THREE-DIMENSIONAL REPRESENTATION OF SPATIAL OBJECT AND TOPOLOGICAL RELATIONSHIPS

Wei GUO

National Key Lab. for Information Engineering in Surveying, Mapping and Remote Sensing
Wuhan Technical University of Surveying and Mapping
39 Luoyu Road, Wuhan 430070 P.R. CHINA
Tel: +86-27-7881292
Fax: +86-27-7884185

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ABSTRACT

Extension of GIS from 2D to 3D is a difficult but necessary step for many GIS applications. Most functions of GIS should be designed based on a data model which is a digital form to represent terrain objects of the real world. Hence it is important to investigate the concept of data modeling for 3D GIS. This paper discusses issues concerning the development of 3D GIS. such as current techniques for representation of spatial object, a topologic model for 3D GIS, the formal description of topological relationships and the implementation of 3D GIS modeling environment on the micro-computer.

1. INTRODUCTION

A Geographic Information Systems (GIS) can be used to store and handle data representing terrain. Although the world we live is three-dimension. The representation of spatial object in GIS is in general restricted to two-dimensional. These 2D data are in digital forms and can be handled, to some extent, efficiently. However, there are many problems that cannot be solved properly by using the 'traditional' 2D GIS software. First of all, successful interpretation of the hydrogeologic setting of a hazardous waste site (HES) requires data detailing 3D distribution of the natural surgical soils and manmade fill material, geologic strata and structure, and ground water conditions both locally and regionally (Fostered al. ,1987), followed by the branch of mine surveyors (Kavouras,1985; Sarkozy,1985), geological modeling (Jones,1989),urban space modeling (Shibasaki,1992; Shaobo,1992), etc. So it is necessary to develop and investigate 3D GIS systems.

In the context of 3D GIS, representation of spatial object and topological relationships play a central role both at the organization of spatial data and at the design of query methods. But the lack of a practical spatial data model and compressive theory of spatial relationships have been a major impediment to 3D GIS implementation..

This paper discusses issues concerning the concept of data modeling for 3D GIS. The remainder of this paper is organized as follows. Section 2 describes current techniques and 3D spatial representation methods. In section 3, a topological model for 3D GIS is presented. In section 4, the formal description of topological relationships has been introduced and a 3D GIS architecture ways has been given. Section 5 then concludes this paper and suggests the future works.

2. 3D SPATIAL REPRESENTATION METHOD

Current techniques for 3D GIS rely heavily on the power of computer graphics capability and stress geometry over interaction between geo-objects, but do attempt to provide solutions for creation, visualization, and analysis of complex geo-object, spatial, and entity-data relationships found in the real world (Fisher and Wales, 1991b). Geoscientific objects are geometrically complete and, thus, the abstract representation of such objects must have similar properties. Most of current 3D spatial representation methods may be categorized as either volume or surface representations (see Fig. 1).

2.1 Volume Representations

Volume representation are octrees and their variations, polytrees (polyhedral trees) and G-(gery-scale) octrees; geo-cellular models; 3D grids and isosurfaces. They represents an object by the union of a set of cells where the cell is a primitive shape which can be either regular or irregular. Cells are adjacent, connected and do not intersect. This method has two distinct advantages: one is space is uniquely defined. i.e. space cannot be occupied by two or more objects; uniqueness of space is assured and need not be verified; the other is cells are spatially indexed, i.e. each cell in the array is spatially indexed and is , therefore , addressable; this allows efficient spatial searching.

The volume representation is an approximation of the object. Because memory requirements, which increase with an increase in domain, dictate the number of cells which can be stored. For complex and large domains unlimited resolution of detail cannot be achieved.

Voxels may be thought of as 3D pixels, and because they bear similarities to 2D raster GIS applications. They are often called "raster" systems.

For raser systems, the conceptual extension from 2D to 3D is rather straightforward. The problems are mainly in the implementation. Because of the large data rates and the related storage and access problems.

2.2 Surface Representations

Surface modeling methods describe solids through various boundary representation techniques.

