

# EQUIVALENT MAPS FOR AREAL ELEMENTS OF ENVIRONMENT ?

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## ABSTRACT

Aerial photographs and electrooptical records from satellites are analyzed from the point of view of cartographic distortions. Existing cartographic databases (maps) are submitted to polemic from the point of view of storage of environment information. It is suitable to store the areal elements of environment in conform maps ? Equivalent representation for the maps of environment is formulated and analyzed.

## 1. INTRODUCTION

Most of information of environment, which are represented in maps, are areal elements. The great part of existing maps in Czech and/or Slovak Republics is in some sort of conform cartographic representations. It is understandable that the areal elements of environment are deformed in the conform maps. This fact leads to idea to define some equivalent representation for the maps of environment.

## 2. CARTOGRAPHIC DISTORTIONS OF SATELLITE IMAGES

The pictures taken from satellites or spaceplanes, either taken

by photographic or optoelectronic apparatuses, can be considered to be a sort of representation. As any other representation also this representation shows the distortions of lengths, or areas, or angles, comparing to the reality. It is necessary to know these distortions, if information from the pictures should be transformed to cartographic databases. The distortions of the satellite pictures had been investigated from different points of view, e.g. (Gonin, 1987), (Konecny, 1976), (Paderes, 1984). The present author investigated the distortions of the satellite pictures with the aim to define the distortions in the way in which the cartographic distortions are defined (Marsik, 1983, 1988). The satellite pictures show all the three distortions: length, area, and angle distortion.

## 2.1 Differential increments of lengths and areas

Fig. 1 is helpful to understand the symbols in following formulas. The Fig. shows the basic geometric relations of one line taken by scanner in the vertical lateral plane. There  $R$  is radius of the reference sphere of the Earth,  $Z$  is the height of flight of the satellite  $S$ ,  $f$  is the focal length of the scanner. In the radial direction, it is in the direction from the nadir  $N$  to the image point  $B$ , there is the differentially small increment  $\delta \bar{s}$  corresponding to the differentially small increment of the angle  $\delta \alpha$ .

$$\delta \bar{s} = \frac{Z}{\cos^2 \alpha} \delta \alpha \quad (1)$$

For the differentially small increment of the arc  $s$  it is

$$\delta s = \frac{Z+z}{\cos \alpha} \delta \alpha (1 + \operatorname{tg}^2 \gamma \operatorname{tg}^2 \alpha + \operatorname{tg}^2 \gamma \operatorname{tg}^2 \alpha + \dots) \frac{1}{\cos \gamma} \delta \alpha \quad (2)$$

In the direction, which is perpendicular to the line  $NB$ , the increments of lines and arcs are corresponding to the differentially small angle  $\delta \beta$ . We can consider that  $\delta \beta = \delta \alpha$ . Then

