendicular to  $\alpha$  through  $S$  and  $S_0$  [1].

2. It is known that points lying on the same ray through  $S$  have a common image. Let  $T \in \alpha$  be a point whose image coincides with  $A'_5(1 : t'_1 : t'_2)$ . The points  $T \in \alpha$  and  $A_5$  in space should lie on the same ray (see fig.(1)).

The Spac coordinates of  $T(X_T, Y_T)$  can be determined using the equation (3), then (17a), and (18a).

3. The line through  $A_5(X_5, Y_5, Z_5)$  and  $T(X_T, Y_T, 0)$  have the parametric equation:

$$\left. \begin{aligned} X &= X_T + (X_T - X_5) t \\ Y &= Y_T + (Y_T - Y_5) t \\ Z &= -Z_5 t \end{aligned} \right\} \quad (29)$$

Substituting into(28), the value of the parameter  $t_s$  for the camera station  $S$  can be determined. When  $t_s$  is substituted for  $t$  into (29) the coordinates  $(X_S, Y_S, Z_S)$  can be found.

4. As a check, the distance between  $S$  and the vanishing line  $v$  can be calculated and compared to  $r$ , deduced from(27). This means that the five control points and their images are subject to a certain constraint.

### 3.8 Determining the rest of the Orientation Parameters

1. The position of  $S(X_S, Y_S, Z_S)$  and the angle  $\chi$  have already been found. The rest of the parameters depend on knowing the position of the

image plane  $\pi$  in space. It is the plane containing  $t$ , parallel to the plane through  $S$  and  $v$ .

Let the direction ratio of its normal vector  $n$  be  $(m, n, l)$ , then

$$m i + n j + l k = \left\| \begin{array}{ccc} i & j & k \\ X_{V_1} - X_S & Y_{V_1} - Y_S & -Z_S \\ X_K - X_L & Y_K - Y_L & 0 \end{array} \right\| \quad (30)$$

hence:

$$\left. \begin{aligned} m &= Z_S (Y_K - Y_L) \\ n &= -Z_S (X_K - X_L) \\ l &= \left. \begin{aligned} (X_{V_1} - X_S)(Y_K - Y_L) \\ -(Y_{V_1} - Y_S)(X_K - X_L) \end{aligned} \right\} \quad (31) \end{aligned} \right\}$$

from which the equation of  $\pi$  has the form:

$$m(X - X_K) + n(Y - Y_K) + lZ = 0 \quad (32)$$

2. The parametric equation of the normal vector  $n$  will be:

$$\left. \begin{aligned} X &= X_S + m t \\ Y &= Y_S + n t \\ Z &= Z_S + l t \end{aligned} \right\} \quad (33)$$

which, when substituted into(32) yields the value  $t_p$  for the parameter  $t$  corresponding to the central point  $P'$ , hence its coordinates will be:

$$\left. \begin{aligned} X_{P'} &= X_S + m t_p \\ Y_{P'} &= Y_S + n t_p \\ Z_{P'} &= Z_S + l t_p \end{aligned} \right\} \quad (34)$$

Finally the focal length  $f = \overline{SP'}$  and the angle  $\omega$  (between  $\pi$  and  $\alpha$ ) are found to be:

$$f = \sqrt{m^2 + n^2 + l^2} t_p$$

$$\omega = \arccos\left(\frac{l}{\sqrt{m^2 + n^2 + l^2}}\right).$$

## 4. RECONSTRUCTION OF SPACE MODELS

A space point lies at the point of intersection of the corresponding pair of rays connecting each station position to the image of that point. After determining the orientation parameters of a camera and the space coordinates of the camera stations( $S_L, S_R$ ) for both photos(left and right) (see fig.(8)), the space coordinate of the intersection point of two corresponding rays can be determined as follow:

1. Let  $A_{\alpha_r}$  be a point in  $\alpha$ , whose image is  $A'_r$  ( in right photo). Its homogeneous coordinates can be found easily by using equation(3). Then the space coordinates can be calculated as mentioned previously. Similarly, the space coordinates for point  $A_{\alpha_l}$ , whose image is  $A'_l$  (in left photo) can be calculated.