Constructive Solid Geometry (CSG) builds complex object using an aggregation of simpler forms (i.e., primitives: cubes, cylinders, cones, etc.). It is a commonly used technique in CAD/CAM because object creation can be achieved interactively with a simple modelling language and the Boolean and geometric operations are computationally simple. But edges and faces in a CSG model are not explicitly defined, therefore it is difficult to generate an image from the representation.

Functional representation such as nonuniform rational *b* splines (NURBS) describes all complex surfaces and provide a single uniform and precise mathematical form capable of representing the common analytical shapes, primitive quadrics, freeform curves, and surfaces necessary for gedogic modeling (Fisher, Wales, 1991). But integral properties and intersection point sets cannot be calculated in all case.

Boundary representation (BR) describes surfaces and solids through use of faces or polygons. Faces, edged, vertices and their relations are explicitly defined. Such information is essential for graphical display; effective visualization of complex objects can be achieved using descriptions in this form. However, calculating the intersection point sets is computationally expensive and reveals a number of disadvantages. Such as the validity of the representation is difficult to verify; Boolean operations are difficult to perform; The calculation of integral properties is a computationally intensive task for complex shapes; Complex algorithms are required for model generation.

Investigation into the available representation techniques has indicated that model generation, object visualization and geometric transformations, for geoscientific feature, can be best achieved using boundary representation (BR). This model grew out of simple vector representation and can used in many current modeling systems.

For vector-based systems, conceptual problems have to be solved first for the extension to 3D. Considering a truly 3D vector model, the geometry of a terrain model has three aspects: topology, shape and size, and position. These three aspects can to a large extent be dealt with independently.

- Volume representation
  - Voxels
    - Octree
      - G-Octree
      - Polytree
    - Geo-Cellular
    - 3D-Grid / Isosurface
- Surface Representation
  - Constructive Solid Geometry (CSG)
  - Boundary Representation (BP)
  - NURBS

Fig. 1: 3D spatial representation methods

3. A TOPOLOGIC MODEL FOR 3D GIS

Prof. Molenaar (1990,1992) has proposed a model to structure a 3D vector GIS which describes a topologic model for handling descriptions of 3D objects in a vector-structure. This model is based on a formal data structure (FDS) (see Fig. 2) .

The representation of terrain by means of a 3D vector model implies the definition of distinct terrain object, e.g. well, road, forest areas and hills. These terrain object can be grouped into four different Geometric Object Types.

---- Point object: zero-dimensional objects which have position but no spatial extension.

---- Line object : one-dimensional objects with length as the only measurable spatial extension, shape and position.

---- Surface object: two-dimensional object with area and perimeter as measurable spatial extensions. They can have a 3D shape.

---- Body object: three-dimensional object with volume and surface area as measurable spatial extensions. They are bordered by a surface.

In the FDS each object is represented by an Object Identifier (OID). There are four OID-set: a Point Object Set , a Line Object set , a Surface Object set and a Body Object set. Each object has its own Object Identifier, referred to respectively as PID, LID, SID or BID. Each OID is linked to two data sets. The first data set contains all thematic data. The second data set contains the geometrical information of terrain object.

In FDS , On the top row there is a thematic class label for each Object Type. The second row gives the four types of the objects of Object Identifiers. The lower part of the diagram gives the geometric structure of the FDS. There are four Geometric Elements which are the elementary data types of the geometrical part of the FDS.

The relationship between Geometric Element and Object Types is determined by the following rules:

---- A Point Object is represented by exactly one node.

---- Line Object are constructed by a chain of arcs. For each Line Object no more than two arcs can be connected at one node , so that there are no Line Objects with loops or multiple branches. A Line Object can be a closed polygon: all nodes connect exactly two arcs.

---- Surface Object are constructed by one or more connected faces.

---- A Body Object is completely described by its bordering faces.

The characteristics of topological relationships between Geometric Object are of importance in GIS data manipulation and analysis. In the FDS, nine sets of topological relationship between the four Geometric Object types can be identified. They are point-line, point-surface, point-body, line-line, line-surface, line-body, surface-surface, surface-body, and body-body. (Table 1.).