$$\delta \bar{s}_\kappa = \frac{Z}{\cos \alpha} \delta \alpha \quad (3)$$

$$\delta s_\kappa = \frac{Z+z}{\cos \alpha} \delta \alpha \quad (4)$$

From the formulae (1) up to (4) it follows, that the increments

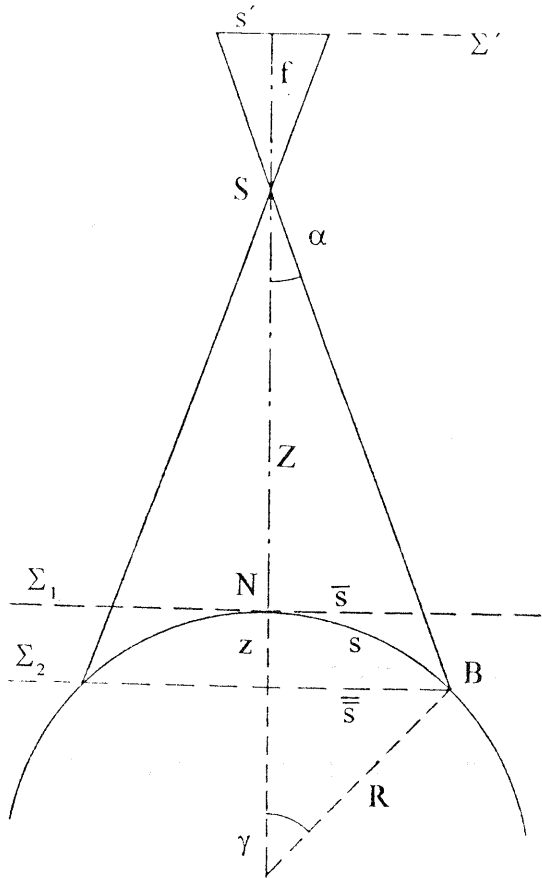


Fig. 1

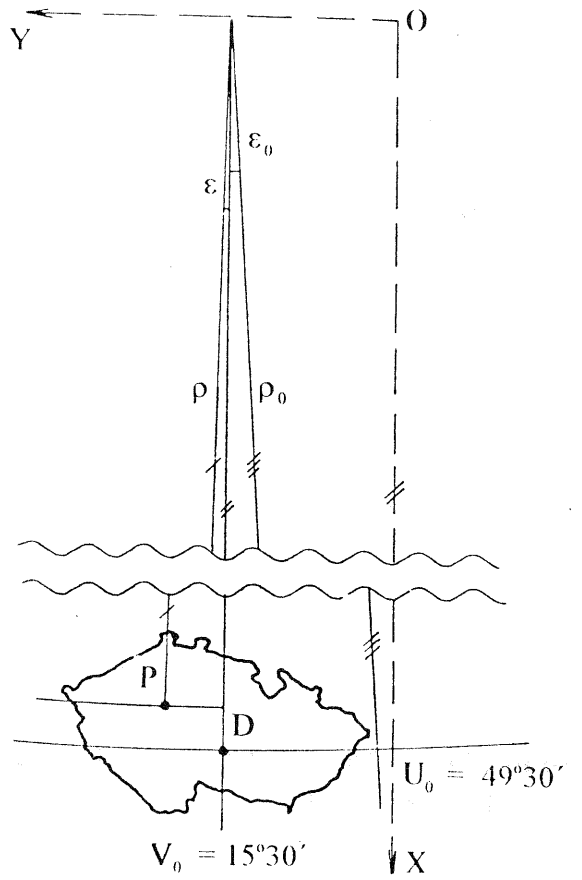


Fig. 2

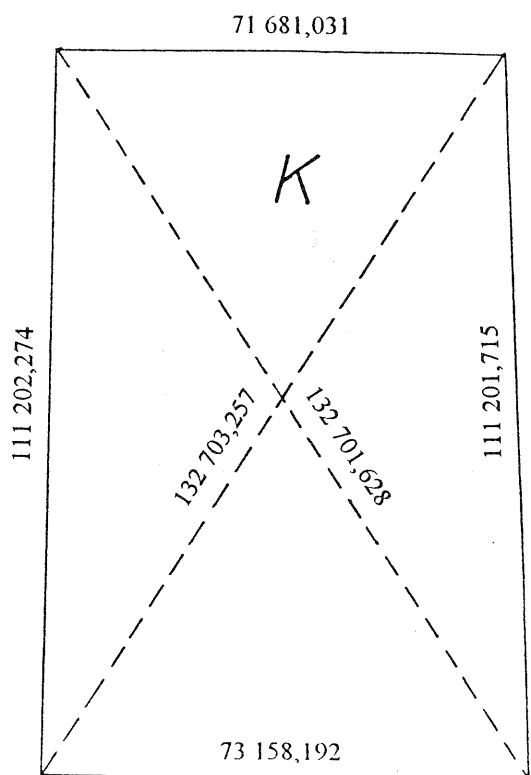
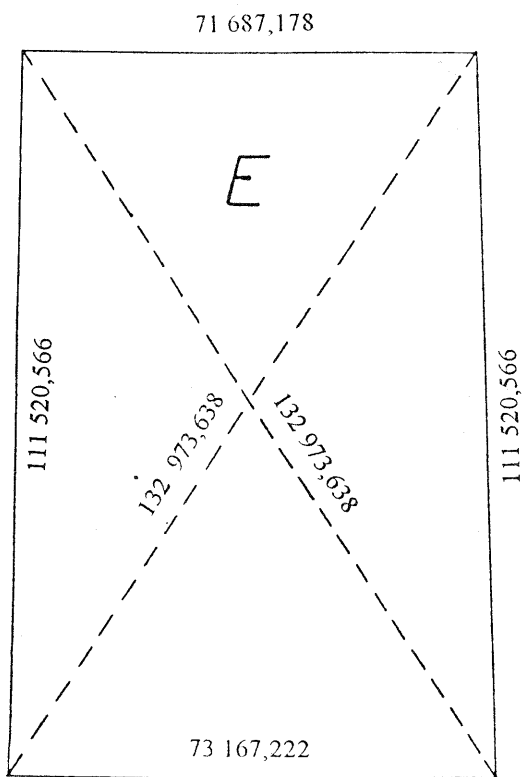