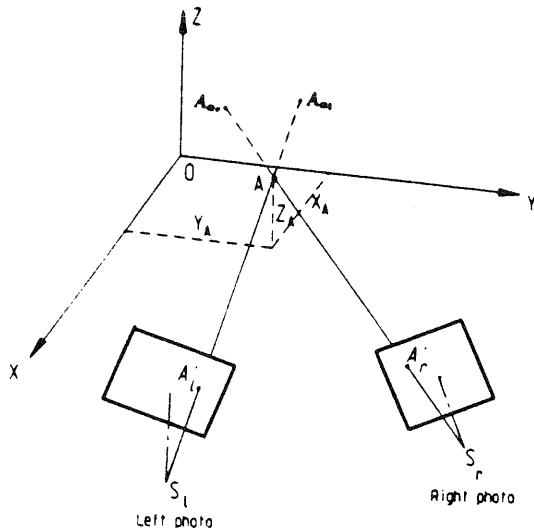


Figure 8: Space intersection with a stereopair of photos

2. The space point  $A$  can be easily found as the point of intersection of two rays  $\overline{S_l A_{\alpha_l}}$  and  $\overline{S_r A_{\alpha_r}}$ .

These steps depend only on knowing the coordinates of both  $S_l$  and  $S_r$ . Hence for the reconstruction of the space model the orientation parameters need not be all calculated.

However, due to errors in measurement, the two corresponding rays are skew in general. Therefore, the shortest distance between them is calculated and the space point  $A$  is assumed to be the midpoint of it.

## 5. COMPUTER PROGRAMS

A computer program was developed using the proposed method. The program takes also into consideration the case of giving more than *four* coplanar points and/ or more than one control points outside, using the least squares technique.

Another computer program relying on the same idea is developed to deal with the reconstruction of a space model, using two images from two different positions using the same camera, without calculating the orientation parameters.

## 6. ACCURACY

For the estimation of the accuracy of the method the Standard deviation for the coordinates of a group of check points (with known space and image coordinates) were calculated.

The test was done with a mathematical simulation of a photo of an object consisting of 30 points (6 points as control points) and the rest as check points [2].

1. The orientation parameters of the camera were regarded as known for right and left photos and

were used to calculate the image coordinates  $X, Y$  of the points.

2. A random numbers were added to each photo coordinates ( $0 - \pm 0.05mm$ ) to simulate the measuring errors and random deformation.
3. The orientation parameters were recalculated from new photo coordinates of the control position which were used to determine the space coordinates of the check points.
4. Standard deviation in  $X, Y$  and  $Z$  direction can be determined by finding the difference between the original, initially given coordinates and the corresponding calculated one [3].

The following Table shows the standard deviation  $\sigma_X, \sigma_Y, \sigma_Z$  and  $\sigma_{XYZ}$  for check points calculated by the proposed method.

St. dev.(in cm)	$\sigma_X$	$\sigma_Y$	$\sigma_Z$	$\sigma_{XYZ}$
Proposed method	0.17	0.18	0.42	0.487

## 7. CONCLUSION

The proposed method is used to determine the orientation parameters of a non-metric camera and also can be used to find the exterior parameters of a metric camera. The main advantage of this method is that the orientation parameters are determined directly without using linearization of equations and they need no complicated technique.

A practical use of this method is when photos of objects contain plane figures such as buildings, interior furniture, ... etc. Old non-metric photos can also be interpreted and photogrammetric measurements can be taken to such objects.

## References

- [1] ELSONBATY A. M.,1992. Using Perspective Projection For Estimating The Accuracy Of Measurements in Photogrammetry. MSC Thesis, Civil Eng. Dept. Assiut University.
- [2] ETHROG, U.,1984. Non-metric Camera Calibration and Photo Orientation Using Parallel and Perpendicular lines of the Photographed Object, *Photogrammetria* 39, pp. 13-22
- [3] MIKHAIL E. H., GRACIES G.,1981. Analysis and Adjustment of Survey Measurements, Litton Educational Publishing.
- [4] WILLIAMSON J. R., BRILL M. H.,1987. Three-dimensional Reconstruction From two-Point Perspective Imagery, *Photogrammetric Eng. and Remote Sensing*, 53 pp. 331-335.