Table 1: Topological Relationships between Geometric Object

	Line	Surface	Body
Point	is-on	is-on	is-on is-inside
Line	intersect	is-on connected	is-inside is-on intersect
Surface		is-neighbor is-inside intersect	is-inside is-on intersect
Body			is-inside is-neighbor

Some topological relationships between Geometric Object listed in Table 1 can be derived easily from the basic data model. But some may be difficult. In order to complete the topological query space and to improve the efficiency of query processes, some topological data structure have built into data model.

The analysis of the topological query space reveals that the two fundamental topological relationships should be added onto the data model. They are the is-on relationship between the Node and Face entities, and the is-inside relationship between the entities of Node and Body Object. Consequently, the complete data model for 3D spatial data will be constructed.

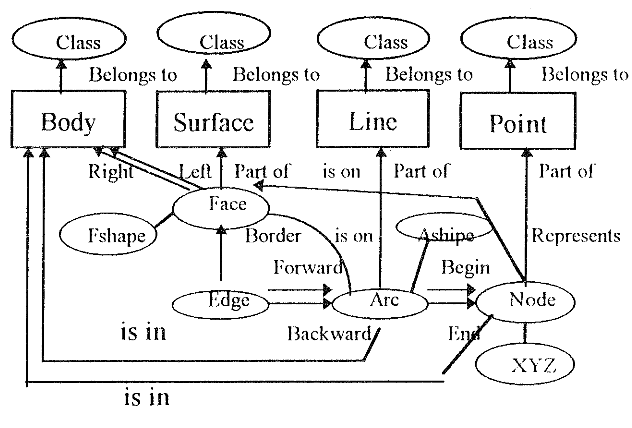

Fig. 2. A formal data structure (FDS) for 3D vector map

This representation of topological spatial relationships stress the organization of the geometric objects and some simple topological relationships. The drawback is that this set of relationships is neither orthogonal nor complete. So it is necessary to investigate the formal description and the smallest set of topological relationships between two geometric objects for 3D GIS. Twelve conventions for FDS have been formulated. Such as the arc are geometrically represented by straight line segments, but arcs are not allowed to form loop; Two arc may not intersect; Faces are planar and should not intersect, etc. These support the unambiguous mapping of the terrain situation to a FDS database.

3D FDS is an attempt in 3D modeling suited for geo-information in many aspects. However, it was not designed to also handle complex objects and interpolation processes efficiently and creating the data model from raw input data and ensuring data consistency is still a big challenge.

4. THE FRAMEWORK OF TOPOLOGICAL RELATIONSHIP AND 3D GIS

People have done many works in description of topological relationships (Guting, 1988; Pullar, 1988; Wagner,1988; Egenhofer and Franzosa, 1991). A drawback of these methods is that they distinguish only between empty or non-empty intersections of the boundaries and interiors of two geometric objects, it is not carried out in all its consequences. For example, no distinction can be made in Fig. 3. and Fig. 4. because both relationships have same description.

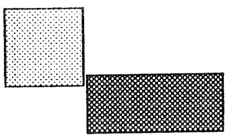

Fig. 3

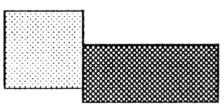

Fig. 4

Clementini et al. adopt the calculus-based method to group together the relationships into a few more general topological relationships: touch, in, cross, overlap and disjoint. This method is a good formal description of topological spatial relationships, but it is in analogy with the 2D situation.

In this paper, we take into account the dimension of the result of the intersection (dimension extended method) in 3D space, then we give the exact semantics to the five basic topological relationships and prove that five topological relationships are mutually exclusive. Finally, we give the framework of 3D GIS modeling environment.