Fig. 3

will be always negative.

## 2.2 Angle distortion

Horizontal angles  $\bar{\sigma}$  could be surveyed in the tangent plane going through the point B. Corresponding angles  $\bar{\sigma}$ , which are measured in the image plane  $\Sigma'$ , are

$$\cotg \bar{\sigma} = \cotg \sigma \frac{\cos \alpha}{\cos(\alpha + \gamma)} \quad (5)$$

Then the angle distortion of the satellite images is

$$\Delta \sigma = \bar{\sigma} - \sigma \quad (6)$$

## 2.3 Cartographic distortion of lengths and areas

Applying the formulae (1) up to (4), the cartographic distortion can be defined:

length distortion in the radial direction  $m = -\frac{\delta \bar{s}}{\delta s} \quad (7)$

length distortion in the perpendicular direction  $m = -\frac{\delta \bar{s}_k}{\delta s_k} \quad (8)$

area distortion  $P = \frac{\delta \bar{s} \cdot \delta \bar{s}_k}{\delta s \cdot \delta s_k} \quad (9)$

## 3. ANALYSIS OF THE EXISTING DATABASES

The present database of the territory, both of Czech and Slovak Republic, are of two types: civil maps and military maps. The civil maps are again of two types: a) large scale maps, mostly only planimetric maps without elevation data, b) topographic maps of medium scales. The military maps are topographic maps of medium and small scales. The planimetric data in the civil medium scale maps are rather poor than in the military maps. It is due to the exaggerated secretiveness in the last decades. The two cartographic databases, civil and military, are different in many respects. First of all, the cartographic representation and coordinate systems are different. The system of the civil maps (abbreviation

S-JTSK) is based on the oblique conic representation (Křovák 1922) The system of military maps (S-42) is based on the transversal cylindrical (Gauss-Krüger) representation. But one thing is common to the civil and military maps: they are based on conform cartographic representation. Several generations of geodesists believed that the conform representation is the best, particularly for the large scale maps. There are many arguments for this statement. At least one should be mentioned here. Already some century ago the geodesists could measure the angles very precisely, with error even less than one second(1"). On the other side, only few years ago they have such devices, which are suitable to measure lengths with errors about 0.1m or less. It has seemed to be logical that the angle elements, which have been measured with high precision are represented in the map without deformation. On the contrary, the length elements, which have been measured with lower precision, cannot be too degraded even if the length distortion exists in the map.