Formal definitions of geometric objects and relationships are based on the point-set approach. The topological space is IR. The boundary and the interior of geometric objects are used in the dimension extended method for describing the topological relationships. The boundary of a Point Object to be always empty; the boundary of a Line Object is an empty set in the case of a circular line while otherwise is the set of the separate end-points; the boundary of an Surface Object is a circular line consisting of all the accumulation points of the Surface; the boundary of a Body Object is a circular area consisting of all the accumulation point of the Body. We consider a function "dim", which returns the dimension of a point-set. In case the point-set consists of multiple parts, then the highest dimension is returned. In the following definition, S is a general point-set, which may consist of several disconnected parts:

$$\dim(S)\begin{cases} \varnothing \text{ if S } \varnothing \\ 0 \text{ if S contains at least a point and no lines , areas or bodies} \\ 1 \text{ if S contains at least a line and no areas or bodies} \\ 2 \text{ if S contains at least a area and no bodies} \\ 3 \text{ if S contains at least a body} \end{cases}$$

The boundary of a Object A is denote by ∂A. The interior of a Object A is denote by A°. It is defined as $A^{\circ}=A-\partial A$. Suppose the intersections of the boundaries and interiors of the Object A and B are represented by the four sets:

$S1=\partial A\cap\partial B$; $S2=\partial A\cap B^{\circ}$; $S3= A^{\circ}\cap\partial B$; $S4= A^{\circ}\cap B^{\circ}$

In the dimension extended method we take into account the dimension of the intersection, so Si (i=1,2,3,4) can be $\varnothing$, 0, 1, . . ., dim(Si). If Si (i=1, 2, 3, 4) have Ni different cases, the topological relationships between the object A and B have $N1 \times N2 \times N3 \times N4 = N$ possible case. Apart from some impossible case, all results in a total of real cases.

Suppose the notation <A, R, B> means that the objects A and B are involved in the relationship R; this triplet is called a fact. fact can be combined through the and($\wedge$), or($\vee$) Boolean operators. For the 3D case, the definition of five relationships are the following:

Definition 1. The *touch* relationship:
$\langle A, \textit{tuoth}, B\rangle \Leftrightarrow (A^{\circ}\cap B^{\circ}=\varnothing)\wedge(A\cap B\neq\varnothing)$

Definition 2. The *in* relationship:
$\langle A, \textit{in}, B\rangle \Leftrightarrow (A^{\circ}\cap B^{\circ}\neq\varnothing)\wedge(A\cap B=A)$

Definition 3. The *cross* relationship:
$\langle A,\textit{cross},B\rangle \Leftrightarrow \dim(A^{\circ}\cap B^{\circ})<(\max(\dim(A^{\circ}),\dim(B^{\circ}))$
$\wedge(A\cap B\neq A)\wedge(A\cap B\neq B)$

Definition 4. The *overlap* relationship:
$\langle A,\textit{overlap}, B\rangle \Leftrightarrow (\dim(A^{\circ}\cap B^{\circ})=\dim(A^{\circ})=\dim(B^{\circ}))$
$\wedge(A\cap B\neq A)\wedge(A\cap B\neq B)$

Definition 5. The *disjoint* relationship:
$\langle A, \textit{disjoint}, B\rangle \Leftrightarrow A\cap B=\varnothing$

Some examples of topological spatial relationships between Line Object with Body Object can be seen in Fig. 5.

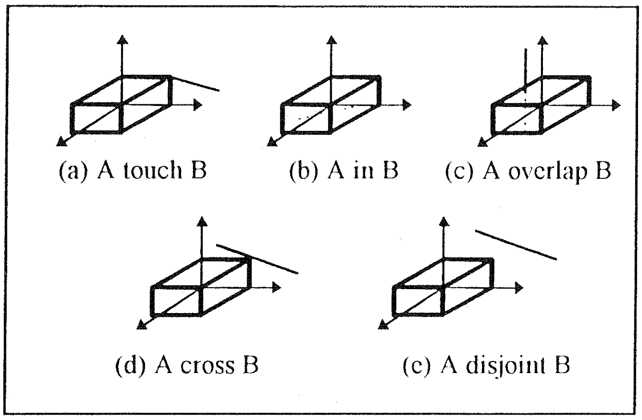

Fig. 5. Some Example

The five relationships are mutually exclusive, that is, it cannot be the case that two different relationships hold between two objects; furthermore, they make a full covering of all possible topological situations, that is, given two objects, the relationships between them must be one of the five.

Given two geometric entities A, B in 3D and a relationship R between them, if <A, *R*, B> holds, then <A, Ri, B> does not hold for every Ri≠R, and there dose not exist a topological situation that falls outside the five relationships.

Proof: "The topological relationship decision" tree (see Fig. 6.) can be constructed as follow.

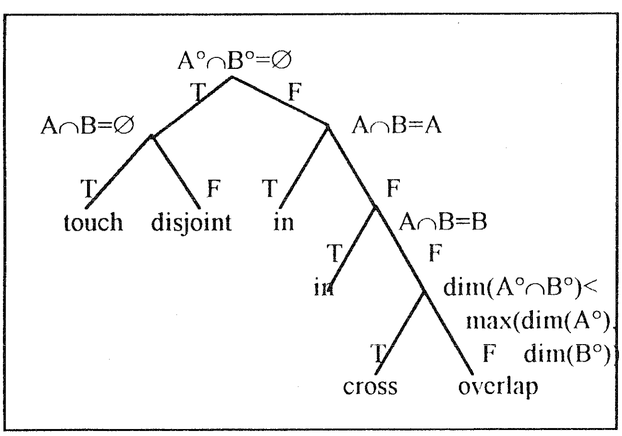

Fig. 6. The topological relationships decision tree

Every internal node in this tree represents a Boolean predicate; if for a certain topological situation, the predicate evaluates to "true" then the left branch is followed, otherwise the right branch is followed. This process is repeated until a leaf node is reached which will indicate to which of the five basic relationships this situation belongs. Now, two different relationships cannot hold between two given objects, because there is only one path to be taken in the topological relationships decision tree. Furthermore, there can be no cases outside five relationship, because every internal node has two branches, so for every value of the predicate there is an appropriate path; and every leaf node has a label that corresponds to one of the five topological relationships.

The five topological relationships are expressive enough to represent all the topological relationships of the dimension extended method. Because each case of the dimension extended method can be specified by the logical conjunction of four terms expressing conditions on the intersection of the boundaries and the interiors of the two objects, in general:

$$T1(\partial A\cap\partial B)\wedge T2(\partial A\cap B^{\circ})\wedge T3(A^{\circ}\cap\partial B)\wedge T4(A^{\circ}\cap B^{\circ}) \quad (1)$$

It is possible to give the equivalencies for every term Ti admissible in the dimension extended method. On the right of each equivalence we have a logic expression Pi making use of the five relationships between objects and between their boundaries. Each equivalence can be easily tested by applying the definitions given for the five relationships. By substituting each Ti with the corresponding Pi, we obtain an expression:

$$P1\wedge P2\wedge P3\wedge P4 \quad (2)$$

expression (2) is equivalent to expression (1). Therefore, the five topological relationships are able to express all situations of the dimension extended method.

Based on the FDS, we decided to design and implement our own 3D GIS modeling environment on the micro-computer. The system architecture consists of major main components:

(1) an relation data base management system (RDBMS);
(2) AutoCAD;
(3) an desktop mapping software ----MapInfo

In our system, Both thematic and spatial data are integrate within objects and stored in the same RDBMS. Reality as capture in the data model is translated directly to the physical RDBMS. Further, the 3D functions are implemented within the database. Visualization and manipulating the image for the interaction between the computer and end-user is implemented in AutoCAD. They are all integrated in MapInfo and can implemented geographical information processing.

With the definitions of formal topological relationships, a set of Boolean functions including *touch, in, cross, overlap and disjoint* can be implemented, returning whether two objects do or do not meet the given topological relationship. The object combinations can be

Point/Point, Point/Line, Point/Surface, Point/Body, Line/Line, Line/Surface, Line/Body, Surface/Surface, Surface/Body, Body/Body.