#### **4. SUGGESTION OF CARTOGRAPHIC REPRESENTATION**

##### **4.1 Characteristics of environment data**

We can consider that the environment is everything in our surroundings, on the Earth and in the atmosphere. The graphical representation of surroundings in maps is very natural, very understandable, and it has a long time historical tradition. Every literate man can read the map, he understands it, and the map is for him the main source of information about environment, about the close surroundings and the far regions. The great part of environment elements, for example woods, agriculture vegetations, water areas, dwelling places, industrial complexes. Those elements are usually the decisive elements for the character of the region, for the character of environment. Now, it seems to be logical to represent these elements in maps without deformation. This leads to an idea of equivalent representation for the maps of environment.

##### **4.2 Theoretical principles of representation**

The shape of our state territory offers the conic representation as the most suitable one. The normal conic representation maintains the rectangularity of the meridians and the parallels also for equivalent representation. The equations of normal conic equivalent representation are given in (Hojovec, 1987).

$$\varphi_0 = R \cdot \cotg U_0 \quad (10)$$

$$\rho^2 = \rho_0^2 + \frac{2R^2}{n} (\sin U_0 - \sin U) \quad (11)$$

$$\epsilon = n \cdot V \quad n = \sin U_0 \quad (12)$$

In the formulae there is: R is the radius of the reference sphere, U, V are geographic latitude and longitude for the Earthsphere.

From the point of view of the mathematical cartography, it might be possible to consider as the best, as the "purest", such a conic representation, where the cone is the tangent cone of the spheroid directly. For the adequate tangent parallel  $\varphi_0 = U_0$  the radius  $R = N_0$ . In a narrow strip along the tangent parallel there can be considered  $(U, V) = (\varphi, \lambda)$ .

The Bessel's ellipsoid, as well as in S-JTSK, has been applied, and  $49^\circ 30'$  has been chosen as the tangent parallel. The origin of the x, y coordinate system has been put to the peak of the cone, x coordinate aiming south in the  $15^\circ$  meridian and y coordinate aiming west (see Fig. 2).

For the tangent parallel  $\varphi_0 = 49^\circ 30'$  the radius R is  $R = N_0 = 6\,389\,738,833$  m, and according the formula (10) it is  $\rho_0 = 5\,457\,357,520$  m. The rectangular coordinate system x, y is defined by formulae

$$x = \rho \cdot \cos \epsilon \quad y = \rho \cdot \sin \epsilon + 500\,000,00 \text{ m} \quad (13)$$

Applying this coordinate system, the area was calculated for the spherical trapezoid defined by  $15^\circ$  and  $16^\circ$  meridians and  $49^\circ$  and  $50^\circ$  parallels. The area is  $P_e = 8\,076\,944\,529 \text{ m}^2$ . The area of the same trapezoid, applying the S-JTSK system, is  $P_k = 8\,053\,027\,614 \text{ m}^2$ . Scheme of the trapezoids is shown in Fig. 3. There E is the trapezoid in the equivalent representation, and K is the trapezoid in the state system S-JTSK. From the distances shown in Fig. 3,

one can see that the compared figure (spherical trapezoid) appears as the plane trapezoid in the equivalent representation (E). On the contrary, the same figure appears as an irregular quadrilateral in the S-JTSK representation (K). Comparing the equivalent representation and the conform S-JTSK system, one can see that each  $1 \text{ km}^2$  is for  $2\,826 \text{ m}^2$  smaller in S-JTSK maps.

The present author is the geodesist and photogrammetrist, and he has left the decision, if it is too much or if it is negligible, to the experts of environment.

## 5. CONCLUSION

The aerial photographs and the satellite records are certainly the main source of information for the environment maps. As shown in the paragraph 2., those records are deformed comparing to the reality in all the three respects: lengths, areas and also angles are deformed. Since the length deformation of the satellite records is different in the satellite path direction than in the perpendicular direction (Marsik, 1988), the areal elements in those records show affinity deformation comparing to the reality. It is difficult to transform the satellite imageries to the conformal maps, however, it is much suitable to transform those imageries to the equivalent maps. The affinity transformation formulae can be applied in the last case.

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