The advantages of this system is consistency and efficiency. It is easy to exchange data sets and to extend the analysis functions.

5. CONCLUSION AND FUTURE WORK

In this paper, The advantages and disadvantages of the abstract representation techniques about geometric objects are discussed. The formal data structure for 3D vector maps (3D FDS; Moleanner,1992) is well suited for the representation of buildings and other man-made objects which can be sampled exhaustively because of their regular shapes. This data model identified nine sets of topological relationships between the four geometric object types, but the sets is neither orthogonal nor complete. So we proposed a formal way of modeling topological relationships suitable for the definition of an actual query language towards GIS and implement the 3D GIS modeling environment on the micro-computer with the integrate of Mapinfo, AutoCAD and RDBMS.

The framework presented is considered a start and further investigations are necessary to verify its suitability. The plans for further work are:

1).extend the framework to encompass also the complex feature;

2). implement more complex analysis functions;

3). check the usefulness of query language.

4). implement spatial reasoning.

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REFERENCES

1. Egenhofer, M.J., and Herring, J. R., 1990, A Mathematical Framework for the Definition of Topological Relationships. In: *Proceedings of 4th International Symposium on Spatial Data Handing* .
2. Egenhofer, M.J., and Franzosa, D. R., 1991, Point-Set Topological Spatial Relations. In: *Int. J. GIS.*
3. Clementini, E. ,and Felice,D.P.,1993,A Small Set of Format Topological Relationships Suitable for End-User Interaction. In: *Advances in Spatial Databases.*
4. Oosterom,P.V., 1994, Integrated 3D Modelling Within GIS. In: *Advanced Geographic Data Modelling* .
5. Yi-Hsing,T. ,and Chang-jan L., 1993, Spatial Data Modeling for 3D GIS.
6. Pigot, S.,1994, A topological Model for a 3D SIS.
7. Peter R. G. Bak and Andrew J. B. Mill, 1993, Three dimensional representation in a Geoscientific Resource Management System for the minerals industry.
8. Fisher, T. R., 1993, Use of 3D Geographic Information Systems in Hazardous Waste Site investigations. *Environmental Modeling with GIS,OXFORO unversity press, pp 238-247*
9. Dieter Fritsch, 1992. Towards three-dimensional data structures in GIS.
10. Martien Molenaar, 1992. A topology for 3D vector maps. *ITC Journal, vol 1.*
11. Martien Molenaar, 1990. A formal data structure for three dimensional vector maps. *Proc 4th int symp on spatial data handling, Zurich.pp 830-843*
12. .Martien Molenaar, 1994. A query oriented implementation of a topologic data structure for 3D vector maps. *INT.J.GIS. vol 8,no 3. pp 243-260*
13. Martien Molenaar, 1993. Object Hierarchies and uncertainly in GIS or why is Standardization so difficult? *GIS. vol 4. pp 22-29*
14. Martien Molenaar, 1995. A synatx for the representation of fuzzy spatial objects. *Advanced Geographic Data Modelling, pp155-169*
15. Rongxing Li, 1994. Three dimensional GIS : A simple extension in the third dimension ?
16. Yi-Hsing Tseng, 1994. Spatial data modeling for 3D GIS.
17. Ferenc Sarkozy, 1994. The GIS concept and 3D modeling. *Comput.,Environ.,and Urban Systems. vol.18. pp 111-121*
18. Eric Carlson, 1993. 3D conceptual modeling of subsurface structures.
19. A.Keith Turner, 1992. 3D GIS for ground-water modeling.
20. Christopher B. Jones, 1989. Data structure for 3D spatial information systems in geology. *INT.J.GIS. vol 3. pp 15-31*
21. Chen, X., and K. Ikeda, 1995. A workstation for Three-dimensional spatial data research. *Proceedings of LIESMARS'95 . pp 42-51*
22. Pilouk, M., and Tempfli, K., 1995. A tetrahedron-based 3D vector data model for Geoinformation. *Advanced Geographic Data Modelling, pp129-140*